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Mathematics and Decision Sciences

Highlights

- Keller Segel Model
- Variable Thermal Conductivity
- Complexity of Trajectorries
- Estimators of Finite Population

Discovering Thoughts, Inventing Future

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Contents of the Volume

i. Copyright Notice
ii. Editorial Board Members
iii. Chief Author and Dean
iv. Table of Contents
v. From the Chief Editor's Desk
vi. Research and Review Papers

1. On some Properties of the Rising Sun Function. *1-12*
2. Some Statistical Properties of Exponentiated Weighted Weibull Distribution. *13-22*
3. A Summation Formula Involving Certain Special Functions. *23-35*
4. Global Existence and Uniqueness of the Weak Solution in Keller Segel Model. *37-45*
5. Effect of Variable Thermal Conductivity & Heat Source/Sink Near a Stagnation Point on a Linearly Stretching Sheet using HPM. *47-53*
6. Homeotopy Groups of 2-Dimensional Manifolds with One Boundary Component. *55-57*
7. Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling. *59-71*

vii. Auxiliary Memberships
viii. Process of Submission of Research Paper
ix. Preferred Author Guidelines
x. Index

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On Some Properties of the Rising Sun Function

By Vajha Srinivasa Kumar

Abstract- This paper studies a few interesting properties of the rising sun function of a bounded real function defined on a closed and bounded interval on the real line. An operator on the space of all bounded real functions defined on a closed and bounded interval is introduced and its properties are investigated.

Keywords: rising sun function, semi-continuity, darboux continuity, lower (upper) semicontinuity, lower (upper) semi-quasicontinuity, symmetric continuity, cliquishness, quasicontinuity, differentiability.

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On Some Properties of the Rising Sun Function

Vajha Srinivasa Kumar

Abstract- This paper studies a few interesting properties of the rising sun function of a bounded real function defined on a closed and bounded interval on the real line. An operator on the space of all bounded real functions defined on a closed and bounded interval is introduced and its properties are investigated.

Keywords : rising sun function, semi-continuity, darboux continuity, lower (upper) semicontinuity, lower (upper) semi-quasicontinuity, symmetric continuity, cliquishness, quasicontinuity, differentiability.

I. Introduction

The rising sun function was used as a tool in the proof of the famous Lebesgue's theorem on the differentiability of a real valued monotone function without using the theory of integration [3]. In this paper, some properties of the rising sun function are presented and an operator on the space of all bounded real functions defined on a closed and bounded interval $[a,b]$ is introduced and its properties are investigated.

In what follows X, $\mathbb{R}$ and $\mathbb{N}$ stand for a topological space, the real line and the set of all positive integers respectively. Also $\mathscr{B}$ stands for the Banach space of all bounded real functions defined on a closed and bounded interval $[a,b]$ where $a,b \in \mathbb{R}$ and $a<b$ under the supremum norm.

II. Preliminaries

1.1 Definition [6]: The rising sun function of a function $f \in \mathscr{B}$ is defined by

$$f_{\odot}(x) = \sup\{f(y) / x \le y \le b\}.$$

1.2 Definition: For $f \in \mathscr{B}$ we define the following.

$$(i) \quad {}_{\odot}f(x) = \sup\{f(y) / a \le y \le x\}$$

$$(ii) \quad f^{\odot}(x) = \inf\{f(y) / x \le y \le b\}$$

$$(iii) \quad {}^{\odot}f(x) = \inf\{f(y) / a \le y \le x\}$$

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Ref

3. **Riesz, F. And B.Sz. Nagy,** *Functional Analysis,* Frederick Ungar Publishing Co., New York, 1995.

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1.3 Definition [5]: A function $f : X \to \mathbb{R}$ is said to be semi-continuous at a point $p \in X$ if for every $\varepsilon > 0$ and every neighborhood U of p in X there exists a non-empty open set $W \subset U$ such that $|f(x) - f(p)| < \varepsilon \ \forall x \in W$. We say that a function f is semi-continuous on X if it is semi-continuous at every point of X.

1.4 Definition [6]: A function $f : X \to \mathbb{R}$ is said to be lower semicontinuous (lsc) at a point $x \in X$ if for every $\varepsilon > 0$ there exists a neighborhood U of x such that

$$f(y) - f(x) > -\varepsilon \ \forall \ y \in U.$$

We say that a function f is lsc on X if it is lsc at every point of X.

1.5 Definition [6]: A function $f : X \to \mathbb{R}$ is said to be upper semicontinuous (usc) at a point $x \in X$ if for every $\varepsilon > 0$ there exists a neighborhood G of x such that

$$f(y) - f(x) < \varepsilon \ \forall \ y \in G.$$

We say that a function f is usc on X if it is usc at every point of X.

1.6 Definition [4]: A function $f : X \to \mathbb{R}$ is said to be lower semi-quasicontinuous $(lsqc)$ at a point $x \in X$ if for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $f(y) - f(x) > -\varepsilon \ \forall \ y \in W$.

We say that a function f is $lsqc$ on X if it is $lsqc$ at every point of X.

1.7 Definition [4]: A function $f : X \to \mathbb{R}$ is said to be upper semi-quasicontinuous $(usqc)$ at a point $x \in X$ if for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $f(y) - f(x) < \varepsilon \ \forall \ y \in W$.

We say that a function f is $usqc$ on X if it is $usqc$ at every point of X.

1.8 Definition [5]: A function $f : X \to \mathbb{R}$ is said to be cliquish at a point $x \in X$ if for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $|f(y) - f(z)| < \varepsilon \ \forall \ y, z \in W$

We say that a function f is cliquish on X if it is cliquish at every point of X.

1.9 Definition: Let $f : [a,b] \to \mathbb{R}$. We define $f(a-) = f(a)$ and $f(b+) = f(b)$.

We say that $f(p+)$ exists at $p \in [a,b)$ and we write $f(p+) = L$, where $L \in \mathbb{R}$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon \ \forall \ x \in (p, p+\delta) \subset [a,b]$. Similarly for $p \in (a,b]$ we write

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$f(p-)=l\in\mathbb{R}$ if for every $\varepsilon>0$ there exists a $\delta>0$ such that

$$|f(x)-l|<\varepsilon \quad \forall\, x\in(p-\delta,p)\subset[a,b].$$

1.10 Definition: A function $f:[a,b]\to\mathbb{R}$ is said to be quasicontinuous at a point $p\in[a,b]$ if $f(p+)$ and $f(p-)$ exist.

1.11 Definition: A function $f:[a,b]\to\mathbb{R}$ is said to be symmetrically continuous at a point $x\in[a,b]$ if $\lim\limits_{h\to 0}\left[f(x+h)-f(x-h)\right]=0$.

1.12 Definition: A function $f:[a,b]\to\mathbb{R}$ is Darboux continuous if for all $p,q\in[a,b]$ and for each c between $f(p)$ and $f(q)$ there is an x between p and q such that $f(x)=c$.

1.13 Definition[2]: An operator P on a linear space L is said to be sublinear if (i) $P(x+y)\le P(x)+p(y)$ $\forall$ $x,y\in$ L and (ii) $P(\lambda x)=\lambda P(x)$ for any positive real number λ and every $x\in$ L.

III. Relations among $f_{\odot}, {}_{\odot}f, f^{\odot}$ and ${}^{\odot}f$

In this section the relations between the rising sun function and its analogues that are introduced are presented in the following propositions.

2.1 Proposition: For $f\in\mathcal{B}$, (a) $f^{\odot}=-(-f)_{\odot}$ and (b) ${}^{\odot}f=-{}_{\odot}(-f)$.

2.2 Proposition: For $f\in\mathcal{B}$, (a) $(f_{\odot})_{\odot}=f_{\odot}$ $\quad(b)$ ${}_{\odot}({}_{\odot}f)={}_{\odot}f$

(c) $\left(f^{\odot}\right)^{\odot}=f^{\odot}$ $\quad(d)$ ${}^{\odot}\left({}^{\odot}f\right)={}^{\odot}f$

2.3 Proposition: For $f\in\mathcal{B}$ and $x\in[a,b]$,

(i) ${}_{\odot}\left(f_{\odot}\right)(x)=f_{\odot}(a)$ $\quad(ii)$ $\left(f_{\odot}\right)^{\odot}(x)=f_{\odot}(b)$ $\quad(iii)$ ${}^{\odot}\left(f_{\odot}\right)(x)=f_{\odot}(x)$

(iv) $\left({}_{\odot}f\right)_{\odot}(x)={}_{\odot}f(b)$ $\quad(v)$ $\left({}_{\odot}f\right)^{\odot}(x)={}_{\odot}f(x)$ $\quad(vi)$ ${}^{\odot}\left({}_{\odot}f\right)(x)={}_{\odot}f(a)$

(vii) $\left(f^{\odot}\right)_{\odot}(x)=f^{\odot}(b)$ $\quad(viii)$ ${}_{\odot}\left(f^{\odot}\right)(x)=f^{\odot}(x)$ $\quad(ix)$ ${}^{\odot}\left(f^{\odot}\right)(x)=f^{\odot}(a)$

(x) $\left({}^{\odot}f\right)_{\odot}(x)={}^{\odot}f(x)$ $\quad(xi)$ ${}_{\odot}\left({}^{\odot}f\right)(x)={}^{\odot}f(a)$ $\quad(xii)$ $\left({}^{\odot}f\right)^{\odot}(x)={}^{\odot}f(b)$

2.4 Remark: In view of the previous propositions it is enough to investigate the properties of the rising sun function and the properties of ${}_{\odot}f$, $f^{\odot}$ and ${}^{\odot}f$ follow analogously.

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IV. Characterisations of $f_{\odot}, {}_{\odot}f, f^{\odot}$ and ${}^{\odot}f$

3.1 proposition: For $f \in \mathcal{B}$, $f_{\odot}$ is the smallest decreasing function dominating f

More precisely,

(a) $f_{\odot}(x) \geq f(x)$ $\forall\, x \in [a,b]$

(b) $f_{\odot}$ is decreasing on $[a,b]$

(c) If g satisfies (a) and (b) above, then $f_{\odot}(x) \leq g(x)$ $\forall\, x \in [a,b]$.

3.2 Proposition: For $f \in \mathcal{B}$, ${}_{\odot}f$ is the smallest increasing function dominating f. More precisely, (a) ${}_{\odot}f(x) \geq f(x)$ $\forall\, x \in [a,b]$

(b) ${}_{\odot}f$ is increasing on $[a,b]$

(c) If g satisfies (a) and (b) above, then ${}_{\odot}f(x) \leq g(x)$ $\forall\, x \in [a,b]$.

3.3 Proposition: For $f \in \mathcal{B}$,

(a) $f^{\odot}$ is the largest increasing function such that $f^{\odot}(x) \leq f(x)$ $\forall\, x \in [a,b]$

(b) ${}^{\odot}f$ is the largest decreasing function such that ${}^{\odot}f(x) \leq f(x)$ $\forall\, x \in [a,b]$

V. The Rising Sun Operator

4.1 Definition: Define $T : \mathcal{B} \to \mathcal{B}$ by $Tf = f_{\odot}$. We call this operator T, the rising sun operator on $\mathcal{B}$.

4.2 Proposition: The rising sun operator T is sublinear on $\mathcal{B}$. More precisely,

(a) $T(f+g) \leq Tf + Tg$ $\forall$ $f, g \in \mathcal{B}$

(b) $T(\lambda f) = \lambda Tf$ for every real number $\lambda > 0$ and every $f \in \mathcal{B}$.

Proof: Let $f, g \in \mathcal{B}$ and $x \in [a,b]$.

(a) For $y \in [x,b]$, $(f+g)(y) = f(y) + g(y) \leq f_{\odot}(x) + g_{\odot}(x) = (f_{\odot} + g_{\odot})(x)$

$$\Rightarrow (f+g)(y) \leq (f_{\odot} + g_{\odot})(x) \;\; \forall \;\; y \in [x,b]$$
$$\Rightarrow (f+g)_{\odot}(x) \leq (f_{\odot} + g_{\odot})(x) \;\; \forall \;\; x \in [a,b]$$

Hence $T(f+g) \leq Tf + Tg$ $\forall$ $f, g \in \mathcal{B}$.

(b) Suppose that λ is a positive real number and $f \in \mathcal{B}$.

Then
$$(\lambda f)_{\odot}(x) = \sup\{(\lambda f)(y) / x \leq y \leq b\}$$
$$= \lambda \sup\{f(y) / x \leq y \leq b\}$$
$$= \lambda\, f_{\odot}(x)$$
$$T\;\lambda f \;= \lambda Tf.$$

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4.3 Remark: From the following example, it is clear that $T(f+g) \neq Tf + Tg$.

4.4 Example: Define $f:[0,1] \to \mathbb{R}$ and $g:[0,1] \to \mathbb{R}$ by $f(x) = x$ and

$$g(x) = 1 - x \ \ \forall\, x \in [0,1] \quad \Rightarrow (f+g)(x) = 1. \forall\, x \in [0,1]$$

Then $\quad (f+g)_{\odot}(x) = 1$ and $(f_{\odot} + g_{\odot})(x) = 2 - x \ \ \forall\, x \in [0,1]$.

Hence $\quad T(f+g) \neq Tf + Tg$.

4.5 Proposition: For $f \in \mathscr{B}$ and $k \in \mathbb{R}$, $T(f+k) = Tf + k$.

4.6 Proposition: For $f, g \in \mathscr{B}$, (a) $f \leq g \Rightarrow Tf \leq Tg$ $\quad (b)$ $T(f \vee g) = Tf \vee tg$

where $(f \vee g)(x) = \max\{f(x), g(x)\}$.

4.7 Proposition: If $\{f_\alpha / \alpha \in \Delta\}$ is a collection of functions in $\mathscr{B}$ and if

$\bigvee_{\alpha \in \Delta} f_\alpha = \sup\{f_\alpha / \alpha \in \Delta\}$ exists in $\mathscr{B}$ then $T\left(\bigvee_{\alpha \in \Delta} f_\alpha\right) = \bigvee_{\alpha \in \Delta} T(f_\alpha)$.

4.8 Remark: From the following example it can be observed that

$T(f \wedge g) \neq Tf \wedge Tg$, where $(f \wedge g)(x) = \min\{f(x), g(x)\}$.

4.9 Example: Define $f:[0,1] \to \mathbb{R}$ and $g:[0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1 - 2x & if \ \ 0 \leq x \leq \frac{1}{2} \\ 2x - 1 & if \ \ \frac{1}{2} \leq x \leq 1 \end{cases} \qquad \text{and} \qquad g(x) = \begin{cases} 2x & if \ \ 0 \leq x \leq \frac{1}{2} \\ -2x + 2 & if \ \ \frac{1}{2} \leq x \leq 1 \end{cases}$$

Then $\quad f_{\odot}(x) = 1 \ \forall\, x \in [0,1]$ and $g_{\odot}(x) = \begin{cases} 1 & if \ \ 0 \leq x \leq \frac{1}{2} \\ -2x + 2 & if \ \ \frac{1}{2} \leq x \leq 1 \end{cases}$

$$\Rightarrow (f_{\odot} \wedge g_{\odot})(x) = g_{\odot}(x) \ \ \forall\, x \in [0,1]$$

Also $\quad (f \wedge g)(x) = \begin{cases} 2x & if \ \ 0 \leq x \leq \frac{1}{4} \\ -2x + 1 & if \ \ \frac{1}{4} \leq x \leq \frac{1}{4} \\ 2x - 1 & if \ \ \frac{1}{2} \leq x \leq \frac{3}{4} \\ -2x + 2 & if \ \ \frac{3}{4} \leq x \leq 1 \end{cases}$

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$$\Rightarrow (f \wedge g)_{\odot}(x) = \begin{cases} \frac{1}{2} & if \quad 0 \le x \le \frac{3}{4} \\ -2x+2 & if \quad \frac{3}{4} \le x \le 1 \end{cases}$$

Hence $\quad (f \wedge g)_{\odot}(x) \ne (f_{\odot} \wedge g_{\odot})(x)$.

4.10 Proposition: T is continuous on $\mathscr{B}$.

Proof: Let $f_n \in \mathscr{B}$ for $n \in \mathbb{N}$ and $f_n \to f$ uniformly on $[a,b]$. Then $f \in \mathscr{B}$.

Let $\varepsilon > 0$ be given. Then there exists a positive integer N such that

$n \ge N \Rightarrow |f_n(x) - f(x)| < \varepsilon \ \forall \ x \in [a,b]$

$\Rightarrow -\varepsilon < f_n(x) - f(x) < \varepsilon \ \ \forall \ x \in [a,b]$

$\Rightarrow f(x) - \varepsilon < f_n(x) < f(x) + \varepsilon \ \ \forall \ x \in [a,b]$.

Let $x \in [a,b]$ and choose $y \in [x,b]$. Then $y \in [a,b]$.

$\Rightarrow f(y) - \varepsilon < f_n(y) < f(y) + \varepsilon \ \ \forall \ n \ge N$

$n \ge N \Rightarrow f_n(y) < f(y) + \varepsilon \le f_{\odot}(x) + \varepsilon$

$\Rightarrow f_n(y) < f_{\odot}(x) + \varepsilon \ \forall \ y \in [x,b]$

$\Rightarrow (f_n)_{\odot}(x) \le f_{\odot}(x) + \varepsilon$.

Similarly $f_{\odot}(x) - \varepsilon < (f_n)_{\odot}(x)$.

Hence $|(f_n)_{\odot}(x) - f_{\odot}(x)| \le \varepsilon \ \ \forall \ n \ge N$ and for every $x \in [a,b]$.

Hence $(f_n)_{\odot} \to f_{\odot}$ uniformly on $[a,b]$.

Hence $f_n \to f$ in $\mathscr{B} \Rightarrow Tf_n \to Tf$ in $\mathscr{B}$.

$\Rightarrow$ T is continuous on $\mathscr{B}$.

4.11Proposition: (a) $T^n f = f_{\odot} \ \ \forall \ f \in \mathscr{B}$ and for every $n \in \mathbb{N}$

(b) For $f \in \mathscr{B}$, the cycle of T is the set $\{f, f_{\odot}\}$.

4.12 Proposition: If $f \in \mathscr{B}$ and f is monotonically decreasing then f is a fixed point of T.

4.13 Remark: The set of all fixed points of T is the set of all monotonically decreasing functions on $[a,b]$. Let F denote the set of all fixed points of T

Then $F = \{f \in \mathscr{B} / Tf = f\} = \{f \in \mathscr{B} / f \text{ is decreasing}\}$.

Notes

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Notes

4.14 Proposition: The set F of all fixed points of the rising sun operator T is closed in $\mathscr{B}$.

4.15 Proposition: Fix $f \in F$. Let $F^* = \{f \in \mathscr{B} / Tf = f\}$.

Then (a) F^* is closed in $\mathscr{B}$.

(b) $(F^*, \leq)$ is a $\vee-$semilattice under the relation $\leq$ defined on F^* by

$$f \leq g \iff f(x) \leq g(x) \ \forall\, x \in [a,b]$$

4.16 Proposition: The operator T is bounded. More precisely,

$$\text{(i)} \ \|Tf\| \leq \|f\| \ \forall \ f \in \mathscr{B} \qquad \text{and} \qquad \text{(ii)} \ \|T\| = 1$$

VI. Invariant Properties

5.1 Proposition: Let $f \in \mathscr{B}$. If f is *usc* at a point $x \in [a,b]$ then so is $Tf = f_{\odot}$.

Proof: Let $\varepsilon > 0$ be given and $x \in [a,b]$. Since f is *usc* at $x \in [a,b]$, there exists a $\delta > 0$ such that $f(t) - f(x) < \varepsilon \ \forall\, t \in (x-\delta, x+\delta) \bigcap [a,b] = U$.

$\Rightarrow f_{\odot}(x) + \varepsilon \geq f(x) + \varepsilon > f(t) \ \forall\, t \in U$

$\Rightarrow f_{\odot}(x) + \varepsilon > f(t) \ \forall\, t \in U$

Suppose that $x < y$.

$$\Rightarrow f_{\odot}(x) + \varepsilon > f_{\odot}(x) \geq f_{\odot}(y) \geq f(y)$$

$$\Rightarrow f_{\odot}(x) + \varepsilon > f(y).$$

If $x < t$ and $y \in [t,b] \Rightarrow x < y$

$$\Rightarrow f_{\odot}(x) + \varepsilon > f(y)$$
$$\Rightarrow f_{\odot}(x) + \varepsilon \geq f_{\odot}(t)$$

Suppose that $t \leq x$. Then $y \in [t,b] \Rightarrow t \leq y \leq x$ or $x < y < b$.

If $t \leq y \leq x$ then $y \in U \ \Rightarrow f_{\odot}(x) + \varepsilon > f(y) \ \Rightarrow f_{\odot}(x) + \varepsilon > f_{\odot}(t)$.

If $x < y \leq b$ then $f_{\odot}(x) + \varepsilon > f_{\odot}(t)$. Hence $f_{\odot}(x) + \varepsilon > f_{\odot}(t) \ \forall\, t \in U$.

$$\Rightarrow f_{\odot} \text{ is } usc \text{ at } x.$$

5.2 Proposition: Let $f \in \mathscr{B}$. If f is *lsc* at a point $x \in [a,b]$ then so is $Tf = f_{\odot}$.

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Proof: Let $\varepsilon > 0$ be given and $x \in [a,b]$. Since f is lsc at $x \in [a,b]$, there exists a $\delta_1 > 0$ such that $f(t) - f(x) > \frac{-\varepsilon}{2}$ $\forall\, t \in (x-\delta_1, x+\delta_1) \bigcap [a,b] = U$

$$\Rightarrow f_{\odot}(t) + \frac{\varepsilon}{2} \geq f(t) + \frac{\varepsilon}{2} > f(x) \;\; \forall\, t \in U$$

$$\Rightarrow f_{\odot}(t) + \frac{\varepsilon}{2} > f(x) \;\; \forall\, t \in U$$

Since $f_{\odot}(x) - \frac{\varepsilon}{2}$ is not an upper bound of the set $\{f(y) / x \leq y \leq b\}$, there exists a point $y \in [x,b]$ such that $f(y) > f_{\odot}(x) - \frac{\varepsilon}{2}$.

If $y = x$ then $f_{\odot}(t) + \frac{\varepsilon}{2} > f(x) > f_{\odot}(x) - \frac{\varepsilon}{2}$ $\forall\, t \in U$

$$\Rightarrow f_{\odot}(t) + \varepsilon > f_{\odot}(x) \;\; \forall\, t \in U.$$

$y \neq x$. If $x < z < y$ then $f_{\odot}(z) \geq f_{\odot}(y) \geq f(y) > f_{\odot}(x) - \frac{\varepsilon}{2}$

$$\Rightarrow f_{\odot}(z) > f_{\odot}(x) - \frac{\varepsilon}{2} > f_{\odot}(x) - \varepsilon.$$

If $z \leq x$ then $f_{\odot}(z) \geq f_{\odot}(x) > f_{\odot}(x) - \varepsilon$.

Choose $\delta > 0$ such that $a \leq x - \delta < x < x + \delta \leq y$.

Then $f_{\odot}(z) > f_{\odot}(x) - \varepsilon$ $\forall\, z \in (x-\delta, x+\delta) \bigcap [a,b]$. Hence $f_{\odot}$ is lsc at x.

5.3 Corollary: Let $f \in \mathcal{B}$. If f is continuous at a point $x \in [a,b]$, then $Tf = f_{\odot}$ is continuous at x.

5.4 Proposition: For any $f \in \mathcal{B}$, $Tf = f_{\odot}$ is $lsqc$ at every $x \in (a,b]$.

Proof: Let $f \in \mathcal{B}$ and $x \in (a,b]$. Let $\varepsilon > 0$ be given and let U be a neighborhood of x in $[a,b]$. Then there exists a $\delta > 0$ such that $(x-\delta, x+\delta) \bigcap [a,b] \subset U$. Choose $W = (x-\delta, x) \bigcap [a,b]$.

W is a non-empty open subset of U.

$$y \in W \Rightarrow x - \delta < y < x$$

$$\Rightarrow f_{\odot}(y) \geq f_{\odot}(x)$$

$$\Rightarrow f_{\odot}(y) + \varepsilon > f_{\odot}(y) \geq f_{\odot}(x)$$

Notes

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Notes

Hence $f_{\odot}(y)+\varepsilon > f_{\odot}(x)\ \forall\ y \in W$.

Thus for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $f_{\odot}(y)+\varepsilon > f_{\odot}(x)\ \forall\ y \in W$.

5.5 Proposition: Let $f \in \mathcal{B}$. If $f:[a,b] \to \mathbb{R}$ is *lsqc* at the point a then $Tf = f_{\odot}$ is *lsqc* at a.

Proof: Let $f \in \mathcal{B}$ and $\varepsilon > 0$ be given. Let U be a neighborhood of a in $[a,b]$. Since f is *lsqc* at a, there exists a non-empty open set $W \subset U$ such that $f(t) - f(x) > -\varepsilon\ \forall\ t \in W$.

$$\Rightarrow f_{\odot}(t) \geq f(t) > f(a) - \varepsilon\ \ \forall\ t \in W$$

$$\Rightarrow f_{\odot}(t) > f(a) - \varepsilon\ \ \forall\ t \in W.$$

Since $f_{\odot}(a) - \varepsilon$ is not an upper bound of $\{f(y) / a \leq y \leq b\}$, there exists a point $y \in [a,b]$ such that $f_{\odot}(a) - \varepsilon < f(y)$.

If $y = a$ then $f_{\odot}(t) + \varepsilon > f(a) > f_{\odot}(a) - \varepsilon\ \ \forall\ t \in W$.

Suppose that $y \neq a$.

Since $a \in U$ and U is open in $[a,b]$, there exists a $\delta_1 > 0$ such that $[a, a+\delta_1) \subset U$. Choose $\delta_2 > 0$ such that $a < a + \delta_2 < y$.

Put $\delta = \min\{\delta_1, \delta_2\}$ and $W_1 = (a, a+\delta)$. Clearly, W_1 is a non-empty open set such that $W_1 \subset U$.

Then $z \in W_1 \Rightarrow z < y$

$$\Rightarrow f_{\odot}(z) \geq f_{\odot}(y) \geq f(y) > f_{\odot}(a) - \varepsilon$$

Hence $f_{\odot}(z) + \varepsilon > f_{\odot}(a)\ \forall\ z \in W_1$

$$\Rightarrow f_{\odot} \text{ is } lsqc \text{ at } a.$$

5.6 Proposition: Let $f \in \mathcal{B}$. Then

(a) $f_{\odot}$ is *usqc* at every $x \in [a,b)$.

(b) If $f:[a,b] \to \mathbb{R}$ is *usqc* at b then so is $f_{\odot}$.

5.7 Proposition [5]: Let $f:[a,b] \to \mathbb{R}$ and $p \in [a,b]$. If $f(p+)$ exists then f is cliquish at p.

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5.8 Corollary: For any $f \in \mathcal{B}$, $Tf = f_{\odot}$ is cliquish on $[a,b]$.

5.9 Proposition: For any $f \in \mathcal{B}$, $Tf = f_{\odot}$ is quasicontinuous on $[a,b]$.

VII. Variant Properties

6.1 Symmetric continuity: It is not necessary that the rising sun function of a symmetrically continuous function is symmetrically continuous. For example, define $f:[-1,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} x^2 & if \quad x \neq 0 \\ 0 & if \quad x = 0 \end{cases}$$

Then
$$f_{\odot}(x) = \begin{cases} x^2 & if \quad -1 \leq x \leq 0 \\ 1 & if \quad 0 < x \leq 1 \end{cases}$$

Clearly f is symmetrically continuous on $[-1,1]$, but $f_{\odot}$ is not.

6.2 Semi-continuity: The semi-continuity of f need not imply the semi-continuity of $f_{\odot}$ as is evident from the following example.

Define $f:[-1,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & if \quad x = 0 \\ (x+1)2^{\frac{-2}{x}} & if \quad 0 < x \leq 1 \\ x+1 & if \quad -1 \leq x < 0 \end{cases}$$

Then
$$f_{\odot}(x) = \begin{cases} 1 & if \quad -1 \leq x \leq 0 \\ \frac{1}{2} & if \quad 0 < x \leq 1 \end{cases}$$

Clearly f is semi-continuous on $[-1,1]$. But $f_{\odot}$ is not semi-continuous at $x = 0$.

6.3 Darboux continuity: It is not necessary that the rising sun function of a Darboux continuous function is Darboux continuous. The function $f:[-1,1] \to \mathbb{R}$ defined in the above example is Dourbox continuous on $[-1,1]$ but its rising sun function is not Dourbox continuous.

6.4 Differentiability: The rising sun function of a differentiable function is not necessarily differentiable as can be observed from the following example.

Notes

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Notes

Define $f:[0,1]\to\mathbb{R}$ by

$$f(x)=\begin{cases}+\sqrt{\frac{2x}{5}-x^2} & if\ \ 0\le x\le\frac{2}{5}\\ -\sqrt{\frac{6x}{5}-\frac{8}{25}-x^2} & if\ \ \frac{2}{5}\le x\le\frac{4}{5}\\ +\sqrt{\frac{9x}{5}-\frac{4}{5}-x^2} & if\ \ \frac{4}{5}\le x\le 1\end{cases}$$

Then $$f_{\odot}(x)=\begin{cases}\frac{1}{5} & if\ \ 0\le x\le\frac{1}{5}\\ f(x) & if\ \ \frac{1}{5}\le x\le\frac{9}{10}\\ \frac{1}{10} & if\ \ \frac{2}{5}\le x\le\frac{9}{10}\\ f(x) & if\ \ \frac{9}{10}\le x\le 1\end{cases}$$

Clearly f is differentiable at $a=0.3732$, but $f_{\odot}$ is not differentiable at this point.

6.5 Pointwise Convergence: If $\{f_n\}$ converges pointwise to f on $[a,b]$, it is not necessary that $\{Tf_n\}$ converges to Tf as can be seen from the following example. Define $f_n:[0,1]\to\mathbb{R}$ by

$$f_n(x)=\begin{cases}1 & if\ \ x\in\left\{\frac{1}{n}/n\in\mathbb{N}\right\}\\ 0 & if\ \ x\notin\left\{\frac{1}{n}/n\in\mathbb{N}\right\}\end{cases}$$

Then $\{f_n\}$ converges pointwise to 0.

But $(f_n)_{\odot}(x)=\begin{cases}1 & if\ \ x=0\\ 0 & if\ \ 0<x\le 1\end{cases}$ does not converge to 0.

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Some Statistical Properties of Exponentiated Weighted Weibull Distribution

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Abstract- This article basically focused on some statistical properties of exponentiated-weighted weibull model which of course numerous authors have written one thing or the other on exponential weibull distribution and not on exponential weighted weibull. This model is established with a view to obtaining a model that is better than both weighted weibull and weibull distribution in terms of the estimate of their characteristics and their parameters using the logit of Beta by Jones (2004). The weighted weibull distribution is proposed by Mahdy (2013) with an additional parameter called "sensitive skewness parameter". Some basic properties of the proposed model including moments and moment generating function (first and second moments about the origin even with standard deviation are derive), survival rate function, hazard rate function, asymptotic behaviours, and the estimation of parameters have been studied. The result from the new model is better representativeness in data and its flexibility and shape.

Keywords: *exponentiated-weighted weibull, hazard rate, moments, weighted-weibull, survival rate.*

GJSFR-F Classification : *MSC 2010: 97K80, 35B40*

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Some Statistical Properties of Exponentiated Weighted Weibull Distribution

Badmus, N. Idowu [α] & Bamiduro, T. Adebayo[σ]

Abstract- This article basically focused on some statistical properties of exponentiated-weighted weibull model which of course numerous authors have written one thing or the other on exponential weibull distribution and not on exponential-weighted weibull. This model is established with a view to obtaining a model that is better than both weighted weibull and weibull distribution in terms of the estimate of their characteristics and their parameters using the logit of Beta by Jones (2004). The weighted weibull distribution is proposed by Mahdy (2013) with an additional parameter called "sensitive skewness parameter". Some basic properties of the proposed model including moments and moment generating function (first and second moments about the origin even with standard deviation are derive), survival rate function, hazard rate function, asymptotic behaviours, and the estimation of parameters have been studied. The result from the new model is better representativeness in data and its flexibility and shape.

Keywords: exponentiated-weighted weibull, hazard rate, moments, weighted-weibull, survival rate.

I. Introduction

In recent time, numerous researchers had used weibull distribution as an alternative to some distribution e.g Gamma and Log-normal distribution in reliability engineering and life testing. The weibull distribution is a well known common distribution and has been a powerful probability distribution in reliability analysis, while weighted distributions are used to adjust the probabilities of the events as observed and recorded. Mahdy applied Azzalini's method to the weibull distribution that produced a new class of weighted weibull distribution as $WW(\lambda,\beta,\alpha)$ distribution with an additional parameter called "Sensitive Skewness Parameter" and the sensitive skewness parameter governs essentially the shape of the probability density function of the $WW(\lambda,\beta,\alpha)$ distribution.

The probability density and the cumulative density function (pdf and cdf) of the new class of weighted weibull distribution by Mahdy (2013) is given by

$$f_{y|\{\lambda,\beta,\alpha\}|}(y) = \frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}, \text{ for } y > 0 \tag{1}$$

and

$$F_{y|\{\lambda,\beta,\alpha\}|}(y) = \frac{\left[\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1\right]}{\alpha^{\beta}} \tag{2}$$

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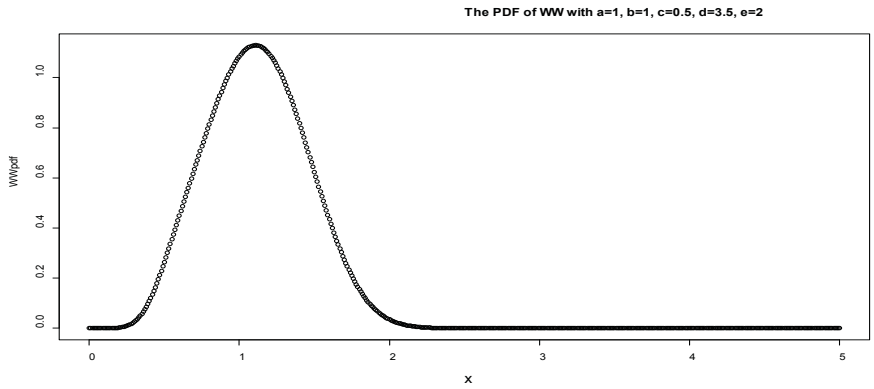

Figure 1 : The Probability Density Function of Weghted Weibull Distribution with a=1, b=1, λ =0.5, β = 3.5, α = 2

Authors on exponenetiated-weighted weibull are very few. The aim of this article is to introduce and investigate this distribution on its statistical properties. The paper is divided as follows: In section 2, we present the proposed distribution exponentiated-weighted weibull distribution. Moments, and moment generating function is studied in section 3, section 4, discussed on the estimation of parameters mathematically and in section 5 we preset the real application to data set and section 6 concluded the research.

II. The Proposed Exponentiated-Weighted Weibull Distribution

Recently, many authors have studied the properties of exponentiated distributions. For instance, Gupta et al (2001) for exponential pareto, Nadarajah and Gupta (2007) for exponential gamma distribution, Mudholkar et al (1995) studied on exponentiated weibull distribution, Salem and Abo-Kasem (2011) based their research on estimation for the parameters of the exponentiated weibull distribution, Gupta and Kundu (2001) they put up a paper on exponentiated exponential etc. Azzalini (1985) first proposed a method of obtaining weighted and the method has been used extensively for several symmetric and non-symmetric distributions. Mahdy (2013) applied the method to study a new class of weighted weibull distribution with an additional parameter called "sensitive skewness parameter". More so, various extensions of weibull and exponential distribution have been proposed in literature. An extension of exponential distribution has been provided by Nadarajah and Kotz (2005) using the logit of Beta distribution and the logit of Beta distribution (the link function of the Beta generalized distribution) is introduced by Jones (2004). Since then extensive work has been done using the logit of beta distribution in literature. For instance, Gupta and Kundu (1999) proposed a generalized exponential distribution which provides an alternative to exponential and weibull distributions. Famoye et al (2005) also introduced the Beta-weibull distribution alongside its major properties and Cordeiro et al (2011) among others.

Now, letting y be a random variable form of the distribution with parameters and defined (1) and (2) using the logit of beta by Jones (2004), we then have

$$f_{EWW}{}^{(y)} = \frac{1}{B(a,b)} [F(y)]^{a-1} \; [1 - F(y)]^{b-1} \;\; f(y) \tag{3}$$

by setting b=1, we get

$$f_{EWW}{}^{(y)} = a[F(y)]^{a-1} \;\; f(y) \tag{4}$$

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Notes

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Putting expressions (1) and (2) in (4) to obtain the probability density function of Exponenetiated-weighted weibull distribution

$$f_{EWW}{}^{(y)} = a\left[\frac{[(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1]}{\alpha^{\beta}}\right]^{a-1}\frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}} \quad (5)$$

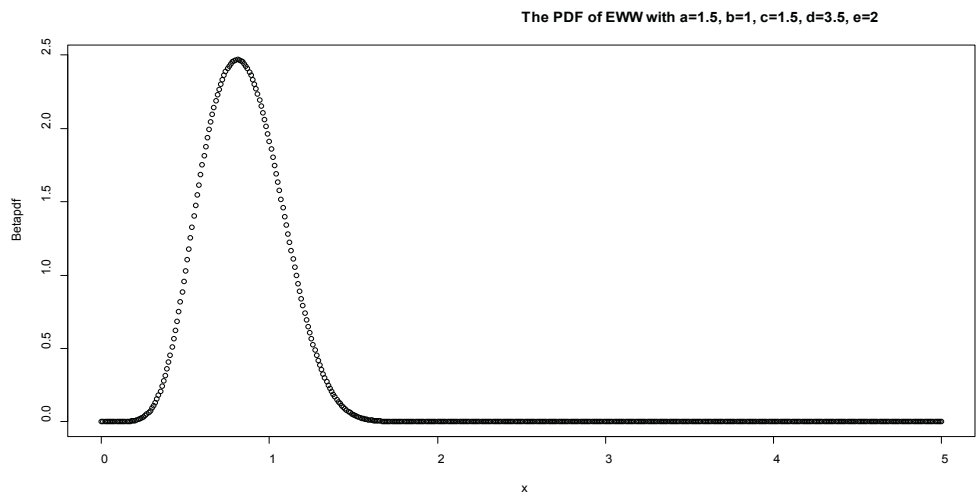

Figure 2 : The PDF of Exponentiated Weighted Weibull Distribution with values of the parameters $(a = 1.5, b = 1, c = \lambda = 1.5, d = \alpha = 3.5, e = \beta = 2.)$ and is rightly skewed

Where $a > 0, \lambda > 0, \beta > 0, \alpha > 0$ *and* $y > 0$ *such that* $Y \sim EWW(a, \lambda, \beta, \alpha)$. Equation (5) is the pdf of Exponenetiated-weighted weibull distribution.

$$\text{set } u(x) = \frac{[(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1]}{\alpha^{\beta}}$$

$$\frac{du}{dy} = \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right] \quad (6)$$

substituting dy into (5), we obtain

$$f_{EWW}{}^{(y)} = a\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right]^{a-1} du \quad (7)$$

$$u = \frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1$$

Now, (5) can becomes

$$f_{EWW}{}^{(y)} = a[U]^{a-1}\frac{du}{dy} \quad (8)$$

a) Cumulative Density Function (cdf)

The probability density function (pdf) of $EWW(a, \lambda, \beta, \alpha)$ given in (7), then expression (7) can be written as

$$F_{EWW}{}^{(y)} = P(Y \le y) = \int_0^y f(u)du$$

$$= \int_0^y a\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right]^{a-1}\frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}} du \quad (9)$$

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$$F_{EWW}{}^{(y)} = P(Y \leq y) = \int_0^y a[U]^{a-1}\, du$$

$= a\int_0^y [U]^{a-1}\, du$ and the cdf is obtained as

$$F_{EWW}{}^{(y)} = -(y)^a \tag{10}$$

b) The Survival Rate Function

The survival rate function of the Exponentiated-weighted weibull distribution is given by $S_{EWW}(y) = 1 - F_{EWW}(y) = 1 - \int_0^y f(u)du$

$$= 1 - a\int_0^y [U]^{a-1}\, du = 1 - [\,-\,(y)^a]$$

$$S_{EWW}(y) = 1 + (y)^a \tag{11}$$

c) The Hazard Rate Function

The hazard rate function of a random variable y with the pdf and cdf is defined by

$$h_{EWW}(y) = \frac{f_{EWW}(y)}{1 - F_{EWW}(y)}$$

Hence, the $EWW(a, \lambda, \beta, \alpha)$ with $f_{EWW}(y)$ and $F_{EWW}(y)$ respectively defined in (4) and (10), the hazard rate function can be expressed as:

$$= \frac{a(u)^{a-1}u'}{S_{EWW}(y)} \tag{12}$$

where U is expression in (5)

To show that $\lim_{y \to \infty} h_{EWW}(y) = 0$ *and* $\lim_{y \to 0} h_{EWW}(y) = 0$, we have the following

$$\lim_{y \to \infty} h_{EWW}(y) = \lim_{y \to \infty} \frac{a(u)^{a-1}u'}{1-(1+y)^a}$$

where, $u' = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1 - e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta(1+\alpha^\beta)} - 1$

$$= \lim_{y \to \infty} \frac{a\left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1\right]^{a-1}\frac{\lambda\beta\left(1+\alpha^\beta\right)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}}{1-(1+x)^a}$$

For simplification on the rigorous mathematics, we take the limit of the following:

When $y \to \infty = 0$ *and* $y \to 0 = 0$

$$= \lim_{y \to \infty} \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta} \quad = \lim_{y \to \infty} \frac{\lambda\beta(1+\alpha^\beta)\infty^{\beta-1}e^{-\lambda\infty^\beta}(1-e^{-\lambda(\alpha\infty)^\beta})}{\alpha^\beta}$$

$$= 0$$

Notes

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$$= \mathrm{y} \xrightarrow{lim} 0 \frac{\lambda\beta\,(1+\alpha^{\beta})0^{\beta-1}e^{-\lambda 0^{\beta}}\,(1-e^{-\lambda(\alpha 0)^{\beta}})}{\alpha^{\beta}} \;= 0$$

As $y \to \infty = 0$ *and* $y \to 0 = 0$, expression (12) above tends to ∞ *and* 0 and equal to zero.

Notes

d) Asymptotic Behaviours

Following the steps in hazard function above taken $\mathrm{y} \xrightarrow{lim} \infty\, f_{EWW}(y)$ *and* $y \xrightarrow{lim} 0\, f_{EWW}(y)$ of the $EWW(a,\lambda,\beta,\alpha)$ distribution is investigated as follows. Now from expression (5),
we have

$$\mathrm{y} \xrightarrow{lim} \infty f_{EWW}(y) = a\left[\frac{[(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}}{\alpha^{\beta}}\right]^{a-1}\frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}$$

taking the limit

$$= \mathrm{y} \xrightarrow{lim} \infty \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}} \quad = \mathrm{y} \xrightarrow{lim} \infty \frac{\lambda\beta(1+\alpha^{\beta})\infty^{\beta-1}e^{-\lambda\infty^{\beta}}(1-e^{-\lambda(\alpha\infty)^{\beta}})}{\alpha^{\beta}}$$

$$= 0$$

$$= \mathrm{y} \xrightarrow{lim} 0 \frac{\lambda\beta\,(1+\alpha^{\beta})0^{\beta-1}e^{-\lambda 0^{\beta}}\,(1-e^{-\lambda(\alpha 0)^{\beta}})}{\alpha^{\beta}} \;= 0$$

From the above results as $y \to \infty = 0$ *and* $y \to 0 = 0$, this shows that the distribution has at least a mode.

III. Moments and Moment Generating Function

Hosking (1990) described in their paper that when a random variable following a generalized beta generated distribution i.e $y \sim GBG(f,a,c)$ *then* $\mu_r' = E[F^{-1}K^{\frac{1}{c}}]^r$ where $K \sim B(a,1), c$ is a constant and $F^{-1}(y)$ is the inverse of CDF of the weighted weibull distribution, since $EWW(a,\lambda,\beta,\alpha)$ distribution is a special form when a=c=1.We then derive the moment generating function (mgf) of the proposed distribution $m(t) = E(e^{ty})$ and the general rth moment of a beta generated distribution is defined by

$$\mu_r' = \frac{1}{B(a,1)}\int_0^1 [F^{-1}(y)]^r\,[y]^{a-1}\,du \tag{13}$$

Also, using the taylor series expansion around the point $E(y_f) = \mu_f$ to obtain

$$\mu_r' = \sum_{u=0}^{r}\binom{r}{k}[F^{-1}(\mu)]^{r-k}\,[F^{-1(1)}(\mu_f)]^k \sum_{k=0}^{n}(-1)^i\binom{r}{i} \tag{14}$$

Cordeiro et al (2011) gave an alternative series expansion for μ_r' in terms of $r(r,U) = E(U^r\,F(U)^Y)$ where k follows the parent distribution then for u = 0, 1, …

$$\mu_r' = \frac{1}{B(a,1)}\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\,r(r,i-1)$$

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They further described another mgf of y for generated beta distribution as

$$M(t) = \frac{1}{B(a,1)} \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \rho(t, i-1) \tag{15}$$

Where,

$$\rho(t,r) = \int_{-\infty}^{\infty} e^{ty} [F(y)]^m f(y)\, dy$$

Therefore,

$$M_y{}^{(t)} = \frac{1}{B(a,1)} \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \int_{-\infty}^{\infty} e^{ty} [F(y)]^{(i+1)-1} f(y)\, dy \tag{16}$$

Substituting both probability density and cumulative density function of the weighted weibull distribution into (16), we obtain

$$M_{EWW(y)}{}^{(t)} = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \int e^{ty} \left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta (1+\alpha^\beta)} - 1\right]^{(i+1)-1} \frac{\lambda\beta(1+\alpha^\beta) y^{\beta-1} e^{-\lambda y^\beta} (1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}\, dy \tag{17}$$

Equation (17) becomes the mgf of Exponentiated-weighted weibull distribution. Then, setting a=1 and i = 0, the same expression (17) is reduced to becomes the parent distribution. To obtain the rth moment of $EWW(a,\lambda,\beta,\alpha)$, the weighted weibull distribution by Mahdy (2013) and is given by

$$M_{(y)}{}^{(t)} = \int_0^{\infty} e^{ty} \frac{\lambda\beta(1+\alpha^\beta) y^{\beta-1} e^{-\lambda y^\beta} \left(1-e^{-\lambda(\alpha y)^\beta}\right)}{\alpha^\beta}\, dy$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!\alpha^\beta} \{\lambda^{-\frac{j}{\beta}} (1+\alpha^\beta) \Gamma(\frac{j+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{j+\beta}{\beta}\right)}\} \tag{18}$$

Equation (17) can be re-written as

$$M_{EWW(y)}{}^{(t)} = a \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{t^j}{j!\alpha^\beta} \{\lambda^{-\frac{j}{\beta}} (1+\alpha^\beta) \Gamma(\frac{j+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{j+\beta}{\beta}\right)}\} \tag{19}$$

The rth moment of the $EWW(a,\lambda,\beta,\alpha)$ distribution can also be written from equation (19) as

$$\mu_r' = E(Y^r) = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{t^r}{r!\alpha^\beta} \{\lambda^{-\frac{r}{\beta}} (1+\alpha^\beta) \Gamma(\frac{r+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \tag{20}$$

Again, putting a = 1 in expression (20) leads to the rth moment of the weighted weibull model by Mahdy (2013) and is given by

$$\mu_r' = E(Y^r) = \frac{1}{\alpha^\beta} \{\lambda^{-\frac{r}{\beta}} (1+\alpha^\beta) \Gamma(\frac{r+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \tag{21}$$

$$\mu_{EWW(r)}' = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{r}{\alpha^\beta} \{\lambda^{-\frac{1}{\beta}} (1+\alpha^\beta) \Gamma(\frac{r+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \tag{22}$$

Notes

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From (22), it is easy to obtain the first and the second mean about the origin e.g when r = 1and the second moment when r = 2, etc.
The first moment of $EWW(a,\lambda,\beta,\alpha)$ is obtain

$$\mu'_{EWW(1)} = a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{1}{\beta}}(1+\alpha^\beta)\Gamma(\frac{1+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{1+\beta}{\beta}\right)}\} \quad (23)$$

Notes

The second moment can also be obtained as follows:

$$\mu'_2 = V(a,1,\lambda,\beta,\alpha)^{(y)} = V_1 - V_2 \quad (24)$$

where,

$$V_1 = E(a,\lambda,\beta,\alpha)^{(Y^2)} = a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{2}{\beta}}(1+\alpha^\beta)\Gamma(\frac{2+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{2+\beta}{\beta}\right)}\}$$

$$V_2 = E(a,\lambda,\beta,\alpha)^{(Y)} = (a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{2}{\beta}}(1+\alpha^\beta)\Gamma\left(\frac{1+\beta}{\beta}\right)\left(1-(1+\alpha^\beta)\right)^{-\left(\frac{1+\beta}{\beta}\right)}\})^2$$

Likewise, the standard deviation is given by

$$SD_{EWW}.(a,\lambda,\beta,\alpha)^{(Y)} = \sqrt{V_1 - V_2}$$

IV. Estimation of Parameter

We show the maximum likelihood estimate (MLEs) of the parameter of $EWW(a,\lambda,\beta,\alpha)$ distribution mathematically following Cordeiro et al (2011) and Shittu and Adepoju (2013) studied on the log-likelihood function for $\omega = (a,c,\varphi)$, where $\varphi = (\lambda,\beta,\alpha)$ and setting ω to be a vector of parameter and is given by

$$L(\omega) = nlogc - nlog[B(a,1)] + \sum_{i=1}^{n}\log[f(y;\varphi)] + (a-1)\sum_{i=1}^{n}\log[F(y;\varphi)] \quad (25)$$

Note that, b = c = 1 (24) becomes $\theta = (a,\varphi)$

$$L(\omega) = -nlog(a,1) + \sum_{i=1}^{n}\log[f(y;\varphi)] + (a-1)\sum_{i=1}^{n}\log[F(y;\varphi)] \quad (26)$$

where, $f(y;\varphi) = \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}$ and

$$F(y;\varphi) = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta(1+\alpha^\beta)} - 1$$

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$$L_{EWW}^{(\omega)} = -nlog(a,1) + \sum_{i=1}^{n} \log\left[\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}\right] + (a-1)\sum_{i=1}^{n}\log\left[\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)} - 1\right] \quad (27)$$

For determining the MLE of $a, \lambda, \beta, \alpha$, we take the partial derivative of the (27) with respect to $(a, \lambda, \beta, \alpha)$ as follows:

$$\frac{L_{EWW}^{(\omega)}}{\partial a} = -nlog(a,1) + (a-1)\sum_{y=1}^{n}\log\left[\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)} - 1\right] \quad (28)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial\lambda} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\lambda}\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}}{\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}}\right] +$$

$$(a-1)\sum_{y=1}^{n}\log\left[\frac{\frac{\partial}{\partial\lambda}\left(\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1\right)}{\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1}\right] \quad (29)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial\beta} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\beta}\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}}{\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}}\right] +$$

$$(a-1)\sum_{y=1}^{n}\log\left[\frac{\frac{\partial}{\partial\beta}\left(1-\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1\right)}{\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1}\right] \quad (30)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial\alpha} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\alpha}\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}}{\frac{\lambda\beta\left(1+\alpha^{\beta}\right)y^{\beta-1}e^{-\lambda y^{\beta}}\left(1-e^{-\lambda(\alpha y)^{\beta}}\right)}{\alpha^{\beta}}}\right] +$$

$$(a-1)\sum_{y=1}^{n}\log\left[\frac{\frac{\partial}{\partial\alpha}\left(1-\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1\right)}{\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}\left(1+\alpha^{\beta}\right)}-1}\right] \quad (31)$$

The equations derive above can also be solved using iteration method (Newton Raphson) to obtain the $\hat{a}, \hat{\lambda}, \hat{\beta}, \hat{\alpha}$ the MLE of $(a, \lambda, \beta, \alpha)$ respectively.

Taking second derivatives of the said equations 28, 29, 30 and 31 with respect to the parameters above, it is possible to derive the interval estimate and hypothesis tests on the model parameter. This may be shown in further research.

V. Real Data Set

The data used in this section was studied by Lemonte in his BJPS Accepted Manuscript on the remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang (2003) to compare and contrast the Exponentiated Weighted Weibull and Weighted Weibull distribution.

Notes

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Notes

R (code) software is used to determine the maximum likelihood estimates and the log-likelihood for the Exponentiated Weighted Weibull distribution are: $\hat{a} = 8.95941$, $\hat{\lambda} = 8.17089$, $\hat{\beta} = 8.36111$, $\hat{\alpha} = 7.91621$ *and* $log_{EWW} = 448.5781$ while the maximum likelihood estimates and the log-likelihood for the Weighted Weibull distribution are: $\hat{\lambda} = 7.90434$, $\hat{\beta} = 8.44282$, $\hat{\alpha} = 8.70033$ and $log_{WW} = 448.4913$, where lg_{LWW} *and* lg_{WW} denote log-likelihood of both Exponentiated Weighted Weibull distribution and Weighted Weibull distribution.

$$\begin{pmatrix} 0.802169648 & -0.02880174 & 0.002221849 & -0.02674447 \\ -0.028801742 & 0.65433442 & -0.025345533 & -0.02069611 \\ 0.002221849 & -0.02534553 & 0.705642669 & -0.02353514 \\ -0.026744474 & -0.02069611 & -0.023535137 & 0.60907454 \end{pmatrix}$$

VI. Conclusion

We investigated on the statistical properties of the proposed distribution e.g moments, moment generating function, estimation of parameters using R (Code) software for data analysis presented in this article. We also upgraded with an additional parameter to the existing three parameters in the weighted weibull distribution and the results from the estimated parameters show that the Exponentiated Weighted Weibull distribution has a better representation of data than weighted weibull distribution.

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A Summation Formula Involving Certain Special Functions

By Salahuddin & Intazar Husain
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Abstract- The main aim of the present paper is to compute a summation formula involving Recurrence relation and Contiguous relation.

Keywords: *gauss second summation theorem, recurrence relation, prudnikov.*

GJSFR-F Classification : *MSC 2010: 33C05 , 33C20 , 33C45 ,33D50 ,33D60*

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A Summation Formula Involving Certain Special Functions

Salahuddin [α] & Intazar Husain [σ]

Abstract- The main aim of the present paper is to compute a summation formula involving Recurrence relation and Contiguous relation.

Keywords : *gauss second summation theorem, recurrence relation, prudnikov.*

I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ b_1, b_2, \cdots, b_B & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $\mid z \mid = 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ c & ; \end{array} z\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b & ; \\ c & ; \end{array} z\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1 & ; \\ c & ; \end{array} z\right] \tag{2} $$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b+1}{2} & ; \end{array} \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b-1}{2} & ; \end{array} \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5} $$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b-1}{2} & ; \end{array} \quad \frac{1}{2}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\ \Gamma(\zeta) \tag{7} $$

II. Main Summation Formula

$$ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ \frac{a+b+45}{2} & ; \end{array} \quad \frac{1}{2}\right] = $$

$$ = \frac{2^b\ \Gamma(\frac{a+b+45}{2})}{(a-b)\ \Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{1}{\left[\prod_{\zeta=1}^{21}\{a-b-(2\zeta-1)\}\right]\left[\prod_{\eta=1}^{22}\{a-b+(2\eta-1)\}\right]}\times\right.\right. $$

$$ \times\Big(2097152a(-1311307045768798860344062 5+3283596050632439519239762 5a $$

$$ -331009024790814617530111 50a^2+188527718414561000489016 30a^3 $$

$$ -699815241779250324351626 1a^4+183098164808805043212494 1a^5 $$

$$ -354501246907809948405480a^6+524622627535035391552 40a^7-606804555261404386542 6a^8 $$

$$ +557078686778195952226a^9-410143375049274102 60a^{10}+243677759464745970 0a^{11} $$

$$ -117140064945846306a^{12}+4552422878866226a^{13}-142380641710440a^{14}+3551773672360a^{15} $$

$$ -69638530941a^{16}+1048818981a^{17}-11702670a^{18}+91070a^{19}-441a^{20}+a^{21} $$

$$ +5374596288262060889316787 5b-4505374366822222400434410 0ab $$

$$ +1138264581361070260751417 10a^2b-3258189634735855999931742 0a^3b $$

$$ +2221353788317106815143953 1a^4b-3437881821120782582829456a^5b $$

$$ +1069513403791665845357128a^6b-1020179620306420732104 80a^7b $$

$$ +1760523830807426405357 4a^8b-1094032303287108057912a^9b+1149343661918490372 36a^{10}b $$

$$ -474270953130853476 0a^{11}b+3170386864943372 62a^{12}b-861215354489940 8a^{13}b $$

$$ +370669544999304a^{14}b-6357587526960a^{15}b+172733256855a^{16}b-1689533124a^{17}b+27096622a^{18}b $$

$$ -114380a^{19}b+903a^{20}b+3418740929846540363004450b^2+1526282071708082046472757 70ab^2 $$

$$ -2966205753801299989533857 4a^2b^2+7392467948905891804601593 8a^3b^2 $$

$$ -9095261732430392323657176a^4b^2+6475045382105375335889480a^5b^2 $$

$$ -5230716994080508599290 16a^6b^2+1712429423513036329105 36a^7b^2 $$

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$$-93368426702716540084 84a^8b^2+16894747707659905960 76a^9b^2-6258856241904048125 2a^{10}b^2$$
$$+6843044908448608444a^{11}b^2-16960975177617234 4a^{12}b^2+1168455696337456 8a^{13}b^2$$
$$-185095028637272a^{14}b^2+8105951624872a^{15}b^2-73945230446a^{16}b^2+2006403306a^{17}b^2$$
$$-7972286a^{18}b^2+123410a^{19}b^2+493028445538106658885759 30b^3+5196235439832751151572788ab^3$$
$$+9173580472480782112160952 6a^2b^3-6747725269089701729734368a^3b^3$$
$$+1643842734922932898254589 6a^4b^3-1038876114726270005445776a^5b^3$$
$$+7352404154768760383126 32a^6b^3-3424182069759708872284 8a^7b^3$$
$$+1122618694511849293985 2a^8b^3-3710784022919059612 24a^9b^3+6724031452151885586 0a^{10}b^3$$
$$-1526743180734652192a^{11}b^3+166445736776215624a^{12}b^3-2459287660143888a^{13}b^3$$
$$+167607787407128a^{14}b^3-1445744454752a^{15}b^3+61843300298a^{16}b^3-235148940a^{17}b^3$$
$$+6096454a^{18}b^3+6523406582853698567485227b^4+4368975768739841479054 7553ab^4$$
$$+1858191737565276546672360a^2b^4+1956654920038543110397 2216a^3b^4$$
$$-6735816903607465827557 56a^4b^4+1596002262124887064872892a^5b^4$$
$$-5686461288277462683172 0a^6b^4+3909647304629472302093 6a^7b^4-1099649665242617000798a^8b^4$$
$$+3520223593464046626 30a^9b^4-715717976953844893 6a^{10}b^4+1266877992724507592a^{11}b^4$$
$$-17271058034689004a^{12}b^4+1832663509135228a^{13}b^4-14882731838680a^{14}b^4+979484169144a^{15}b^4$$
$$-3560764597a^{16}b^4+145008513a^{17}b^4+6412982275781442369534447b^5$$
$$+3020391903873160874624880ab^5+1107254450671742504094 3320a^2b^5$$
$$+2514095028329299405808 48a^3b^5+1850900598832213039588308a^4b^5$$
$$-3336218352868369614318 4a^5b^5+7663514407813279848487 2a^6b^5-1614107103242780383952a^7b^5$$
$$+1064600574662833613834a^8b^5-1828540502332801400 0a^9b^5+5631584078513951272a^{10}b^5$$
$$-68669517587706672a^{11}b^5+11692563911936180a^{12}b^5-87796234586480a^{13}b^5+8925930851320a^{14}b^5$$
$$-30677356528a^{15}b^5+1917334783a^{16}b^5+6045737459344011910059 12b^6$$
$$+2761535361221837965136664ab^6+4464098364110469077513 68a^2b^6$$
$$+1155941471891438860601896a^3b^6+1562421487601667973935 2a^4b^6$$
$$+8731368014856515011183 2a^5b^6-8746496076437123724 72a^6b^6+1943115716512939776808a^7b^6$$
$$-2452088236644689042 4a^8b^6+1539602176685380007 2a^9b^6-157355447527654376a^{10}b^6$$
$$+46179770718580408a^{11}b^6-309271965286648a^{12}b^6+50125620668424a^{13}b^6-159286274280a^{14}b^6$$
$$+15338678264a^{15}b^6+2317243390781757061501 36b^7+1538113765276058881172 64ab^7$$
$$+3749217757600593098902 00a^2b^7+2883134725423221639843 2a^3b^7$$
$$+5811854531770033541340 0a^4b^7+4904682108611848923 36a^5b^7+2185851421255096595000a^6b^7$$
$$-1243823631333643360 0a^7b^7+2667252758559901168 8a^8b^7-1969902989223929 76a^9b^7$$

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$$+117137843987075912a^{10}b^{7}-652631982303392a^{11}b^{7}+181277011099912a^{12}b^{7}-512075874352a^{13}b^{7}$$
$$+78378960360a^{14}b^{7}+15128932813380875312110b^{8}+5688148102699391131057 8ab^{8}$$
$$+1303935804572137494342 0a^{2}b^{8}+2213910491782167157267 6a^{3}b^{8}+9214719430968641392 66a^{4}b^{8}$$
$$+1520878767836758616286a^{5}b^{8}+8095043487442343432a^{6}b^{8}+29720901376476436984a^{7}b^{8}$$
$$-94427080896983406a^{8}b^{8}+195113449113625614a^{9}b^{8}-774287072717444a^{10}b^{8}$$
$$+434411052282116a^{11}b^{8}-1013931747010a^{12}b^{8}+265182149218a^{13}b^{8}+3024008960876320420246b^{9}$$
$$+2332408473401733698504ab^{9}+4569477008635213654148a^{2}b^{9}+4926782541836963544 88a^{3}b^{9}$$
$$+6459455372184446766 42a^{4}b^{9}+1532914988399567572 8a^{5}b^{9}+2135754030064486031 2a^{6}b^{9}$$
$$+69758491996612848a^{7}b^{9}+215842089833789562a^{8}b^{9}-353214178214040a^{9}b^{9}$$
$$+702101231167908a^{10}b^{9}-1152680301864a^{11}b^{9}+608359048206a^{12}b^{9}+1371048711540959426 04b^{10}$$
$$+4542220139072028439 00ab^{10}+1206150548918758877 72a^{2}b^{10}+1619498366344827065 24a^{3}b^{10}$$
$$+9180121926490903960a^{4}b^{10}+9807216639522886584a^{5}b^{10}+1324057231861444 08a^{6}b^{10}$$
$$+158910354362756568a^{7}b^{10}+291326223911580a^{8}b^{10}+772341543530700a^{9}b^{10}$$
$$-503154100020a^{10}b^{10}+960566918220a^{11}b^{10}+1611628006384226953 2b^{11}$$
$$+1326652400993417045 6ab^{11}+2246863049429253678 8a^{2}b^{11}+2744654110430536928a^{3}b^{11}$$
$$+2812829209563463896a^{4}b^{11}+8599935821006420 8a^{5}b^{11}+7733116772117245 6a^{6}b^{11}$$
$$+552383916933472a^{7}b^{11}+579276741224844a^{8}b^{11}+457412818200a^{9}b^{11}+1052049481860a^{10}b^{11}$$
$$+5044648084479651 18b^{12}+1520074513892383306ab^{12}+4223904815550140 24a^{2}b^{12}$$
$$+483431815242444952a^{3}b^{12}+29546096072985492a^{4}b^{12}+24458385708121820a^{5}b^{12}$$
$$+381004267695688a^{6}b^{12}+294616546783832a^{7}b^{12}+864510226398a^{8}b^{12}+800472431850a^{9}b^{12}$$
$$+3684291919673911 0b^{13}+30556991340393392ab^{13}+46327319945209016a^{2}b^{13}$$
$$+5648934122340208a^{3}b^{13}+4881179381635732a^{4}b^{13}+144878605202576a^{5}b^{13}$$
$$+100261443987096a^{6}b^{13}+623957998160a^{7}b^{13}+421171648758a^{8}b^{13}+777692306266456b^{14}$$
$$+2164283489573848ab^{14}+578915870956584a^{2}b^{14}+585771144020552a^{3}b^{14}+32252520795880a^{4}b^{14}$$
$$+22313057280808a^{5}b^{14}+256037937176a^{6}b^{14}+151532656696a^{7}b^{14}+358579489724 08b^{15}$$
$$+28280942023504ab^{1}5+39037774410808a^{2}b^{15}+4143128700320a^{3}b^{15}+3132630060840a^{4}b^{15}$$
$$+63714509712a^{5}b^{15}+36576848168a^{6}b^{15}+484185248451b^{16}+1250881854197ab^{16}$$
$$+288251667106a^{2}b^{16}+262400960302a^{3}b^{16}+9586673915a^{4}b^{16}+5752004349a^{5}b^{16}+13811946279b^{17}$$
$$+9489432124ab^{17}+11986499486a^{2}b^{17}+837826964a^{3}b^{17}+563921995a^{4}b^{17}+106564578b^{18}$$
$$+254376218ab^{18}+39191490a^{2}b^{18}+32224114a^{3}b^{18}+1743994b^{19}+839188ab^{19}+962598a^{2}b^{19}$$
$$+5719b^{20}+12341ab^{20}+43b^{21})\Big)+\frac{1}{\Big[\prod_{\mu=1}^{22}\{a-b-(2\mu-1)\}\Big]\Big[\prod_{\xi=1}^{21}\{a-b+(2\xi-1)\}\Big]}\times$$

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$$\Big(2097152b(-1311307045768798860344062 5+5374596288262060889316787 5a$$
$$+3418740929846540363004450a^2+4930284455381066588857593 0a^3$$
$$+6523406582853698567485227a^4+6412982275781442369534447a^5+6045737459344011910059 12a^6$$
$$+2317243390781757061501 36a^7+1512893281338087531211 0a^8+3024008960876320420246a^9$$
$$+1371048711540959426 04a^{10}+1611628006384226953 2a^{11}+5044648084479651 18a^{12}$$
$$+3684291919673911 0a^{13}+777692306266456a^{14}+35857948972408a^{15}+484185248451a^{16}$$
$$+13811946279a^{17}+106564578a^{18}+1743994a^{19}+5719a^{20}+43a^{21}+3283596050632439519233976 25b$$
$$-4505374366822222400434410 0ab+1526282071708082046472757 70a^2b$$
$$+5196235439832751151572788a^3b+4368975768739841479054755 3a^4b$$
$$+3020391903873160874624880a^5b+2761535361221837965136664a^6b$$
$$+1538113765276058881172 64a^7b+5688148102699391131057 8a^8b+2332408473401733698504a^9b$$
$$+4542220139072028439 00a^{10}b+1326652400993417045 6a^{11}b+1520074513892383306a^{12}b$$
$$+3055699134039339 2a^{13}b+2164283489573848a^{14}b+28280942023504a^{15}b+1250881854197a^{16}b$$
$$+9489432124a^{17}b+254376218a^{18}b+839188a^{19}b+12341a^{20}b-3310090247908146175301115 0b^2$$
$$+1138264581361070260751417 10ab^2-2966205753801299989533857 4a^2b^2$$
$$+9173580472480782112160952 6a^3b^2+1858191737565276546672360a^4b^2$$
$$+1107254450671742504094332 0a^5b^2+4464098364110469077513 68a^6b^2$$
$$+3749217757600593098902 00a^7b^2+1303935804572137494342 0a^8b^2+4569477008635213654148a^9b^2$$
$$+1206150548918758877 72a^{10}b^2+2246863049429253678 8a^{11}b^2+4223904815550140 24a^{12}b^2$$
$$+4632731994520901 6a^{13}b^2+578915870956584a^{14}b^2+39037774410808a^{15}b^2+288251667106a^{16}b^2$$
$$+11986499486a^{17}b^2+39191490a^{18}b^2+962598a^{19}b^2+1885277184145610004890163 0b^3$$
$$-3258189634735855999931742 0ab^3+7392467948905891804601593 8a^2b^3$$
$$-6747725269089701729734368a^3b^3+1956654920038543110397221 6a^4b^3$$
$$+2514095028329299405808 48a^5b^3+1155941471891438860601896a^6b^3$$
$$+2883134725423221639843 2a^7b^3+2213910491782167157267 6a^8b^3+4926782541836963544 88a^9b^3$$
$$+1619498366344827065 24a^{10}b^3+2744654110430536928a^{11}b^3+4834318152424449 52a^{12}b^3$$
$$+5648934122340208a^{13}b^3+585771144020552a^{14}b^3+4143128700320a^{15}b^3+262400960302a^{16}b^3$$
$$+837826964a^{17}b^3+32224114a^{18}b^3-6998152417792503243516261b^4$$
$$+2221353788317106815143953 1ab^4-9095261732430392323657176a^2b^4$$
$$+1643842734922932898254589 6a^3b^4-6735816903607465827557 56a^4b^4$$
$$+1850900598832213039588308a^5b^4+1562421487601667973935 2a^6b^4$$
$$+5811854531770033541340 0a^7b^4+9214719430968641392 66a^8b^4+6459455372184446766 42a^9b^4$$

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$$+9180121926490903960a^{10}b^4+2812829209563463896a^{11}b^4+29546096072985492a^{12}b^4$$
$$+4881179381635732a^{13}b^4+32252520795880a^{14}b^4+3132630060840a^{15}b^4+9586673915a^{16}b^4$$
$$+563921995a^{17}b^4+1830981648088050432124941b^5-3437881821120782582829456ab^5$$
$$+6475045382105375335889480a^2b^5-10388761147262700054457 76a^3b^5$$
$$+1596002262124887064872892a^4b^5-3336218352868369614318 4a^5b^5$$
$$+8731368014856515011183 2a^6b^5+4904682108611848923 36a^7b^5+1520878767836758616286a^8b^5$$
$$+153291498839956757 28a^9b^5+980721663952288658 4a^{10}b^5+85999358210064208a^{11}b^5$$
$$+24458385708121820a^{12}b^5+144878605202576a^{13}b^5+22313057280808a^{14}b^5+63714509712a^{15}b^5$$
$$+5752004349a^{16}b^5-3545012469078099484054 80b^6+1069513403791665845357128ab^6$$
$$-5230716994080508599290 16a^2b^6+7352404154768760383126 32a^3b^6$$
$$-5686461288277462683172 0a^4b^6+7663514407813279848487 2a^5b^6-8746496076437123724 72a^6b^6$$
$$+2185851421255096595000a^7b^6+8095043487442343432a^8b^6+2135754030064486031 2a^9b^6$$
$$+132405723186144408a^{10}b^6+77331167721172456a^{11}b^6+381004267695688a^{12}b^6$$
$$+100261443987096a^{13}b^6+256037937176a^{14}b^6+36576848168a^{15}b^6+5246226275350353915524 0b^7$$
$$-1020179620306420732104 80ab^7+1712429423513036329105 36a^2b^7$$
$$-3424182069759708872284 8a^3b^7+3909647304629472302093 6a^4b^7-1614107103242780383952a^5b^7$$
$$+194311571651293977680 8a^6b^7-1243823631333643360 0a^7b^7+2972090137647643698 4a^8b^7$$
$$+69758491996612848a^9b^7+158910354362756568a^{10}b^7+552383916933472a^{11}b^7$$
$$+294616546783832a^{12}b^7+623957998160a^{13}b^7+151532656696a^{14}b^7-6068045552614043865426b^8$$
$$+1760523830807426405357 4ab^8-9336842670271654008484a^2b^8+1122618694511849293985 2a^3b^8$$
$$-1099649665242617000798a^4b^8+1064600574662833613834a^5b^8-2452088236644689042 4a^6b^8$$
$$+2667252758559901168 8a^7b^8-94427080896983406a^8b^8+215842089833789562a^9b^8$$
$$+291326223911580a^{10}b^8+579276741224844a^{11}b^8+864510226398a^{12}b^8+421171648758a^{13}b^8$$
$$+5570786867781959522226b^9-1094032303287108057912ab^9+1689474770765990596076a^2b^9$$
$$-3710784022919059612 24a^3b^9+3520223593464046626 30a^4b^9-1828540502332801400 0a^5b^9$$
$$+1539602176685380007 2a^6b^9-1969902989223929 76a^7b^9+195113449113625614a^8b^9$$
$$-353214178214040a^9b^9+772341543530700a^{10}b^9+457412818200a^{11}b^9+800472431850a^{12}b^9$$
$$-4101433750492741026 0b^{10}+1149343661918490372 36ab^{10}-6258856241904048125 2a^2b^{10}$$
$$+6724031452151885586 0a^3b^{10}-7157179769538448936a^4b^{10}+5631584078513951272a^5b^{10}$$
$$-157355447527654376a^6b^{10}+117137843987075912a^7b^{10}-774287072717444a^8b^{10}$$
$$+702101231167908a^9b^{10}-503154100020a^10b^{10}+1052049481860a^{11}b^{10}+2436777594647459700b^{11}$$
$$-4742709531308534760ab^{11}+6843044908448608444a^2b^{11}-1526743180734652192a^3b^{11}$$

Notes

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$$+1266877992724507592a^4b^{11} - 68669517587706672a^5b^{11} + 46179770718580408a^6b^{11}$$
$$-652631982303392a^7b^{11} + 434411052282116a^8b^{11} - 1152680301864a^9b^{11} + 960566918220a^{10}b^{11}$$
$$-1171400649458463 06b^{12} + 317038686494337262ab^{12} - 169609751776172344a^2b^{12}$$
$$+166445736776215624a^3b^{12} - 17271058034689004a^4b^{12} + 11692563911936180a^5b^{12}$$
$$-309271965286648a^6b^{12} + 181277011099912a^7b^{12} - 1013931747010a^8b^{12} + 608359048206a^9b^{12}$$
$$+45524228788662 26b^{13} - 8612153544899408ab^{13} + 11684556963374568a^2b^{13}$$
$$-2459287660143888a^3b^{13}+1832663509135228a^4b^{13}-87796234586480a^5b^{13}+50125620668424a^6b^{13}$$
$$-512075874352a^7b^{13} + 265182149218a^8b^{13} - 1423806417104 40b^{14} + 370669544999304ab^{14}$$
$$-185095028637272a^2b^{14} + 167607787407128a^3b^{14} - 14882731838680a^4b^{14} + 8925930851320a^5b^{14}$$
$$-159286274280a^6b^{14} + 78378960360a^7b^{14} + 3551773672360b^{15} - 6357587526960ab^{15}$$
$$+8105951624872a^2b^{15} - 1445744454752a^3b^{15} + 979484169144a^4b^{15} - 30677356528a^5b^{15}$$
$$+15338678264a^6b^{15}-69638530941b^{16}+172733256855ab^{16}-73945230446a^2b^{16}+61843300298a^3b^{16}$$
$$-3560764597a^4b^{16} + 1917334783a^5b^{16} + 1048818981b^{17} - 1689533124ab^{17} + 2006403306a^2b^{17}$$
$$-235148940a^3b^{17} + 145008513a^4b^{17} - 11702670b^{18} + 27096622ab^{18} - 7972286a^2b^{18} + 6096454a^3b^{18}$$
$$+91070b^{19} - 114380ab^{19} + 123410a^2b^{19} - 441b^{20} + 903ab^{20} + b^{21})\Big)\Big\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{4194304}{\Big[\prod_{\zeta=1}^{21}\{a-b-(2\zeta-1)\}\Big]\Big[\prod_{\eta=1}^{22}\{a-b+(2\eta-1)\}\Big]}\Big(1311307045768798860344 0625$$
$$+5374596288262060889316 7875a - 3418740929846540363004450a^2$$
$$+4930284455381066588857 5930a^3 - 6523406582853698567485227a^4$$
$$+6412982275781442369534447a^5-6045737459344011910059 12a^6+2317243390781757061501 36a^7$$
$$-1512893281338087531211 0a^8 + 3024008960876320420246a^9 - 1371048711540959426 04a^{10}$$
$$+1611628006384226953 2a^{11} - 504464808447965118a^{12} + 368429191967391 10a^{13}$$
$$-777692306266456a^{14}+35857948972408a^{15}-484185248451a^{16}+13811946279a^{17}-106564578a^{18}$$
$$+1743994a^{19}-5719a^{20}+43a^{21}+3283596050632439519239 7625b+4505374366822222400434 4100ab$$
$$+1526282071708082046472 75770a^2b - 5196235439832751151572788a^3b$$
$$+4368975768739841479054 7553a^4b - 3020391903873160874624880a^5b$$
$$+2761535361221837965136664a^6b - 1538113765276058881172 64a^7b$$
$$+5688148102699391131057 8a^8b - 2332408473401733698504a^9b + 4542220139072028439 00a^{10}b$$
$$+2164283489573848a^{14}b - 28280942023504a^{15}b + 1250881854197a^{16}b - 9489432124a^{17}b$$
$$+254376218a^{18}b - 839188a^{19}b + 12341a^{20}b + 3310090247908146175301 1150b^2$$

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$$+1138264581361070260751417 10ab^2+2966205753801299989533857 4a^2b^2$$
$$+917358047248078211216095 26a^3b^2-1858191737565276546672360a^4b^2$$
$$+1107254450671742504094332 0a^5b^2-4464098364110469077513 68a^6b^2$$
$$+3749217757600593098902 00a^7b^2-1303935804572137494342 0a^8b^2$$
$$+4569477008635213654148a^9b^2-1206150548918758877 72a^{10}b^2+2246863049429253678 8a^{11}b^2$$
$$-422390481555014024a^{12}b^2+46327319945209016a^{13}b^2-578915870956584a^{14}b^2$$
$$+39037774410808a^{15}b^2-288251667106a^{16}b^2+11986499486a^{17}b^2-39191490a^{18}b^2+962598a^{19}b^2$$
$$+18852771841456100048901630b^3+3258189634735855999931742 0ab^3$$
$$+7392467948905891804601593 8a^2b^3+6747725269089701729734368a^3b^3$$
$$+1956654920038543110397221 6a^4b^3-2514095028329299405808 48a^5b^3$$
$$+1155941471891438860601896a^6b^3-2883134725423221639843 2a^7b^3$$
$$+2213910491782167157267 6a^8b^3-4926782541836963544 88a^9b^3+1619498366344827065 24a^{10}b^3$$
$$-2744654110430536928a^{11}b^3+483431815242444952a^{12}b^3-5648934122340208a^{13}b^3$$
$$+585771144020552a^{14}b^3-4143128700320a^{15}b^3+262400960302a^{16}b^3-837826964a^{17}b^3$$
$$+32224114a^{18}b^3+6998152417792503243516261b^4+2221353788317106815143953 1ab^4$$
$$+9095261732430392323657176a^2b^4+1643842734922932898254589 6a^3b^4$$
$$+6735816903607465827557 56a^4b^4+1850900598832213039588308a^5b^4$$
$$-1562421487601667973935 2a^6b^4+5811854531770033541340 0a^7b^4-9214719430968641392 66a^8b^4$$
$$+6459455372184446766 42a^9b^4-9180121926490903960a^{10}b^4+2812829209563463896a^{11}b^4$$
$$-29546096072985492a^{12}b^4+4881179381635732a^{13}b^4-32252520795880a^{14}b^4$$
$$+3132630060840a^{15}b^4-9586673915a^{16}b^4+563921995a^{17}b^4+1830981648088050432124941b^5$$
$$+3437881821120782582829456ab^5+6475045382105375335889480a^2b^5$$
$$+1038876114726270005445776a^3b^5+1596002262124887064872892a^4b^5$$
$$+3336218352868369614318 4a^5b^5+8731368014856515011183 2a^6b^5-4904682108611848923 36a^7b^5$$
$$+1520878767836758616286a^8b^5-1532914988399567572 8a^9b^5+9807216639522886584a^{10}b^5$$
$$-85999358210064208a^{11}b^5+24458385708121820a^{12}b^5-144878605202576a^{13}b^5$$
$$+22313057280808a^{14}b^5-63714509712a^{15}b^5+5752004349a^{16}b^5+3545012469078099484054 80b^6$$
$$+1069513403791665845357128ab^6+5230716994080508599290 16a^2b^6$$
$$+7352404154768760383126 32a^3b^6+5686461288277462683172 0a^4b^6$$
$$+7663514407813279848487 2a^5b^6+8746496076437123724 72a^6b^6+2185851421255096595000a^7b^6$$
$$-8095043487442343432a^8b^6+2135754030064486031 2a^9b^6-1324057231861444 08a^{10}b^6$$
$$+77331167721172456a^{11}b^6-381004267695688a^{12}b^6+100261443987096a^{13}b^6-256037937176a^{14}b^6$$

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$$+36576848168a^15b^6+524622627535035391552 40b^7+102017962030642073210480ab^7$$
$$+171242942351303632910536a^2b^7+342418206975970887228 48a^3b^7$$
$$+390964730462947230209 36a^4b^7+161410710324278038395 2a^5b^7+194311571651293977680 8a^6b^7$$
$$+124382363133364336003 600a^7b^7+297209013764764369 84a^8b^7-69758491996612848a^9b^7$$
$$+158910354362756568a^{10}b^7-552383916933472a^{11}b^7+294616546783832a^{12}b^7$$
$$-623957998160a^{13}b^7+151532656696a^{14}b^7+606804555261404386542 6b^8$$
$$+176052383080742640535 74ab^8+933684267027165400848 4a^2b^8+112261869451184929398 52a^3b^8$$
$$+109964966524261700079 8a^4b^8+106460057466283361383 4a^5b^8+245208823664468904 24a^6b^8$$
$$+266725275855990116 88a^7b^8+94427080896983406a^8b^8+215842089833789562a^9b^8$$
$$-291326223911580a^{10}b^8+579276741224844a^{11}b^8-864510226398a^{12}b^8+421171648758a^{13}b^8$$
$$+557078686778195952226b^9+109403230328710805791 2ab^9+168947477076599059607 6a^2b^9$$
$$+371078402291905961224a^3b^9+352022359346404662630a^4b^9+182854050233280140 00a^5b^9$$
$$+153960217668538000 72a^6b^9+196990298922392976a^7b^9+195113449113625614a^8b^9$$
$$+353214178214040a^9b^9+772341543530700a^{10}b^9-457412818200a^{11}b^9+800472431850a^{12}b^9$$
$$+410143375049274102 60b^{10}+114934366191849037236ab^{10}+625885624190404812 52a^2b^{10}$$
$$+672403145215188558 60a^3b^{10}+715717976953844893 6a^4b^{10}+563158407851395127 2a^5b^{10}$$
$$+157355447527654376a^6b^{10}+117137843987075912a^7b^{10}+774287072717444a^8b^{10}$$
$$+702101231167908a^9b^{10}+503154100020a^10b^{10}+1052049481860a^{11}b^{10}+243677759464745970 0b^{11}$$
$$+474270953130853476 0ab^{11}+684304490844860844 4a^2b^{11}+152674318073465219 2a^3b^{11}$$
$$+126687799272450759 2a^4b^{11}+686695175877066 72a^5b^{11}+461797707185804 08a^6b^{11}$$
$$+652631982303392a^7b^{11}+434411052282116a^8b^11+1152680301864a^9b^{11}+960566918220a^{10}b^{11}$$
$$+117140064945846306b^{12}+317038686494337262ab^{12}+169609751776172344a^2b^{12}$$
$$+166445736776215624a^3b^{12}+172710580346890 04a^4b^{12}+116925639119361 80a^5b^{12}$$
$$+309271965286648a^6b^{12}+181277011099912a^7b^{12}+1013931747010a^8b^{12}+608359048206a^9b^{12}$$
$$+455242287886622 6b^{13}+861215354489940 8ab^{13}+116845569633745 68a^2b^{13}$$
$$+245928766014388 8a^3b^{13}+183266350913522 8a^4b^{13}+87796234586480a^5b^{13}$$
$$+50125620668424a^6b^{13}+512075874352a^7b^{13}+265182149218a^8b^{13}+142380641710440b^{14}$$
$$+370669544999304ab^{14}+185095028637272a^2b^{14}+167607787407128a^3b^{14}+148827318386 80a^4b^{14}$$
$$+892593085132 0a^5b^{14}+159286274280a^6b^{14}+78378960360a^7b^{14}+3551773672360b^{15}$$
$$+6357587526960ab^{15}+8105951624872a^2b^{15}+1445744454752a^3b^{15}+979484169144a^4b^{15}$$
$$+30677356528a^5b^{15}+15338678264a^6b^{15}+69638530941b^{16}+172733256855ab^{16}$$
$$+73945230446a^2b^{16}+61843300298a^3b^{16}+3560764597a^4b^{16}+1917334783a^5b^{16}+1048818981b^{17}$$

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$$+1689533124ab^{17}+2006403306a^2b^{17}+235148940a^3b^{17}+145008513a^4b^{17}+11702670b^{18}$$
$$+27096622ab^{18}+7972286a^2b^{18}+6096454a^3b^{18}+91070b^{19}+114380ab^{19}+123410a^2b^{19}+441b^{20}$$
$$+903ab^{20}+b^{21}\Big)+\frac{4194304}{\Big[\prod_{\mu=1}^{22}\{a-b-(2\mu-1)\}\Big]\Big[\prod_{\xi=1}^{21}\{a-b+(2\xi-1)\}\Big]}\times$$
$$\times\Big(1311307045768798860344062 5+3283596050632439519239762 5a$$
$$+3310090247908146175301115 0a^2+1885277184145610004890163 0a^3$$
$$+6998152417792503243516261a^4+1830981648088050432124941a^5$$
$$+3545012469078099484054 80a^6+5246226275350353915524 0a^7+6068045552614043865426a^8$$
$$+5570786867781959522 26a^9+4101433750492741026 0a^{10}+2436777594647459700a^{11}$$
$$+1171400649458463 06a^{12}+4552422878866226a^{13}+142380641710440a^{14}+3551773672360a^{15}$$
$$+69638530941a^{16}+1048818981a^{17}+11702670a^{18}+91070a^{19}+441a^{20}+a^{21}$$
$$+5374596288262060889316787 5b+4505374366822222400434410 0ab$$
$$+1138264581361070260751417 10a^2b+3258189634735855999931742 0a^3b$$
$$+2221353788317106815143953 1a^4b+3437881821120782582829456a^5b$$
$$+1069513403791665845357128a^6b+1020179620306420732104 80a^7b$$
$$+1760523830807426405357 4a^8b+1094032303287108057912a^9b+1149343661918490372 36a^{10}b$$
$$+4742709531308534760a^{11}b+3170386864943372 62a^{12}b+8612153544899408a^{13}b$$
$$+370669544999304a^{14}b+6357587526960a^{15}b+172733256855a^{16}b+1689533124a^{17}b$$
$$+27096622a^{18}b+114380a^{19}b+903a^{20}b-3418740929846540363004450b^2$$
$$+1526282071708082046472757 70ab^2+2966205753801299989533857 4a^2b^2$$
$$+7392467948905891804601593 8a^3b^2+9095261732430392323657176a^4b^2$$
$$+6475045382105375335889480a^5b^2+5230716994080508599290 16a^6b^2$$
$$+1712429423513036329105 36a^7b^2+9336842670271654008484a^8b^2$$
$$+1689474770765990596076a^9b^2+6258856241904048125 2a^{10}b^2+6843044908448608444a^{11}b^2$$
$$+169609751776172344a^{12}b^2+1168455696337456 8a^{13}b^2+185095028637272a^{14}b^2$$
$$+8105951624872a^{15}b^2+73945230446a^{16}b^2+2006403306a^{17}b^2+7972286a^{18}b^2+123410a^{19}b^2$$
$$+4930284455381066588857593 0b^3-5196235439832751151572788ab^3$$
$$+9173580472480782112160952 6a^2b^3+6747725269089701729734368a^3b^3$$
$$+1643842734922932898254589 6a^4b^3+1038876114726270005445776a^5b^3$$
$$+7352404154768760383126 32a^6b^3+3424182069759708872284 8a^7b^3$$
$$+1122618694511849293985 2a^8b^3+3710784022919059612 24a^9b^3+6724031452151885586 0a^{10}b^3$$

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$$+1526743180734652192a^{11}b^3+16644573677621 5624a^{12}b^3+2459287660143888a^{13}b^3$$
$$+167607787407128a^{14}b^3+1445744454752a^{15}b^3+61843300298a^{16}b^3+235148940a^{17}b^3$$
$$+6096454a^{18}b^3-65234065828536985674852 27b^4+436897576873984147905475 53ab^4$$
$$-1858191737565276546672360a^2b^4+1956654920038543110397221 6a^3b^4$$
$$+673581690360746582755756a^4b^4+1596002262124887064872892a^5b^4$$
$$+568646128827746268317 20a^6b^4+3909647304629472302093 6a^7b^4$$
$$+1099649665242617000798a^8b^4+3520223593464046626 30a^9b^4+7157179769538448936a^{10}b^4$$
$$+1266877992724507592a^{11}b^4+17271058034689004a^{12}b^4+1832663509135228a^{13}b^4$$
$$+14882731838680a^{14}b^4+979484169144a^{15}b^4+3560764597a^{16}b^4+145008513a^{17}b^4$$
$$+641298227578144236953444 7b^5-3020391903873160874624880ab^5$$
$$+1107254450671742504094 3320a^2b^5-2514095028329299405808 48a^3b^5$$
$$+1850900598832213039588308a^4b^5+3336218352868369614318 4a^5b^5$$
$$+7663514407813279848487 2a^6b^5+1614107103242780383952a^7b^5+1064600574662833613834a^8b^5$$
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$$-1562421487601667973935 2a^4b^6+8731368014856515011183 2a^5b^6+8746496076437123724 72a^6b^6$$
$$+1943115716512939776808a^7b^6+2452088236644689042 4a^8b^6+1539602176685380007 2a^9b^6$$
$$+157355447527654376a^{10}b^6+46179770718580408a^{11}b^6+309271965286648a^{12}b^6$$
$$+50125620668424a^{13}b^6+159286274280a^{14}b^6+15338678264a^{15}b^6+2317243390781757061501 36b^7$$
$$-1538113765276058881172 64ab^7+3749217757600593098902 00a^2b^7$$
$$-2883134725423221639843 2a^3b^7+5811854531770033541340 0a^4b^7-4904682108611848923 36a^5b^7$$
$$+2185851421255096595000a^6b^7+1243823631333643360 0a^7b^7+2667252758559901168 8a^8b^7$$
$$+196990298922392976a^9b^7+117137843987075912a^{10}b^7+652631982303392a^{11}b^7$$
$$+181277011099912a^{12}b^7+512075874352a^{13}b^7+78378960360a^{14}b^7-1512893281338087531211 0b^8$$
$$+5688148102699391131057 8ab^8-1303935804572137494342 0a^2b^8+2213910491782167157267 6a^3b^8$$
$$-9214719430968641392 66a^4b^8+1520878767836758616286a^5b^8-8095043487442343432a^6b^8$$
$$+2972090137647643698 4a^7b^8+9442708089698340 6a^8b^8+1951134491136256 14a^9b^8$$
$$+774287072717444a^{10}b^8+434411052282116a^{11}b^8+1013931747010a^{12}b^8+265182149218a^{13}b^8$$
$$+3024008960876320420246b^9-2332408473401733698504ab^9+4569477008635213654148a^2b^9$$
$$-4926782541836963544 88a^3b^9+6459455372184446766 42a^4b^9-1532914988399567572 8a^5b^9$$

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$$+21357540300644860312a^6b^9 - 69758491996612848a^7b^9 + 215842089833789562a^8b^9$$
$$+353214178214040a^9b^9 + 702101231167908a^{10}b^9 + 1152680301864a^{11}b^9 + 608359048206a^{12}b^9$$
$$-137104871154095942604b^{10} + 454222013907202843900ab^{10} - 1206150548918758877 72a^2b^{10}$$
$$+161949836634482706524a^3b^{10} - 918012192649090396 0a^4b^{10} + 9807216639522886584a^5b^{10}$$
$$-132405723186144408a^6b^{10} + 158910354362756568a^7b^{10} - 291326223911580a^8b^{10}$$
$$+772341543530700a^9b^{10}+503154100020a^10b^{10}+960566918220a^{11}b^{10}+161162800638422695 32b^{11}$$
$$-132665240099341704 56ab^{11} + 224686304942925367 88a^2b^{11} - 274465411043053692 8a^3b^{11}$$
$$+281282920956346389 6a^4b^{11} - 85999358210064208a^5b^{11} + 77331167721172456a^6b^{11}$$
$$-552383916933472a^7b^{11} + 579276741224844a^8b^{11} - 457412818200a^9b^{11} + 1052049481860a^10b^{11}$$
$$-504464808447965118b^{12} + 152007451389238330 6ab^{12} - 422390481555014024a^2b^{12}$$
$$+483431815242444952a^3b^{12} - 295460960729854 92a^4b^{12} + 244583857081218 20a^5b^{12}$$
$$-381004267695688a^6b^{12} + 294616546783832a^7b^{12} - 864510226398a^8b^{12} + 800472431850a^9b^{12}$$
$$+368429191967391 10b^{13} - 305569913403933 92ab^{13} + 463273199452090 16a^2b^{13}$$
$$-564893412234020 8a^3b^{13} + 488117938163573 2a^4b^{13} - 144878605202576a^5b^{13}$$
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$$+2164283489573848ab^{14} - 578915870956584a^2b^{14} + 585771144020552a^3b^{14}$$
$$-32252520795880a^4b^{14} + 22313057280808a^5b^{14} - 256037937176a^6b^{14} + 151532656696a^7b^{14}$$
$$+358579489724 08b^{15} - 282809420235 04ab^{15} + 390377744108 08a^2b^{15} - 4143128700320a^3b^{15}$$
$$+3132630060840a^4b^{15} - 63714509712a^5b^{15} + 36576848168a^6b^{15} - 484185248451b^{16}$$
$$+1250881854197ab^{16} - 288251667106a^2b^{16} + 262400960302a^3b^{16} - 9586673915a^4b^{16}$$
$$+5752004349a^5b^{16} + 13811946279b^{17} - 9489432124ab^{17} + 11986499486a^2b^{17} - 837826964a^3b^{17}$$
$$+563921995a^4b^{17} - 106564578b^{18} + 254376218ab^{18} - 39191490a^2b^{18} + 32224114a^3b^{18}$$
$$+1743994b^{19} - 839188ab^{19} + 962598a^2b^{19} - 5719b^{20} + 12341ab^{20} + 43b^{21}\Big)\Big\}\Big] \qquad (8)$$

III. Derivation of the Summation Formula

Substituting $c = \frac{a+b+45}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b & ; \\ \frac{a+b+45}{2} & ; \end{matrix}\ \frac{1}{2}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b & ; \\ \frac{a+b+45}{2} & ; \end{matrix}\ \frac{1}{2}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; \\ \frac{a+b+45}{2} & ; \end{matrix}\ \frac{1}{2}\right]$$

Now involving the formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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Global Existence and Uniqueness of the Weak Solution in Keller Segel Model

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Abstract- This paper deals with the global existence, uniqueness and boundedness of the weak solution for the chemotaxis system **(P)** defined as

$$\begin{cases} u_t - \Delta u + \operatorname{div}(u\nabla c) = 0 & (t,x) \in \mathbb{R}^+ \times \Omega \\ -\Delta c + \tau c = 0 & x \in \Omega \\ u = 0,\ c = g & \Gamma \\ u(0,x) = u_0 & x \in \Omega \end{cases} \qquad (\mathrm{P})$$

The system **(P)** is under homogeneous Dirichlet boundary conditions in a convex smooth bounded domain $\Omega \in \mathbb{R}^n$ with smooth boundary $\Gamma\ (\in H^{\frac{3}{2}}(\Gamma)$and $u_0 \in H^{\frac{1}{2}}(\Omega)$ Based on Galerkin's method, Lax -Milgran's and maximum principle, a prove of the existence and uniqueness of a global solution for the system **(P)** is determined. Moreover we show that the unique solution is positive.

Keywords and Phrases: *chemotaxis, global existence, boundedness, positive solution.*

GJSFR-F Classification : *MSC 2010: 35K58, 45G05, 65C35, 82C22, 82C31,82C80, 92C17*

GLOBALEXISTENCEANDUNIQUENESSOFTHEWEAKSOLUTIONINKELLERSEGELMODEL

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Abstract- This paper deals with the global existence, uniqueness and boundedness of the weak solution for the chemotaxis system **(P)** defined as

$$\begin{cases} u_t - \Delta u + \operatorname{div}(u\nabla c) = 0 & (t,x) \in \mathbb{R}^+ \times \Omega \\ -\Delta c + \tau c = 0 & x \in \Omega \\ u = 0,\ c = g & \Gamma \\ u(0,x) = u_0 & x \in \Omega \end{cases} \qquad (\mathrm{P})$$

The system **(P)** is under homogeneous Dirichlet boundary conditions in a convex smooth bounded domain $\Omega \in \mathbb{R}^n$ with smooth boundary Γ ($\in H^{\frac{3}{2}}(\Gamma)$and $u_0 \in H^{\frac{1}{2}}(\Omega)$ Based on Galerkin's method, Lax-Milgran's Theorem and maximum principle, a prove of the existence and uniqueness of a global solution for the system **(P)** is determined. Moreover we show that the unique solution is positive.

Keywords and Phrases: *chemotaxis, global existence, boundedness, positive solution.*

I. Introduction

Chemotaxis is an important means for cellular communication. It is the influence of chemical substances in the environment on the movement of mobile species. This can lead to strictly oriented movement or to partially oriented and partially tumbling movement. The movement towards a higher concentration of the chemical substance is called positive chemotaxis whereas the movement towards regions of lower chemical concentration is called negative chemotactical movement.

The classical chemotaxis model — the so-called Keller–Segel model — system defined in (0.1) was first introduced by Paltak [11] (1953), E. Keller and L. Segel [9] (1970)

$$\begin{aligned} u_t - \nabla(a\nabla u) + \nabla(\chi u \nabla c) = 0 & \qquad (t,x) \in \mathbb{R}^+ \times \mathbb{R}^d \\ \alpha c_t - \Delta c + \tau c + \beta u = 0 & \qquad x \in \mathbb{R}^d \end{aligned} \qquad (0.1)$$

where $u(t,x)$ denotes the density of bacteria in the position $x \in \mathbb{R}^d$ and at time t, c the concentration of chemical signal substance, $\alpha \geq 0$ the relaxation time, the parameter χ the sensitivity of cells to the chemoattractant and $a, \tau,\ \beta$ are given smooth functions. As it can be seen, when $\alpha \neq 0$ the model is

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called Parabolic-Parabolic while it is an Elliptic-Parabolic model when $\alpha = 0$. This modelling is very simple, it exhibits a profound mathematical structure and mostly only dimension 2 is understood, especially chemotactic collapse. The proposed model has been extensively studied in the last few years (see ([7]-[8],[12],[13]) for a recent survey articles).

The Parabolic-Parabolic model has been investigated by many authers (see for examples Refs [8] and [13]), I. Fatkullin [10] had developed numerical method (a composite particle-grid) with adaptive time stepping which allows us to resolve and propagates singular solutions when with Neumann boundary condition.

The Elliptic-Parabolic model has been investigated by many authers (see for examples Refs [3] and [4]). This model have been carried out where the main concern is whether the solution of model is bounded or blow-up. It has been proved that the solution strongly depends on the spatial dimension. It does not occur in one-dimensional problems, and it occurs conditionally in higher dimensional situations. More precisley, see [2] in case higher dimensions ($n \geq 3$), if the norm of initial condition u_0 is small in space $L^{\frac{n}{2}}(\mathbb{R}^n)$, then there are global weak solutions and if $\left(\int x^2 u_0\right)^{d-2} \leq C \|u_0\|^n_{L^1(\mathbb{R}^n)}$with C is small ,then there is blow up in a finite time T^*. But in two dimension, (see [5]), if $\|u_0\|_{L^1(\mathbb{R}^2)} < \frac{8\pi}{\chi}$, there are smooth solutions, and if $\|u_0\|_{L^1(\mathbb{R}^2)} > \frac{8\pi}{\chi}$, there is creation of a singular measure (blow-up) in finite time.

In this paper we demonstrate the global existence and uniqueness of weak positive solution for the elliptic-parabolic model's problem defined as

$$(P)\begin{cases} (P_1)\begin{cases} u_t - \Delta u + \operatorname{div}(u\nabla c) = 0 & (t,x) \in \mathbb{R}^+ \times \Omega \\ u = 0 & \Gamma \\ u(0,x) = u_0 & x \in \Omega \end{cases} \\ (P_2)\begin{cases} -\Delta c + \tau c = 0 & x \in \Omega \\ c = g & \Gamma \end{cases} \end{cases}$$

Where $u(t,x)$ is a function denotes the density of bacteria in the position $x \in \Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$, Ω is a bounded convex domain with smooth boundary Γ, c denotes the concentration of chemical signal that stimulates the bacteria. The parameter τ is a time constant and it is expressed on the one hand the movement of bacteria (representing a random distribution side and a deterministic drift in the direction of high concentrations) and secondly the diffusion degradation of c.

To simplify the solution of the system (P), a decomposition of (P) into two subsystems (P_1) and (P_2) are adopted. Lax-Milgram's Theorem is very important theorem which we help us to demonstrate the existence and uniqueness of a weak solution for the system (P_2). However this theorem can not be applied directly because it is a nonhomogenous system. For this raison an adoptation of Trace Theorem is used to simplify the system (P_2), and together with Galerking method we can demonstrate the existence and uniquness of a weak solution for the system (P_1). Therefore we have the existence and uniquness for the problem (P). Moreover we show that the solution is positive. The following initial-boundary conditions on u_0 and g assumptions are used to prove the proposed solution of (P)

H_1	$g \in L^{\frac{1}{2}}(\Gamma)$
H_2	$g \in L^{\frac{3}{2}}(\Gamma)$
H_3	$u_0 \in L^2(\Omega)$
H_4	$u_0 \geq 0$ and $g \geq 0$.

Ref

[8] T. Hillen, K. Painter, Global existence for a parabolic chemotaxis model with prevention of overcrowding. Adv. Appl. Math. 26 (2001) 280-301.

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If the hypothesis H_1 is satisfies and using the theorem of trace, one can find a lifting of this trace which we denote $R(g) \in H_0^1(\Omega)$. Thus by definition it verifies $\gamma_0(R(g)) = g$. Now looking for c having the form $c = \widetilde{c} + R(g)$ reduces the problem (P_2) to $\widetilde{c}$.

$$\left(\widetilde{P}_2\right) \begin{cases} -\Delta\widetilde{c} + \tau\widetilde{c} - \Delta R(g) + \tau R(g) = 0 & \text{in } x \in \Omega \\ \widetilde{c} = 0 & \text{on } \Gamma \end{cases}$$

Definition 1 *We say $(u, \widetilde{c}) \in L^2\left(0, T; H_0^1(\Omega)\right) \times H_0^1(\Omega)$ with $u_t \in L^2\left(0, T; H^{-1}(\Omega)\right)$ is a weak solution of the problem (P) if and only if*

$$\langle u_t, v\rangle + B(u, v, t) = 0 \tag{0.2}$$

$$a(\widetilde{c}, q, t) = l(q) \tag{0.3}$$

where

$$\begin{cases} B(u, v, t) = \int_\Omega (\nabla u \nabla v + \nabla c \nabla u v + \tau c u v)\, dx \\ a(\widetilde{c}, q) = \int_\Omega (\nabla \widetilde{c} \nabla q + \tau \widetilde{c} q)\, dx \\ l(q) = -\int_\Omega (\nabla R(g) \nabla q + \tau R(g) q)\, dx \end{cases}$$

for all $(v, q) \in \left(H_0^1(\Omega)\right)^2$, $0 \le t \le T$, and

$$u(0, x) = u_0 \in L^2(\Omega) \tag{0.4}$$

Remark 2 *Note that $u \in C\left([0, T]; L^2(\Omega)\right)$ as $u \in L^2\left(0, T; H_0^1(\Omega)\right)$ and $u_t \in L^2\left(0, T; H^{-1}(\Omega)\right)$. Then equality (0.4) makes sense.*

II. Existence of Weak Solution of the Problem (P)

In this section, use the Theorem of Lax- Milgran to study the existence and uniqueness of weak solution of problem (P_2), which its variational formulat is given by equation (0.3) and use the method of Galerking to study the existence and uniqueness of weak solution of problem (P_1), which its variational formulat is given by equation (0.2). So we have the existence and uniqueness of weak solution of problem (P).

a) Existence of weak solution of the problem (P2)

Theorem 3 *If the hypothesis H_1 holds. Then the problem (P_2) has only one solution $c \in H^1(\Omega)$ for any $q \in H^1(\Omega)$.*

By applying the Theorem of Lax-Milgran, the solution $\widetilde{c}$ of the problem (0.3) exists and it is unique. So (P_2) has unique solution.

Remark 4 *Elliptic regularity Theorem remains valid provided that the boundary condition g is in the space $L^{\frac{3}{2}}(\Gamma)$ which is the image by the operator trace space $H^2(\Omega)$.*

Remark 5 [6] *If $c \in H^2(\Omega)$ and (c is a solution of problem (P_2)) this implies that $c \in W^{1,q}(\Omega)$ $(H^2(\Omega) \hookrightarrow W^{1,q}(\Omega)$ for $1 \le q \le 2^*)$.*

Using the Maximum Principle one can show that the solution of the problem (P_2) is positive as follows. Multiplying the first equation of (P_2) by $q \in H_0^1(\Omega)$, we obtain other variational formulat for problem (P_2)

$$\left(\widetilde{P}_3\right) \int_\Omega (\nabla c \nabla q + \tau c q)\, dx = 0.$$

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Proposition 6 [1] *If* $g \in L^{\frac{3}{2}}(\Gamma)$ *and* $c \in H^1(\Omega) \cap C(\overline{\Omega})$ *then the problem* $\left(\widetilde{P}_3\right)$ *have a positive solution* c.

Proof. As Γ is smooth enough and $g \in L^{\frac{3}{2}}(\Gamma)$ then $c \in H^2(\Omega)$. And as $\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$, by embedding of Sobolev spaces ($H^2(\overline{\Omega}) \hookrightarrow C(\overline{\Omega})$) this implies that $c \in C(\overline{\Omega})$. If $c = g \geq 0$ on Γ, then $c^- = \min(c, 0) \in H_0^1(\Omega)$. So, we have

$$\begin{aligned} \int_\Omega cc^- dx &= \int_\Omega (c^-)^2 dx \\ \int_\Omega \nabla c \nabla c^- dx &= \int_\Omega (\nabla c^-)^2 dx, \end{aligned}$$

Since the support of functions $c^-$and $c^+ = \max(c, 0)$ is set $A(x) = \{x/u(x) = 0\}$. This implies that $\nabla u = 0$ on $A(x)$. As $c = c^+ + c^-$, thus we have

$$0 = \int (\nabla c^-)^2 + \tau (c^-)^2 dx \geq \min(1, \tau) \left\|c^-\right\|^2_{H_0^1(\Omega)}$$

Finally, we find $c^- = 0$.

b) *Existence of a weak solution of the problem (P1)*

Before proving the existence and uniqueness of weak solution of problem (P_1), we need the following lemma

Lemma 7 *i) For all* $v \in H_0^1(\Omega)$ *then* $B(.,.,t)$ *is continuous in* $H_0^1(\Omega) \times H_0^1(\Omega)$, *there exists a constant positive* C *such that*

$$|B(u, v, t)| \leq C \|u\|_{H^1(\Omega)} \|v\|_{H^1(\Omega)} \tag{1.1}$$

ii) For any $u \in H_0^1(\Omega)$ *and* H_2 *is hold. Then there exists a constant positive* β *such that*

$$\beta \|u\|^2_{H_0^1(\Omega)} \leq B(u, u, t)$$

Proof. i) We use the Cauchy-Shwarz inequality and $c \in H^2(\Omega) \hookrightarrow L^q(\Omega)$ for any $q \in \left[1, \frac{2n}{n-2}\right[$ with $n = 2$ or $n = 3$, we obtain i) as follows

$$\begin{aligned} B(u, v, t) &\leq \|\nabla u\|_{L^2(\Omega)} \|\nabla v\|_{L^2(\Omega)} + \|\nabla c\|_{L^4(\Omega)} \|u\|_{L^2(\Omega)} \|v\|_{L^4(\Omega)} \\ &\quad + \tau \|c\|_{L^4(\Omega)} \|u\|_{L^2(\Omega)} \|v\|_{L^4(\Omega)} \\ &\leq C \|u\|_{H^1(\Omega)} \|v\|_{H^1(\Omega)} \end{aligned}$$

ii) Making use of $-\Delta c + \tau c = 0$ the expression of $B(u, u, t)$ becomes

$$\begin{aligned} B(u, u, t) &= \int (\nabla u)^2 + \frac{\nabla c}{2} \nabla u^2 + \tau c u^2 dx \\ &= \int (\nabla u)^2 + \left(\tau c - \frac{\Delta c}{2}\right) u^2 dx \\ &= \int (\nabla u)^2 + \frac{1}{2} \tau c u^2 dx \geq \|\nabla u\|^2_{L^2(\Omega)}. \end{aligned}$$

Notes

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Finally, by Poincarre inequality yields

$$B(u,u,t) \geq \beta \|u\|^2_{H^1_0(\Omega)}.$$

To demonstrate the existence of weak solution of (P_1) via the method of Galerking, we assume $w_k = w_k(x)$ are smooth functions verifying

$$\{w_k\}_{k=1}^{\infty} \quad \text{is an orthogonal basis of } H^1_0(\Omega) \tag{1.2}$$

and

$$\{w_k\}_{k=1}^{\infty} \quad \text{is an orthonormal basis of } L^2(\Omega). \tag{1.3}$$

Consider a positive integer m. We will look for a function $u_m : [0\ T] \to H^1_0(\Omega)$ of the form

$$u_m(t) := \sum_{k=1}^{m} d_m^k(t)\, w_k \tag{1.4}$$

which satisfies

$$d_m^k(0) = (u_0, w_k) \tag{1.5}$$

and

$$\langle u'_m, w_k\rangle + B(u_m, w_k, t) = 0, \quad 0 \leq t \leq T \text{ and } \quad k = 1, ..., m \tag{1.6}$$

where $u' = u_t$ and here $(.,.)$ denotes the scalar product in $L^2(\Omega)$.

Theorem 8 *(construction of the approximate solution) For each integer m, there exists a unique function u_m of the form* (1.4) *satisfying* (1.5) *and* (1.6).

Proof. Assuming u_m has the structure (1.4). Substituting (1.4) into (1.5) and using (1.3) we obtained

$$d'^k_m(t) + \sum_{l=1}^{m} d^l_m B(w_l, w_k, t) = 0 \qquad 0 \leq t \leq T \ \text{ and } \ k = 1, ..., m \tag{1.7}$$

According to standard existence theory for ordinary differential equations, there exists a unique absolutely continuous functions $d_m(t) = \left(d^1_m, d^2_m, ..., d^m_m\right)$ satisfying (1.5) and (1.7). So u_m of the form (1.4) satisfies (1.5) and (1.6) for all $t \in [0\ T]$.

c) *Energy estimates*

We propose now to send m to infinity and show a subsequence of our solutions u_m of the approximation problems (1.5) and (1.6) converges to a weak solution of (P_1). For this we will need some uniform estimates.

Theorem 9 (*Energy estimates*) [6]. *There exists a constant C, depending only on Ω, T and c, such that*

$$\max_{0\leq t\leq T} \|u_m\|_{L^2(\Omega)} + \|u_m\|_{L^2\left(0,T;\ H^1_0(\Omega)\right)} + \|u'_m\|_{L^2(0,T;\ H^{-1}(\Omega))} \leq C\, \|u_0\|_{L^2(\Omega)} \tag{1.8}$$

for $m = 1, 2, ...$

Proof. 1. Multiplying equation (1.6) by $d^k_m(t)$, summing for $k = 1, ..., m$, and then recalling (1.4) we find

$$(\acute{u}_m, u_m) + B(u_m, u_m, t) = 0 \tag{1.9}$$

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for all $0 \leq t \leq T$. From Lemma 7, there exists constant $\beta > 0$ such that

$$\beta \left\| u_m \right\|^2_{H^1_0(\Omega)} \leq B\left(u_m, u_m, t\right) \tag{1.10}$$

for all $0 \leq t \leq T$. Consequently (1.10) yields the inequality

$$\frac{d}{dt}\left(\left\| u_m \right\|^2_{L^2(\Omega)}\right) + \beta \left\| u_m \right\|^2_{H^1_0(\Omega)} \leq 0 \quad \text{for all } 0 \leq t \leq T. \tag{1.11}$$

This implies that

$$\left\| u_m \right\|^2_{L^2(\Omega)} \leq \left\| u_m\left(0\right) \right\|^2_{L^2(\Omega)} \leq \left\| u_0 \right\|^2_{L^2(\Omega)} \quad \text{for all } 0 \leq t \leq T. \tag{1.12}$$

So we have

$$\max_{0 \leq t \leq T} \left\| u_m \right\|_{L^2(\Omega)} \leq \left\| u_0 \right\|_{L^2(\Omega)}. \tag{1.13}$$

2. Integrate inequality (1.11) from 0 to T and we employ the inequality (1.13) to find

$$\left\| u_m \right\|^2_{L^2\left(0,T;\ H^1_0(\Omega)\right)} = \int_0^T \left\| u_m \right\|^2_{H^1_0(\Omega)} dt \leq C \left\| u_0 \right\|^2_{L^2(\Omega)}.$$

3. Fix any $v \in H^1_0\left(\Omega\right)$, with $\left\| v \right\|^2_{H^1_0(\Omega)} \leq 1$, and write $v = v^1 + v^2$, where $v^1 \in span\left(w_k\right)_{k=1}^{k=m}$, and $\left(v^2, w_k\right) = 0$ $(k = 1, ..., m)$. We use (1.6), we deduce for all $0 \leq t \leq T$ that

$$\left(u'_m, v^1\right) + B\left(u_m, v^1, t\right) = 0.$$

Then (1.4) implies

$$\left\langle u'_m, v \right\rangle = \left(u'_m, v\right) = \left(u'_m, v^1\right) = -B\left(u_m, v^1, t\right),$$

consequently

$$\left| \left\langle u'_m, v \right\rangle \right| \leq C \left\| u_m \right\|_{H^1_0(\Omega)}.$$

Since $\left\| v^1 \right\|^2_{H^1_0(\Omega)} \leq \left\| v \right\|^2_{H^1_0(\Omega)} \leq 1$. Thus

$$\left\| u'_m \right\|_{H^{-1}(\Omega)} \leq C \left\| u_m \right\|_{H^1_0(\Omega)},$$

and therefore

$$\left\| u'_m \right\|^2_{L^2(0,T;\ H^{-1}(\Omega))} = \int_0^T \left\| u'_m \right\|^2_{H^{-1}(\Omega)} dt \leq C \int_0^T \left\| u_m \right\|^2_{H^1_0(\Omega)} dt \leq C \left\| u_0 \right\|^2_{L^2(\Omega)}.$$

d) Existence and uniqueness

Next we pass to limits as $m \to \infty$, to build a weak solution of our initial boundary-value problem (P_1).

Theorem 10 *(Existence of weak solution). Under hypothesis H_2 and H_3, there exists a weak solution of (P_1).*

Proof. *1. According to the energy estimates (1.8), we see that the sequence $\{u_m\}_{m=1}^{\infty}$ is bounded in $L^2\left(0,T;H^1_0\left(\Omega\right)\right)$ and $\{u'_m\}_{m=1}^{\infty}$ is bounded in $L^2\left(0,T;H^{-1}\left(\Omega\right)\right)$. Consequently there exists a subsequence which is also noted by $\{u_m\}_{m=1}^{\infty}$ and a function $u \in L^2\left(0,T;H^1_0\left(\Omega\right)\right)$, with $u' \in L^2\left(0,T;H^{-1}\left(\Omega\right)\right)$, such that*

$$\begin{aligned} u_m &\rightharpoonup u \quad \text{weakly in } L^2\left(0,T;H^1_0\left(\Omega\right)\right) \\ u'_m &\rightharpoonup u' \quad \text{weakly in } L^2\left(0,T;H^{-1}\left(\Omega\right)\right). \end{aligned} \tag{1.14}$$

2. Next fix an integer N and choose a function $v \in C^1\left(0,T;H^1_0\left(\Omega\right)\right)$ having the form

$$v\left(t\right) = \sum_{k=1}^{N} d^k\left(t\right) w_k \tag{1.15}$$

Notes

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Notes

where $\{d^k\}_{k=1}^N$ *are given smooth functions. We choose* $m \geq N$*, multiply equation* (1.6) *by* $d^k(t)$*, sum for* $k = 1, ..., N$*, and then integrate with respect to* t *to find*

$$\int_0^T \langle u_m', v\rangle + B(u_m, v, t)\, dt = 0. \tag{1.16}$$

We recall (1.14) *to find upon passing to weak limits that*

$$\int_0^T \langle u', v\rangle + B(u, v, t)\, dt = 0 \qquad \forall v \in L^2\left(0, T; H_0^1(\Omega)\right). \tag{1.17}$$

As functions of the form (1.15) *are dense in* $L^2\left(0, T; H_0^1(\Omega)\right)$*. Hence in particular*

$$\langle u', v\rangle + B(u, v, t)\, dt = 0 \qquad \forall v \in H_0^1(\Omega) \text{ and } \forall t \in [0\ T], \tag{1.18}$$

and from Remark 2 we have $u \in C\left(0, T; L^2(\Omega)\right)$.
3. In order to prove $u(0) = u_0$*, we first note from* (1.17) *that*

$$\int_0^T -\langle u, v'\rangle + B(u, v, t)\, dt = (u(0), v(0)) \tag{1.19}$$

for each $v \in C^1\left(0, T; H_0^1(\Omega)\right)$ *with* $v(T) = 0$*. Similary, from* (1.16) *we deduce*

$$\int_0^T -\langle u_m, v'\rangle + B(u_m, v, t)\, dt = (u_m(0), v(0)). \tag{1.20}$$

We use again (1.14)*, we obtain*

$$\int_0^T -\langle u, v'\rangle + B(u, v, t)\, dt = (u_0, v(0)), \tag{1.21}$$

since $u_m(0) \to u_0$ *in* $L^2(\Omega)$*. Comparing* (1.19) *and* (1.21)*, we conclude* $u(0) = u_0$.

Theorem 11 *(Uniqueness of weak solutions) A weak solution of* (P_1) *is unique.*
Proof. *We suppose there exists two weak solution* u_1 *and* u_2*. We put*

$$U = u_2 - u_1$$

then U *is also a solution of* (P_1)*with* $U_0 = (u_2 - u_1)(0) \equiv 0$*. Setting* $v = U$ *in identity* (1.18) *we have*

$$\frac{d}{dt}\left(\frac{1}{2}\|U\|_{L^2(U)}^2\right) + B(U, U, t) = 0.$$

From Lemma 7 we have $B(U, U, t) \geq \beta \|U\|_{H_0^1(U)}^2 \geq 0$*, so* $\frac{d}{dt}\left(\frac{1}{2}\|U\|_{L^2(U)}^2\right) \leq 0$*, then integrate with respect to* t *to find*

$$\|U\|_{L^2(\Omega)}^2 \leq \|U_0\|_{L^2(\Omega)}^2 = 0,$$

thus $U \equiv 0$.

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e) Global solution of problem (P)

Our main results in this paper are stated as follows.

Theorem 12 *i) if $c \geq c_0 > 0$. Then la solution (u, c) of problem (P) is global*
ii) if $c \geq c_0 > 0$. Then la solution (u, c) of problem (P) is global. Furthermore there exists $\tau_0 > 0$ such that $\|u\|_{L^2} \leq e^{-\tau_0 t} \|u_0\|_{L^2}$.

Proof. We put

$$E(t) = \frac{1}{2} \int_\Omega u^2 dx \tag{1.22}$$

We derivate the equation (1.22) and we use firsts equations of (P_1) and (P_2) to find
i) We have

$$\frac{dE}{dt} = -B(u, u, t) \leq 0,$$

therefore

$$E(t) \leq E(0).$$

ii) We have

$$\frac{dE}{dt} = -B(u, u, b, t) = -\int \left((\nabla u)^2 + \frac{1}{2} \tau c u^2 \right) dx \leq \frac{-1}{2} \tau c_0 \|u\|^2_{L^2(\Omega)} = -\tau_0 E(t).$$

This implies that

$$E(t) \leq E(0) e^{-\tau_0 t}.$$

Proposition 13 [1] *Let $u_0 \in L^2(\Omega)$ and $u \in C([0, T]; L^2(\Omega)) \cap L^2(0, T; H_0^1(\Omega))$ is the unique weak solution of (P_1). If $u_0 \geq 0$ in Ω, then $u \geq 0$ in $]0, T[\times \Omega$.*
Proof. *If $u_0 \geq 0$ on Γ. Therefore $u^- = \min(u, 0) \in L^2(]0, T[; H_0^1(\Omega))$. A reasoning similar to the Proposition 6, we obtain for all $0 \leq t \leq T$*

$$\frac{1}{2} \frac{d}{dt} \int_\Omega (u^-)^2 dx + \int_\Omega B(u^-, u^-, t) dx = 0.$$

Using the Lemma 7 and integrating with respect to τ from 0 to t, we get

$$\frac{1}{2} \int_\Omega (u^-)^2 dx + \beta \int_0^t \|u(s)\|^2_{H_0^1(U)} ds \leq \frac{1}{2} \int_\Omega (u^-(0))^2 dx = 0.$$

Since $u^-(0) = (u_0)^- = 0$. So $u^- = 0$.

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Effect of Variable Thermal Conductivity & Heat Source/Sink Near a Stagnation Point on a Linearly Stretching Sheet using HPM

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GJSFR-F Classification : MSC 2010: 00A69

EFFECTOFVARIABLETHERMALCONDUCTIVITYHEATSOURCESINKNEARASTAGNATIONPOINTONALINEARLYSTRETCHINGSHEETUSINGHPM

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Effect of Variable Thermal Conductivity & Heat Source/Sink Near a Stagnation Point on a Linearly Stretching Sheet using HPM

Vivek Kumar Sharma [α] & Aisha Rafi [σ]

Abstract- Aim of the paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet. The equations of continuity, momentum and energy are transformed into ordinary differential equations and solved numerically using Similarity transformation and Homotopy Perturbation Method. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived, discussed numerically and their numerical values for various values of physical parameter are presented through Tables.

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I. Introduction

Study of heat transfer in boundary layer find applications in extrusion of plastic sheets, polymer, spinning of fibers, cooling of elastic sheets etc. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. Liquid metals have small Prandtl number of order 0.01~ 0.1(e.g. Pr = 0.01 is for Bismuth, Pr = 0.023 for Mercury etc.) and are generally used as coolants because of very large thermal conductivity.

Aim of the present paper is to investigate effects of variable thermal conductivity, heat source/sink and variable free stream on flow of a viscous incompressible electrically conducting fluid and heat transfer on a non-conducting stretching sheet. Linear stretching of the sheet is considered because of its simplicity in modelling of the flow and heat transfer over stretching surface and further it permits the similarity solution, which are useful in understanding the interaction of flow field with temperature field. The heat source and sink is included in the work to understand the effect of internal heat generation and absorption [Chaim (1998)].

The Homotopy Perturbation Method is a combination of the classical perturbation technique and homotopy technique, which has eliminated the limitations of the traditional perturbation methods. This technique can have full advantage of the traditional perturbation techniques. J. H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media. J. H. He, A coupling method of homotopy technique and perturbation technique for nonlinear problems. To illustrate the basic idea of the Homotopy Perturbation Method for solving nonlinear differential equations, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0 \tag{1}$$

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Subject to boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \tag{2}$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytic function, and Γ is the boundary of the domain Ω.

The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Equation can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

By the homotopy technique, we construct a homotopy $V(r,p) : \Omega^*(0,1) \rightarrow R$ which satisfy

$$H(V,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{4}$$

$$H(V,p) = L(v) - L(u_0) + p\, L(u_0) + p[N(v) - f(r)] = 0 \tag{5}$$

Where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of which satisfies the boundary conditions.

$$H(V,0) = L(v) - L(u_0)$$

$$H(V,1) = A(v) - f(r)] \tag{6}$$

Thus, the changing process of p from zero to unity is just that of $v(r, p)$ from $u_0(r)$ to $u(r)$. In Topology, this is called deformation and $L(v) - L(u_0)$, $A(v) - f(r)$ are called homotopic. According to the HPM, we can first use the embedding parameter p as a "small parameter," and assume that the solution of can be written as a power series in p:

$$V = V_0 + pV_1 + p^2 V_2 + \ldots\ldots\ldots\ldots\ldots \tag{7}$$

Setting p=1 results in the approximate solution of

$$u = \lim_{p \rightarrow 1} V = V_0 + V_1 + \ldots\ldots\ldots\ldots\ldots\ldots \tag{8}$$

The series is convergent for most cases; however, the convergent rate depends upon the Nonlinear operator $A(V)$. The second derivative of $N(V)$ with respect to V must be small because the parameter may be relatively large; that is, $p \rightarrow 1$.

In this paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet.

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II. Formulation of the Problem

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity in the vicinity of a stagnation point on a non-conducting stretching sheet It is assumed that external field is zero, the electric field owing to polarization of charges and Hall Effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x-axis is taken along the sheet The fluid occupies the upper half plane i.e. $y > 0$. The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2}, \tag{10}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right) + Q\,(T - T_\infty)\,, \tag{11}$$

where ε -perturbation parameter, η-similarity parameter $\{ = (c/\nu)^{1/2}\, y\}$, η_∞-value of η at which boundary conditions is achived, κ-uniform thermal conductivity, κ^*-variable thermal conductivity, ν-kinematic viscosity, ρ-density of fluid, ψ-stream function, σ-electrical conductivity, θ-dimensionless temperature$\{ = (T - T_\infty) / (T_w - T_\infty)\}$, τ_w-shear stress, S-heat source/sink parameter $\{= Q/\rho\, C_p\, c\}$, T-fluid temperature.

The second derivatives of u and T with respect to x have been eliminated on the basis of magnitude analysis considering that Reynolds number is high. Hence the Navier-Stokes equation modifies into Prandtl's boundary layer equation.
The boundary conditions are.

$$\left.\begin{array}{lll} y = 0: & u = u_w(x) = c \quad v = 0, & T = T_w \\ y \to \infty: & u = U(x) = bx\,, & T = T_\infty \end{array}\right\} \tag{12}$$

Introducing the stream function $\psi\,(x, y)$ as defined by

$$u = \frac{\partial \psi}{\partial y}\,, \qquad v = -\frac{\partial \psi}{\partial x}, \tag{13}$$

the similarity variable $\eta = (c/\nu)^{1/2}\, y$ and

$$\Psi\,(x, y) = (c\,\nu)^{1/2}\; x\, f(\eta), \tag{14}$$

into the equations (3) and (5), we get

$$f''' + ff'' - (f')^2 + \lambda^2 = 0\,, \tag{15}$$

And

$$(1+\varepsilon)\,\theta'' + \varepsilon\,(\theta')^2 + \Pr\theta' f + \Pr S\theta = 0.$$

The governing boundary layer and thermal boundary layer equations (15) and (16) with the boundary conditions (17) are solved using Homotopy Perturbation Method.

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Equations (15) and (16) are non-linear coupled differential equation. To solve these equations, we introduce the following Homotopy.

$$D(f,p) = (1-p)\left[\left(\frac{d^3f}{d\eta^3}\right) - \left(\frac{d^3f_I}{d\eta^3}\right)\right] + p\left[\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} - \left(\frac{df}{dn}\right)^2 + \lambda^2\right] = 0 \quad (16)$$

$$D(\theta,p) = (1-p)\left[\left(\frac{\partial^2\theta}{\partial\eta^2} + P_rS\theta\right) - \left(\frac{\partial^2\theta_I}{\partial\eta^2} + P_rS\theta_I\right)\right]$$

$$+P\left[(1+\varepsilon\theta)\frac{\partial^2\theta}{\partial n^2} + \varepsilon\left(\frac{\partial\theta}{dn}\right)^2 + P_r\frac{\partial\theta}{\partial n}f + P_rS\theta\right] = 0 \quad (17)$$

With the following assumption

$$f = f_0 + pf_1 + p^2f_2 + \cdots \quad (18)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 \quad (19)$$

Using equation (18),(19) into equation (10) and (11) and on comparing the like powers of p, we get the zeoth order equation,

$$\left[\left(\frac{d^3f_0}{d\eta^3}\right) - \left(\frac{d^3f_I}{d\eta^3}\right)\right] + \left[\frac{d^3f_0}{d\eta^3} + f_0\frac{d^2f_0}{d\eta^2} - \left(\frac{df_0}{dn}\right)^2 + \lambda^2\right] = 0, \quad (21)$$

$$\left[\left(\frac{\partial^2\theta_0}{\partial\eta^2} + P_rS\theta_0\right) - \left(\frac{\partial^2\theta_I}{\partial\eta^2} + P_rS\theta_I\right)\right] + \left[(1+\varepsilon\theta_0)\frac{\partial^2\theta_0}{\partial n^2} + \varepsilon\left(\frac{\partial\theta_0}{dn}\right)^2 + P_r\frac{\partial\theta_0}{\partial n}f + P_rS\theta_0\right]$$

$$= 0, \quad (21)$$

with the corresponding boundary conditions are of zeroth order equations are:

$$\eta = 0:\ f_0 = 0,\ f_0' = 1, \theta_0 = 1\ ; \quad (22)$$

$$\eta = \infty:\quad f_0' = \lambda, \qquad \theta_0 = 0 \quad (23)$$

$$\frac{d^3f_1}{d\eta^3} + (e^{-n} - \lambda e^{-\eta}) + (\lambda\eta - e^{-n} + \lambda e^{-n} - \lambda + 1)(\lambda e^{-\eta} - e^{-\eta}) - (\lambda + e^{-n} - \lambda e^{-n})(\lambda + e^{-n} - \lambda e^{-n}) + \lambda^2 = 0 \quad (24)$$

$$\frac{\partial^2\theta_I}{\partial\eta^2} + P_rS\theta_I + e^{-\eta}(1+\varepsilon e^{-\eta}) + \epsilon\, e^{-2\eta} - P_re^{-\eta}(\lambda\eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1) - P_rSe^{-\eta} = 0 \quad (25)$$

With the corresponding boundary conditions are of first order equations are:

$$\left.\begin{aligned} \boldsymbol{\eta = 0}:\ \boldsymbol{f_0 = 0},\ \boldsymbol{f_0' = 0}, \boldsymbol{\theta_0 = 0} \\ \boldsymbol{\eta} = \infty:\quad \boldsymbol{f_0' = 0}, \qquad \boldsymbol{\theta_0 = 0}\ ; \end{aligned}\right\} \quad (26)$$

Solving equations with corresponding boundary conditions, the following functions can be obtained successively, by summing up the results, and $p \to 1$ we write the f($\boldsymbol{\eta}$) , $\boldsymbol{\theta}(\boldsymbol{\eta})$, profile as:

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$$\boldsymbol{f(\eta) = \lambda\eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1 + (\eta e^{-\eta} + 4e^{-\eta} + 3\eta - 4)} \tag{27}$$

$$\theta(\eta) = e^{-\eta} + \frac{A_2 e^{-\eta}}{(A_1)^2+1} + \frac{A_3 e^{-2\eta}}{(A_1)^2+4} + \frac{A_4 e^{-\eta}}{(A_1)^2}\left(\eta - \frac{\eta-2}{(A_1)^2}\right) - \left(\frac{2A_4}{(A_1)^4} + \frac{A_3}{(A_1)^2+4} + \frac{A_2}{(A_1)^2+1}\right)e^{-\eta} \tag{28}$$

Notes

where $A_1 = \sqrt{P_r S}$: $A_2 = P_r S + P_r - \lambda P_r - 1$: $A_3 = \lambda P_r - P_r - 2$: $A_4 = \lambda P_r$ (29)

Skin-Friction: Skin-friction coefficient at the sheet is given by

$$C_f = x f''(0) \tag{30}$$

Nusselt Number: The rate of heat transfer in terms of the Nusselt number at the sheet is given by

$$Nu = -\theta'(0) \tag{31}$$

III. Conclusion

It is observed from Table 1 as L increases, the numerical values of $f''(0)$ also increase. It is noted from Table 2 that the numerical values of -θ ′(0) increase when λ increases and -θ ′(0) decreases when ε increases. The skin-friction coefficient and Nusselt number are presented by equations (30) and (31) and they are directly proportional to $f''(0)$ and $-\theta'(0)$ respectively. The effects of ε, Pr and S on Nusselt number have been presented through Table 3 respectively.

Table 1

λ	f''(0)	λ	f''(0)
0.0	-1	0.1	-1.0800
0.01	-1.0098	1.0	0.0004
0.05	-1.0450	2.0	2.0175

Table 2

λ	$-\theta'(0)$	ε	$-\theta'(0)$
0.1	.81235	0.0	0.223558
0.5	.13629	0.05	0.215792
2.0	.24133	0.1	0.204672

Table 3

ε	S	P_r	Nu
0	0	0.5	2.010357
0.1	0	0.5	3.542315
0	-0.1	1	2.565423
0.1	-0.1	1	2.845633

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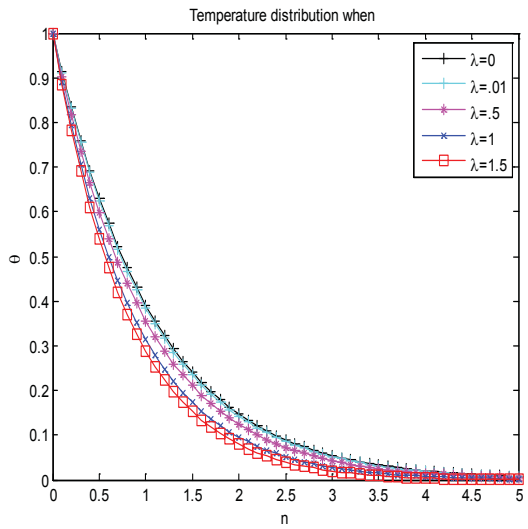


Figure 1

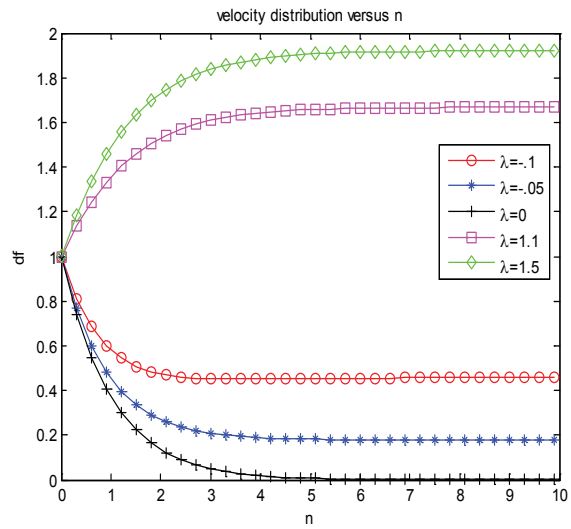


Figure 2

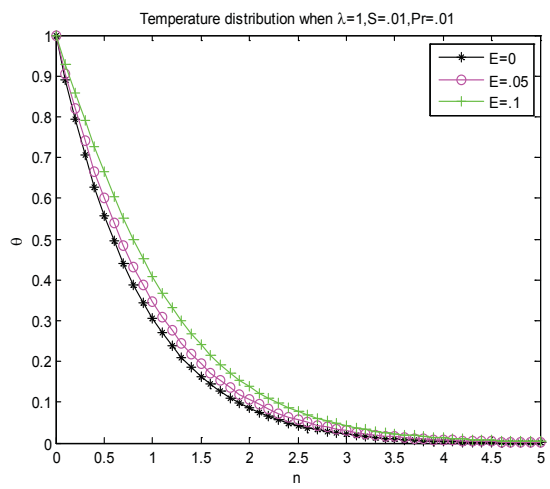
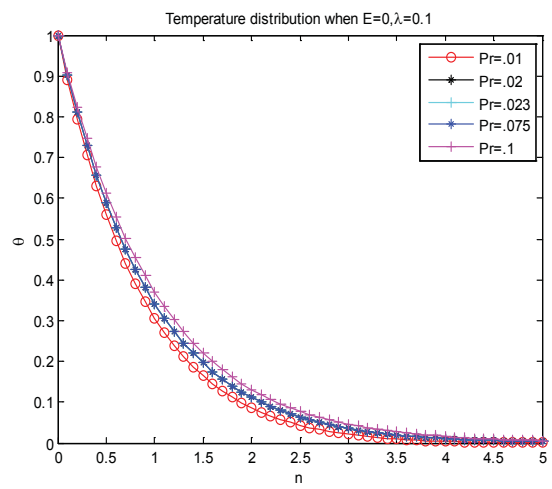


Figure 3



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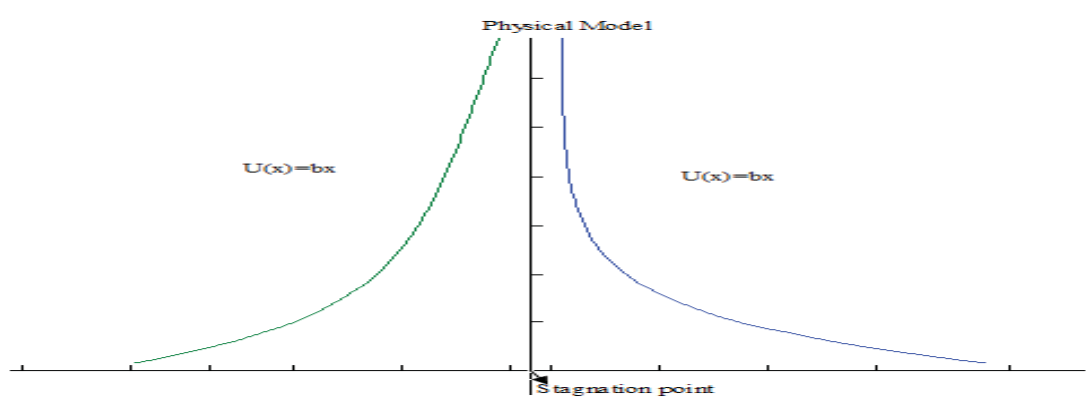

Figure 5

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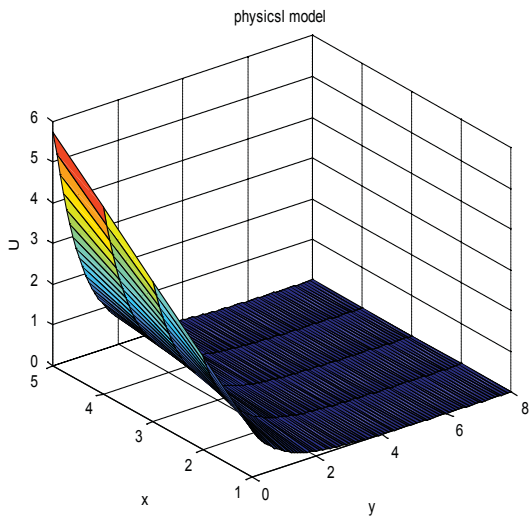

Figure 6

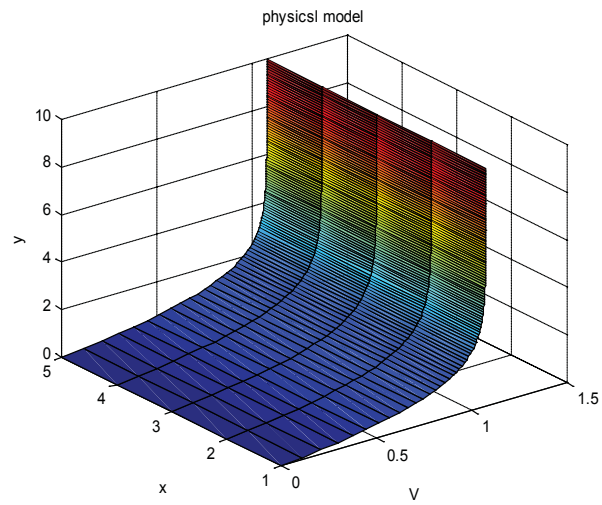

Figure 7

From figure 1, we observe that as λ increases, value of f' also increases. From figure 2 it is observed that when λ increase simultaneously θ also increases. In figure3, λ, S and Pr are constant but when ε increases θ will also increased.It is observed in figure 4, s, ε and λ are constant, when Pr increases, θ will also increase. Figure 5 is a physical model which becomes clearer from figure, 6 and 7.

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Homeotopy Groups of 2–Dimensional Manifolds with One Boundary Component

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Introduction- Let Y be a compact, connected 2—dimensional manifold with boundary. The homeotopy group of Y, denoted H(Y), is defined to be the group of isotopy classes in the space of all homeomorphisms of Yonto Y. This group (also known as the mapping class group) has been studied for various manifolds (see, for example, [2] and [3]). It is also possible to consider "subhomeotopy groups" where there are restrictions placed on the action of the homeomorphisms on the boundary of Y (see, for example, [7] and [8]). In this note we will consider the special case of a compact, connected manifold with exactly on boundary component. For the remainder of this paper we will assume Y represents a compact, connected manifold with exactly one boundary component and we willlet X denote the closed 2—manifold obtained by sewing a disk to the boundary of Y. Let Aut $\pi_1(X,x_0)$ denote the group of automorphisms of $\pi_1(X,x_0)$ where $x_0 \varepsilon$— Bd(Y). In this paper we establish the following result.
Theorem. If Y is not aMoebius band or a disk, then H(Y)=Aut π_1 (X, x0).

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HOMEOTOPYGROUPSOF2DIMENSIONALMANIFOLDSWITHONEBOUNDARYCOMPONENT

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Theorem. If Y is not aMoebius band or a disk, then H(Y) =Aut π_1 (X, x_0)

II. Proof of the Theorem

Let $[(X, x_0),(X,x_0)]$ denote the set of homotopy classes (rel x_0) of maps from (X, x_0) to (X,x_0) and let [f] denote the homotopy class (rel x_0) of a mapping f from (X,x_0) to (X,x_0). Let End π_1 (X, x_0) denote the set of endomorphisms of $\pi_1(X,x_0)$ with the operation of composition.

<u>Lemma 1</u>. If X is a closed 2-manifo1d with $\pi_2(X, x_0) = 0$ and $\phi : [(X, x_0), (X, x_0)] \rightarrow$ End $\pi_1(X,x_0)$ is given by ϕ ([f]) =f_* , then ϕ is a bijection which preserves the operation of composition.

Proof of lemma 1.

1. Clearly ϕ is well defined and $\phi([f \circ g]) = \phi([f]) \circ \phi([g])$
2. Claim ϕ is a surjection.

Let F in END $\pi_1(X,x_0)$. We define f: $(X,x_0) \rightarrow (X, x_0)$ as follows:

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7. D. Sprows, "Local sub homotopy groups of bounded surfaces" International Journal of Math & Math Sci. 24 (2000), 251-255.

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Take x_0 to be vertex of the triangulation of X and let T be a maximal tree in X. If $x \in T$, we let $f(x) = x_0$. Now, suppose s is a 1—simplex not in T with h: [0,1] = s. Let Γ_i, be a path in T from x_0 to h(i), i = 0, 1. Define the loop α_s at x_0 by letting $\alpha_s(t) = \Gamma_0(t)$ if $-1 \le t \le 0$, $\alpha_s(t) = h(t)$ if $0 \le t \le 1$, and $\alpha_s(t) = \Gamma_0^{-1}(t)$ if $1 \le t \le 2$. If $F[\alpha_s] = [\beta]$, we let $f/s = \beta \circ h^{-1}s$. This defines f on the 1-skeleton of X.

Finally, if Δ is a 2-simplex with edges s1, s2, and s3, then $[\alpha_{S1} * \alpha_{S2} * \alpha_{S3}] = 1$. This means $f/\delta\Delta$ is null homotopic, i.e., f extends to Δ. Hence, the mapping of f defined on the 1-skeleton as above, extends to a mapping defined on all of the 2-dimensional manifold X. Note that by construction, $f_*[\alpha_s] = F[\alpha_s]$ and since $\{[\alpha_s]$: S is a 1-simplex of X″ generates $\pi_1(X,x_0)$ we have $f_* = F$.

III. Claim ϕ is an Injection

Suppose $f_* = g_*$. As in Part 2), let T be a maximal tree. f/T is homotopic (rel x_0) to a map which sends T to x_0 (just use the retraction of T to x_0). Hence, by the homotopy extension property, f is homotopic (rel x_0) to a map f' with $f'(T) = x_0$. Therefore, we can assume $f(T) = g(T) = x_0$. In particular, for each 1-simplex s, f/s and g/s are loops at x_0. Since $f_* = g_*$ this means f/s is homotopic to g/s (rel x_0). Thus, for each 2—simplex Δ we have a map H: $\delta(\Delta x I) \to X$ where $H/\Delta x0 = f/\Delta$, $H/\Delta xl = f/\Delta$ and for each 1-simplex s in δΔ, H/SxI is a homotopy from f/s to g/s. Since $\pi 2(X,x_0) = 0$ and $\delta(\Delta x I) = S^2$, this map H can be extended to all of ΔxI. Fitting together each of these H's we get a homotopy (rel x_0) from f to g, i.e., [f] = [g].

Lemma 2 : If X is a closed 2—manifold and $h:(X,x_0) \to (X,x_0)$ is a homeomorphism which is homotopic to the identity (rel x_0) then h is isotopic to the identity (rel x_0).

Proof. This is a special case of Theorem 6.3 of [1].

Lemma 3. If X is a closed 2—manifold with $x_0 \in X$ and G is an automorphism of $\pi 1(X,x_0)$, then there exists a homeomorphism h: $(X,x_0) + (X,x_0)$ with $h_* = G$.

Proof. This result is proved in [4].

Let $H(X,x_0)$ denote the group of isotopy classes (rel x_0) in the space of all homeomorphisms of (X,x_0) onto (X,x_0).

By Lemma 2 the function from $H(X,x_0)$ to $[(X,x_0)]$ which sends the isotopy class (rel x_0) of a homeomorphism to its homotopy class (rel x_0) is an injection. By Lemma 1 the composition $H(X,x_0) \to [(X,x_0)\ ,\ (X,x_0)] \to \text{End}\ \pi 1(X,x_0)$ is a monomorphism of the group $H(X,x_0)$ onto a subgroup of Aut $\pi 1(X,x_0)$. Finally Lemma 3 shows that this monomorphism is an isomorphism onto Aut $\pi_1(X,x_0)$. Thus the proof of the theorem reduces to showing that H(Y) is isomorphic to $H(X,x_0)$. This is done by the following lemma.

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Lemma 4. If Y is a compact, connected 2—dimensional manifold with one boundary component and X is the closed 2—manifold obtained by sewing a disk to the boundary of Y and $x_0 \in X\text{-}Bd(Y)$then $H(X,x_0) = H(Y)$.

Proof. This result is a special case of Theorem 6 of [6].

Remark 3. If Y is a Moebius band or disk, then H(Y) is not isomorphic to Aut $\pi 1(X,x0)$. In the case Y is a disk, so that $X = S^2$, we have Aut $\pi 1\ (X,x_0) = 1$ while $H(Y) = Z_2$(see Theorem 4.2 of [4]). In the case Y is a Moebius band, so that $X = P^2$, we have Aut $\pi 1$, $(X,x_0) = 1$ while $H(Y) = Z_2$ (see Theorem 8.1 of[4]).

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Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling

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Abstract- In the present study, we have proposed a class of ratio-cum-product estimators for estimating finite population mean of the study variable in two phase sampling. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Comparison of the proposed class of estimators with other estimators is also worked out theoretically to demonstrate the superiority of the proposed estimator over the other estimators.

Keywords: ratio-cum-product, mean, two phase sampling, asymptotically optimum estimator, bias, mean square error.

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Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling

Waikhom Warseen Chanu [α] & B. K. Singh [σ]

Abstract- In the present study, we have proposed a class of ratio-cum-product estimators for estimating finite population mean $\bar{Y}$ of the study variable y in two phase sampling. The bias and mean square error of the proposed estimator have been obtained. The asymtotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Comparison of the proposed class of estimators with other estimators is also worked out theoretically to demonstrate the superiority of the proposed estimator over the other estimators.

Keywords: ratio-cum-product, mean, two phase sampling, asymptotically optimum estimator, bias, mean square error.

I. Introduction

The literature on survey sampling describes a great variety of techniques for using auxiliary information in order to obtain improved estimators for estimating some most common population parameter such as population total, population mean, population proportion, population ratio etc. More often we are interested in the estimation of the mean of a certain characteristic of a finite population on the basis of a sample taken from the population following a specified sampling procedure.

Use of auxiliary information has shown its significance in improving the efficiency of estimators of unknown population parameters. Cochran (1940) used auxiliary information in the form of population mean of auxiliary variate at estimation stage for the estimation of population parameters when study and auxiliary variate are positively correlated. In case of negative correlation between study variate and auxiliary variate, Robson (1957) defined product estimator for the estimation of population mean which was revisited by Murthy (1967). Ratio estimator performs better than simple mean estimator in case of positive correlation between study variate and auxiliary variate.

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For further discussion on ratio cum product estimator, the reader is referred to Singh (1967), Shah and Shah (1978), Singh and Tailor (2005), Tailor and Sharma (2009), Tailor and Sharma (2009), Sharma and Tailor (2010), Choudhury and Singh (2011) etc.

Notes

When the population mean $\bar{X}$ of the auxiliary variable x is unknown before start of the survey, it is estimated from a preliminary large sample on which only the auxiliary characteristic x is observed. The value of X in the estimator is then replaced by its estimate. After then a smaller second-phase sample of the variate of interest (study variate) y is then taken. This technique is known as double sampling or two-phase sampling. Neyman (1938) was the first to give the concept of double sampling in connection with collecting information on the strata sizes in a stratified sampling

Consider a finite population $U = (u_1, u_2, u_3, ..., u_N)$ of size N units, y and x are the study and auxiliary variate respectively. When the population mean $\bar{X}$ of x is not known, a first phase sample of size n_1 is drawn from the population on which only the x characteristic is measured in order to furnish a good estimate of $\bar{X}$. After then a second-phase sample of size n $(n < n_1)$ is drawn on which both the variates y and x are measured.

The usual ratio and product estimators in double sampling are:

$$\bar{Y_R}^d = \bar{y}\frac{\bar{x_1}}{\bar{x}}$$

and

$$\bar{Y_P}^d = \bar{y}\frac{\bar{z}}{\bar{z_1}}$$

where $\bar{x}, \bar{y}$ and $\bar{z}$ are the sample mean of x, y and z respectively based on the sample of size n out of the population N units and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ and $\bar{z_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} z_i$ denote the sample mean of x and z based on the first- phase sample of the size n_1.

Singh (1967) improved the ratio and the product methods of estimation by studying the ratio cum product estimator for estimating $\bar{Y}$ as

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$$\bar{Y}_{RP} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\frac{\bar{z}}{\bar{Z}}\right)$$

Notes

Motivated by Singh (1967), Choudhury and Singh (2011) proposed a modified class of ratio cum product type of estimator for estimating population mean $\bar{Y}$ as

$$\bar{Y}_{RP}^{(\alpha)} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\frac{\bar{z}}{\bar{Z}}\right)^{\alpha}$$

Motivated by **?**) and as an extension to the work of Choudhury and Singh (2011), we have developed an improved class of ratio-cum-product estimators in double sampling to estimate the population mean $\bar{Y}$ theoretically and studied the properties of the proposed estimator.

II. The Proposed Estimator

The proposed improved class of ratio-cum-product estimators of population mean $\bar{Y}$ in two-phase sampling is given as

$$\bar{Y}_{RP}^{w(d)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}.\frac{\bar{z}}{\bar{z}_1}\right)^{\alpha}. \tag{1}$$

where α is a suitably chosen constant.

To obtain the bias and MSE of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation, we write

$$e_0 = \left(\bar{y} - \bar{Y}\right)/\bar{Y}, \quad e_1 = \left(\bar{x} - \bar{Y}\right)/\bar{X}, \quad e_2 = \left(\bar{x}_1 - \bar{X}\right)/\bar{X}, \quad e_3 = \left(\bar{z} - \bar{Z}\right)/\bar{Z},$$
$$e_4 = \left(\bar{z}_1 - \bar{Z}\right)/\bar{Z}.$$

Expressing $Y_{RP}^{\bar{w}(d)}$ in terms of e's and neglecting higher power of e's, we have

$$Y_{RP}^{\bar{w}(d)} = \bar{Y}\left(1+e_0\right)\left\{\left(1+e_2\right)\left(1+e_1\right)^{-1}\left(1+e_3\right)\left(1+e_4\right)^{-1}\right\}^{\alpha}.$$

Assuming the sample size to be large enough so that $|e_1| < 1$, $|e_4| < 1$ and expanding $(1+e_1)^{-1}$, $(1+e_4)^{-1}$ in powers of e_1, e_4, multiplying out and neglecting higher powers of $e's$, we have

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$$
\begin{aligned}
Y_{RP}^{\bar{w}(d)} =& \bar{Y}\left(1+e_0\right)\left[1-\left\{e_1-e_2-e_3+e_4-e_1^2-e_4^2+e_1e_2-e_1e_4\right.\right.\\
&\left.\left.+e_2e_4+e_1e_3-e_2e_3+e_3e_4\right\}\right]^{\alpha}\\
\bar{Y}_{RP}^{w(d)}-\bar{Y} =& \bar{Y}\left[e_0-\alpha\left(e_1-e_2-e_3+e_4-\frac{e_1^2}{2}+\frac{e_2^2}{2}+\frac{e_3^2}{2}-\frac{e_4^2}{2}+e_0e_1\right.\right.\\
&\left.-e_0e_2+e_0e_4-e_0e_3\right)+\alpha^2\left(\frac{e_1^2}{2}+\frac{e_2^2}{2}+\frac{e_3^2}{2}+\frac{e_4^2}{2}-e_1e_2\right.\\
&\left.\left.-e_1e_3+e_1e_4+e_2e_3-e_2e_4-e_3e_4\right)\right] \qquad (2)
\end{aligned}
$$

The following two cases will be considered separately.

Case I: When the second phase sample of size n is subsample of the first phase of size n_1.

Case II: when the second phase sample of size n is drawn independently of the first phase sample of size n_1.

CASE I

III. Bias, mse and Optimum Value of $\bar{Y}_{RP}^{wd}$ in Case I

In this case, we have

$$
\begin{aligned}
&E\left(e_0\right)=E\left(e_1\right)=E\left(e_2\right)=E\left(e_3\right)=E\left(e_4\right)=0;\\
&E\left(e_0^2\right)=\left(\frac{1-f}{n}\right)C_Y^2;\quad E\left(e_1^2\right)=\left(\frac{1-f}{n}\right)C_X^2;\quad E\left(e_3^2\right)=\left(\frac{1-f}{n}\right)C_Z^2;\\
&E\left(e_2^2\right)=E\left(e_1e_2\right)=\left(\frac{1-f^*}{n}\right)C_X^2;\quad E\left(e_4^2\right)=E\left(e_3e_4\right)=\left(\frac{1-f^*}{n}\right)C_Z^2;\\
&E\left(e_0e_1\right)=\left(\frac{1-f}{n}\right)\rho_{YX}C_YC_X;\quad E\left(e_0e_2\right)=\left(\frac{1-f^*}{n}\right)\rho_{YX}C_YC_X;\\
&E\left(e_0e_3\right)=\left(\frac{1-f}{n}\right)\rho_{YZ}C_YC_Z;\quad E\left(e_0e_4\right)=\left(\frac{1-f^*}{n}\right)\rho_{YZ}C_YC_Z;\\
&E\left(e_1e_3\right)=\left(\frac{1-f}{n}\right)\rho_{XZ}C_XC_Z;\\
&E\left(e_1e_4\right)=E\left(e_2e_3\right)=E\left(e_2e_4\right)=\left(\frac{1-f^*}{n}\right)\rho_{XZ}C_XC_Z \qquad (3)
\end{aligned}
$$

where $f=\frac{n}{N}$ is the sampling fraction, $f^*=\frac{n_1}{N}$, $C_X^2=\frac{S_X^2}{\bar{X}^2}$, $C_Y^2=\frac{S_Y^2}{\bar{Y}^2}$, $C_Z^2=\frac{S_Z^2}{\bar{Z}^2}$.

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Taking expectations on both the sides and using the results of (3) in (2), we get the bias of $\bar{Y}_{RP}^{w(d)}$ as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_I = \bar{Y}\left(\frac{1-f_1}{2n}\right)\left[\alpha^2\left\{C_X^2 + C_Z^2\left(1-2K_{XZ}\right)\right\} + \alpha\left\{C_X^2\left(1-2K_{YX}\right) - C_Z^2\left(1-2K_{YZ}\right)\right\}\right] \quad (4)$$

$$= \bar{Y}\left(\frac{1-f_1}{2n}\right)\left[\alpha^2 K_3 + \alpha\left(K_1 - K_2\right)\right] \quad (5)$$

where $f_1 = \frac{n}{n_1}$, $K_{YX} = \rho_{YX}\frac{C_Y}{C_X}$, $K_1 = C_X^2\left(1-2K_{YX}\right)$, $K_2 = C_Z^2\left(1-2K_{YZ}\right)$, $K_3 = C_X^2 + C_Z^2\left(1-2K_{XZ}\right)$.

Now from equation (2), we have

$$\bar{Y}_{RP}^{w(d)} - \bar{Y} = \bar{Y}\left[e_0 - \alpha\left(e_1 - e_2 - e_3 + e_4\right)\right].$$

Squaring both the sides and taking expectations in the above equation and using the results of (3), we get the mean square error of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation as

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_I = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[\alpha^2\left\{C_X^2 + C_Z^2\left(1-2K_{XZ}\right)\right\} - 2\alpha\left(C_{YX} - C_{YZ}\right)\right]$$

$$= \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[\alpha^2 K_3 - 2\alpha S_1\right] \quad (6)$$

where $S_1 = C_{YX} - C_{YZ}$, $C_{YZ} = \rho_{YZ}C_Y C_Z$, $C_{YX} = \rho_{YX}C_Y C_X$

Differentiating $M\left(\bar{Y}_{RP}^{w(d)}\right)$ w.r.t α and equating to zero, we get

$$\alpha = \frac{S_1}{K_3}. \quad (7)$$

Now putting the optimum value of α from (7) in the proposed estimator (1), we get the asymptotically optimum estimator(AOE) as

$$\left(\bar{Y}_{RP}^{w(d)}\right)_{I(opt)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}\frac{\bar{z}}{\bar{z}_1}\right)^{I\alpha(opt)}.$$

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Therefore, after putting the value of α in (4) and (6), we obtain the optimum bias and MSE of $\bar{Y}_{RP}^{w(d)}$ respectively as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)} = \bar{Y}\left(\frac{1-f_1}{2n}\right)\frac{S_1}{K_3}\left[K_1' - K_2'\right]$$

where $$K_1' = C_X^2\left(1-K_{YX}\right), \quad K_2' = C_Z^2\left(1-K_{YZ}\right).$$

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 - \bar{Y}^2\left(\frac{1-f_1}{n}\right)\frac{S_1^2}{K_3}. \tag{8}$$

Remark 1 *For $\alpha = 1$, the estimator reduces to ratio cum product estimator in double sampling. The bias and MSE of $\bar{Y}_{RP}^{(d)}$ are obtained by putting $\alpha = 1$ in relation (4) and (6) as follows*

$$B\left(\bar{Y}_{RP}^{(d)}\right)_I = \bar{Y}\left(\frac{1-f}{n}\right)\left[K_1' - C_{XZ} + C_{YZ}\right] \tag{9}$$

where $$K_1' = C_X^2\left(1-K_{YX}\right)$$

and $$MSE\left(\bar{Y}_{RP}^{(d)}\right)_I = \left(\frac{1-f}{n}\right)S_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left(K_1+K_4\right) \tag{10}$$

where $$K_4 = C_Z^2\left(1-2K_{XZ}+2K_{YZ}\right).$$

Remark 2 *For $\alpha = 1$ and when the auxiliary variate z is not used, i.e if z is non-zero constant, the proposed estimator reduces to the usual ratio estimator in two phase sampling. The bias and mean square error of $\bar{Y}_R^d$ can be obtained by putting $\alpha = 1$ and omitting the terms of z in equation (4) and (6), respectively as*

$$B\left(\bar{Y}_R^d\right)_I = \bar{Y}\left(\frac{1-f_1}{n}\right)K_1'$$

and

$$MSE\left(\bar{Y}_R^d\right)_I = \bar{Y}\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}\left(\frac{1-f_1}{n}\right)K_1. \tag{11}$$

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Notes

Remark 3 *For $\alpha = 1$ and when the auxiliary variate x is not used, i.e if x is non-zero, the proposed estimator reduces to the usual product estimator in two phase sampling. The bias and mean square error of $\bar{Y}_P^d$ can be obtained by putting $\alpha = 1$ and omitting the terms of x in equation (4) and (6), respectively as*

$$B\left(\bar{Y}_P^d\right)_I = \bar{Y}\left(\frac{1-f_1}{n}\right)C_{YZ}$$

$$\text{and} \quad M\left(\bar{Y}_P^d\right)_I = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)K_5 \tag{12}$$

where $K_5 = C_X^2\left(1+2K_{YX}\right)$.

IV. Efficiency Comparisons

Compairison of the optimum proposed estimator $\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)}$

a) with sample mean per unit estimator $\bar{y}$

The MSE of sample mean $\bar{y}$ under SRSWOR sampling scheme is given by

$$V\left(\bar{y}\right) = \left(\frac{1-f}{n}\right)S_Y^2. \tag{13}$$

From equation (13) and (8), we observed

$$V\left(\bar{y}\right) - M\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha I(opt)} = \frac{S_1^2}{K_3} > 0. \tag{14}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

b) with ratio estimator in double sampling

From equation (11) and (8), we observed

$$M\left(\bar{Y}_R^d\right) - M\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left(K_1 + \frac{S_1^2}{K_3}\right) > 0. \tag{15}$$

if $K_1 > 0$, $K_3 > 0$ i.e. $K_{YX} < 1/2$, $K_{XZ} < 1/2$.

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c) with product estimator in double sampling

From (12) and (8), we observed

$$M\left(\bar{Y}_P^d\right) - MSE\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[K_5 + \frac{S_1^2}{K_3}\right] > 0 \tag{16}$$

if $K_3 > 0$, $K_5 > 0$ i.e. $K_{XZ} < 1/2$.

d) with ratio cum product estimators in double sampling

From (10) and (8), we observed

$$M\left(\bar{Y}_{RP}^d\right) - M\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left[K_1 + K_5 + \frac{S_1^2}{K_3}\right] > 0 \tag{17}$$

if $K_1 > 0$, $K_3 > 0$, $K_5 > 0$ i.e. $K_{YX} < 1/2$, $K_{XZ} < 1/2$, $K_{XZ} - K_{YZ} < 1/2$.

Now we state the theorem

Theorem 4 *To the first degree of approximation, the proposed class of estimators* $\bar{Y}_{Rd}^{w(d)}$ *under the optimality* (7) *is consider to be more efficient than* $\bar{Y}_R^d$, $\bar{Y}_P^d$, $\bar{Y}_{RP}^d$ *and* $\bar{y}$ *under the given conditions* K_1, K_3, K_4, *and* $K_5 > 0$, *where* $K_1 = C_X^2\left(1 - 2K_{YX}\right)$, $K_3 = C_X^2 + C_Z^2\left(1 - 2K_{XZ}\right)$, $K_4 = C_Z^2\left(1 - 2K_{XZ} + 2K_{YZ}\right)$ *and* $K_5 = C_X^2\left(1 + 2K_{YX}\right)$.

V. Bias, mse and Optimum Value of $\bar{Y}_{RP}^{w(d)}$ in Case II

In this case, we have

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = E\left(e_4\right) = 0;$$

$$E\left(e_0^2\right) = \left(\frac{1-f}{n}\right)C_Y^2; \quad E\left(e_1^2\right) = \left(\frac{1-f}{n}\right)C_X^2; \quad E\left(e_3^2\right) = \left(\frac{1-f}{n}\right)C_Z^2;$$

$$E\left(e_2^2\right) = E\left(e_1e_2\right) = \left(\frac{1-f^*}{n}\right)C_X^2; \quad E\left(e_4^2\right) = E\left(e_3e_4\right) = \left(\frac{1-f^*}{n}\right)C_Z^2;$$

$$E\left(e_0e_1\right) = \left(\frac{1-f}{n}\right)\rho_{YX}C_YC_X; \quad E\left(e_0e_3\right) = \left(\frac{1-f}{n}\right)\rho_{YZ}C_YC_Z;$$

$$E\left(e_1e_3\right) = \left(\frac{1-f}{n}\right)\rho_{XZ}C_XC_Z; \quad E\left(e_2e_4\right) = \left(\frac{1-f^*}{n}\right)\rho_{XZ}C_XC_Z;$$

$$E\left(e_0e_2\right) = E\left(e_0e_4\right) = E\left(e_1e_2\right) = E\left(e_1e_4\right) = E\left(e_3e_2\right) = E\left(e_3e_4\right) = 0. \tag{18}$$

Notes

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Notes

Taking expectations in (2) and using the results of (18), we get the bias of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}\left[\alpha^2 N_1 K_3 + \alpha\left\{f''\left(C_X^2 - C_Z^2\right) - f' S_1\right\}\right].$$

where $\quad f' = \frac{1-f}{n}, \quad f'' = \frac{1-f_1}{2n}, \quad N_1 = \frac{1}{2}\left(\frac{1}{n} + \frac{1}{n_1} - \frac{2}{N}\right).$

Squaring and taking expectations in both the sides of (2) and using the results of (18), we obtain the MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)$ to the first degree of approximation as

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}^2 f' C_Y^2 + \bar{Y}^2 2\left[\alpha^2 N_1 K_3 - \alpha f' S_1\right]. \tag{19}$$

Minimization of (19) is obtained with optimum value of α as

$$\alpha = \frac{f' S_1}{2 N_1 K_3} = \alpha_{II(opt)}. \tag{20}$$

Substituting the value of α from (20) in (1) gives the AOE of (1) as

$$\left\{\bar{Y}_{RP}^{w(d)}\right\}_{II(opt)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}} . \frac{\bar{z}}{\bar{z}_1}\right)^{\alpha_{II(opt)}}. \tag{21}$$

Thus, the resulting bias and MSE of (21) are, respectively given as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{II\alpha(opt)} = \bar{Y}\frac{f' S_1}{2 N_1 K_3}\left[f'' C_1 - \frac{f' S_1}{2 N_1 K_1}\right]$$

where $\quad C_1 = C_X^2 - C_Z^2$ and

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II\alpha(opt)} = \bar{Y}^2 f' C_Y^2 - \bar{Y}^2 \frac{f'^2 S_1^2}{2 N_1 K_3}.$$

For simplicity, we assume that the population size N is large enough as compared to the sample sizes n and n_1 so that the finite population correction (FPC) terms $1/N$ and $2/N$ are ignored.

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Ignoring the FPC in (19), the MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)_{II}$ reduces to

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}\frac{C_Y^2}{n} + \bar{Y}^2\left[\alpha^2\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \alpha\frac{S_1}{n}\right]$$

which is minimized for

$$\alpha = \frac{n_1 S_1}{(n+n_1)K_3} = \alpha_{II(opt)}^* \quad \text{(say)} \tag{22}$$

Substituting the value of α from (22) in (1), we obtained AOE of (1) as

$$\left(\bar{Y}_{RP}^{w(d)}\right)^* = \bar{Y}\left[\frac{x}{x_1}.\frac{z_1}{z}\right]^{\alpha_{II(opt)}^*}.$$

Therefore, the resulting MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)^*$ is

$$M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{C_Y^2}{n} - \bar{Y}^2\frac{n_1 S_1^2}{n(n+n_1)K_3}. \tag{23}$$

Remark 5 *For $\alpha = 1$, the proposed estimator reduces to ratio cum product estimator in double sampling and MSE is given as*

$$M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2 2\left[\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \frac{S_1}{n}\right]. \tag{24}$$

Ignoring the FPC, the variance of $\bar{y}$ under SRSWOR is given by

$$V(y)_{II} = \bar{Y}^2\frac{C_Y^2}{n}. \tag{25}$$

and the MSE of $\left(\bar{Y}_{Rd}\right)_{II}$ and $\left(\bar{Y}_{Pd}\right)_{II}$ are given by

$$M\left(\bar{Y}_{Rd}\right)_{II} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2\left[C_X^2\left(\frac{2}{n}-\frac{1}{n_1}-\frac{2}{n}K_{YX}\right)\right] \tag{26}$$

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and

$$\left(\bar{Y}_{Pd}\right)_{II} = \bar{Y}^2 \frac{C_Y^2}{n} + \bar{Y}^2 \left[\frac{C_X^2}{n} + \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n_1}\right) C_Z^2 + \frac{2C_{YZ}}{n}\right] \tag{27}$$

respectively.

Notes

VI. Efficiency Copmparisons

Compairison of the optimum proposed estimator $\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)}$

a) with sample mean per unit estimator

From (25) and (23), we observed that

$$V\left(\bar{y}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2 \frac{n_1 S_1^2}{n\left(n+n_1\right) K_3} > 0 \tag{28}$$

if $$K_3 > 0 \text{ i.e } K_{XZ} < 1/2.$$

b) with ratio estimator in double sampling

From (26) and (23), we observed that

$$M\left(\bar{Y}_{Rd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2 \left[\frac{n_1 S_1^2}{n\left(n+n_1\right) K_3} + C_X^2 P\right] > 0 \tag{29}$$

if $$K_3 > 0,\ P > 0 \text{ i.e } K_{XZ} < 1/2,\ K_{YX} < 1 - \tfrac{n}{2n_1},$$

where $$P = \tfrac{2}{n}\left(1 - K_{YX}\right) - \tfrac{1}{n_1}.$$

c) with product estimator in double sampling

From (27) and (23), we observed that

$$M\left(\bar{Y}_{Pd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2 \left[Q - \frac{C_Z^2}{2n_1} + \frac{n_1 S_1^2}{n\left(n+n_1\right) K_3}\right] > 0 \tag{30}$$

if $$K_3 > 0 \text{ i.e } K_{XZ} < 1/2, \text{ where } Q = C_X^2 + \tfrac{C_Z^2}{2} + 3C_{YX}.$$

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d) with ratio cum product estimator in double sampling

From (24) and (23), we observed that

$$M\left(\bar{Y}_{RPd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^{*}_{II\alpha(opt)} = \bar{Y}^{2}\left[\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \frac{S_1}{2n} + \frac{n_1 S_1^2}{n\left(n+n_1\right)K_3}\right] > 0 \tag{31}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

VII. Conclusion

We have developed an efficient class of ratio-cum-product estimators in two phase sampling. The comparative study shows that the proposed estimator $\bar{Y}_{RP}^{w(d)}$ established their superiority over sample mean $\bar{Y}$, ratio estimator $\bar{Y}_R^d$, product estimator$\bar{Y}_P^d$ and ratio-cum-product estimator $\bar{Y}_{RP}^d$ in two-phase sampling under the given conditions. Hence from the resulting equation (14), (15), (16) and (17), we conclude that under the given conditions the proposed estimator is consider to be the best estimator.

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References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

Index

D

Disquisitiones · 27
Darboux Continuity · 19
Dynamical · 43, 51, 52

E

Epidemiology · 30, 46
Exponentiated · 1, 24, 26, 28, 30, 34, 39, 40, 41, 42

G

Geodetic · 65, 66, 67, 68, 69, 70, 71, 73

O

Oscillator · 102, 104, 105, 107, 108, 114, 116, 118
Quasicontinuous · 3, 5, 19

P

Perturbation · 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 102, 116, 118

T

Thermophoresis · 87
Trajectorries · 1, 43

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