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Highlights

- Hypergeometric Function
- Kinematic Wave Equation
- Stiff Differential Equations
- Countable Boolean Lattice

Discovering Thoughts, Inventing Future

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Characterization of Partial Lattices on Countable Boolean Lattice

By D.V.S.R. Anil Kumar, Y.V.Seshagiri Rao, Y Narasimhulu & Venkata Sundaranand Putcha

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Abstract - In this paper new concepts countable join property, countable meet property, P_σ – lattice and P_δ – lattice are introduced. We established that P_σ – lattice and P_δ – lattice are measureable partial lattices and characterized partial lattices of a lattice through countable join and meet properties. We also established some interesting result on the injective property of the lattice measurable functions defined over countable Boolean lattices.

Keywords : lattice, partial lattice, σ- algebra, measure.

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CHARACTERIZATION OF PARTIAL LATTICES ON COUNTABLE BOOLEAN LATTICE

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Characterization of Partial Lattices on Countable Boolean Lattice

D.V.S.R. Anil Kumar [α], Y.V.Seshagiri Rao [σ], Y Narasimhulu [ρ] & Venkata Sundaranand Putcha [ω]

Abstract - In this paper new concepts countable join property, countable meet property, P_σ–lattice and P_δ –lattice are introduced. We established that P_σ–lattice and P_δ –lattice are measureable partial lattices and characterized partial lattices of a lattice through countable join and meet properties. We also established some interesting result on the injective property of the lattice measurable functions defined over countable Boolean lattices.

Keywords : *lattice, partial lattice, σ -algebra, measure.*

I. Introduction

The origin of a lattice concept can be traced back to Boole's analysis of thought and Dedekind's study of divisibility, Schroder and Pierce contributed substantially to this area. Though some of the work in this direction was done around 1930, much momentum was gained in 1967 with the contributions of Birkhoff's [2]. In 1963, Gabor szasz [9] introduced the generalization of the lattice measure concepts. To study σ-additive set functions on a lattice of sets, Gena A. DE Both [3] introduced σ - lattice in 1973. The concept of partial lattices was introduced by George Gratzer [5] in 1978. In 2000, Pao - Sheng Hus [8] characterized outer measures associated with lattice measure. The Hann decomposition theorem of a signed lattice measure by Jun Tanaka [10] defined a signed lattice measure on a lattice σ - algebras and the concept of sigma algebras are extensively studied by [4]. D.V.S.R. Anil Kumar etal [1] introduce the concept of measurable Borel lattices, σ - lattice and δ –lattice to characterize a class of Measurable Borel Lattices. This paper is organized as follows. Section 2 presents the preliminaries definitions and results. In Section 3 we proved that

P_σ – lattice and P_δ –lattice are measurable partial lattices and all partial lattices of a lattice satisfy both countable join and meet properties. Some interesting result on the injective property of the lattice measurable functions defined over countable Boolean lattices are established in Section 4.

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2. Birkhoff. G, Lattice Theory 3rd ed., AMS Colloquim Publications, Providence, RI, 1967.

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II. Preliminaries

Consider a lattice $(L,\wedge,\vee)$ with the operations meet $\wedge$ and join $\vee$ and usual ordering $\leq$, where L is a collection of subset of a non empty set X. Now this lattice $(L,\wedge,\vee)$ is denoted by L and satisfy the commutative law, the associative law and the absorption law. A lattice L is called distributive if the distributive law is satisfied. The zero and one elements of the lattice L are denoted by 0 and 1 respectively. A distributive lattice L is called a Boolean lattice if for any element x in L, there exists a unique complement x^c such that $x \vee x^c = 1$ and $x \wedge x^c = 0$. An operator C: $L \to L$, where L is a lattice is called a lattice complement in L if the law of complementation, the law of contra positive and the law of double negation are satisfied. The following are very important examples of Boolean lattice.

Example2.1. Let $(\{0,1\},\leq)$ be the set consisting of the two elements 0,1 equipped with the usual order relation $0 \leq 1$.This poset is a Boolean lattice with respect to the operations presented in the tables below (at the left the lattice operations and at the right the complementation):

a	b	$a \wedge b$	$a \vee b$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

x	x^c
0	1
1	0

This is usually known as the two valued or two elements Boolean lattice, denoted by $B= (\{0,1\},\vee, \wedge, ^c,0,1)$.

Example2.2. The power set P(X) of a universe X a Boolean lattice if we choose the set theoretic complement $A^c = X\backslash A:=\{x \in X: x \in X \text{ and } x \notin A\}$ as the complement of a given set A in the universe X. Such a Boolean lattice is $P = (P(X), \vee, \wedge, ^c, \phi, X)$.

Example2.3. $E = (2^X ,\vee, \wedge,^c 0, 1)$ is the collection 2^X of all two valued functional on the universe X is a Boolean lattice if we choose the functional $\chi^c = 1-\chi$ as the complement of a given functional χ.

Example2.4. Let $(D, \vee, \wedge, ^c,1,70)$ is a Boolean lattice where D={1,2,5,7,10,14,35,70} is the set of all divisors of 70, $x \wedge y$ = Greatest Common Devisor of x and y, $x \vee y$ = Least Common Multiple of x and y and $x^c = \frac{70}{x}$.

Definition2.1. A Boolean lattice L is called a countable Boolean lattice if L is closed under countable join and is denoted by σ (L).

Example2.5. {empty setϕ, X}, Power set of X, Let $X = \Re$, L = {measurable subsets of $\Re$} with usual ordering ($\leq$) are all countable Boolean lattice.

Definitition2.2. The entire set X together with countable Boolean lattice is called lattice measurable space and is denoted by the ordered pair (X, σ (L)).

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Example2.6. $X = \Re$, where $\Re$ is extended real number system and L = {All Lebesgue measurable sub sets of $\Re$ },($\Re$,σ (L)) is a lattice measurable space.

Definitition2.3. If μ: σ (L) → R ∪ {∞} satisfies the following properties (i) μ(ϕ) = μ (0) = 0 (ii) for all h, g ∈ σ (L), such that μ (h), μ (g) ≥ 0; h ≤ g ⇒ μ (h) ≤ μ (g) (iii) for all h, g ∈ σ (L): μ (h ∨ g) + μ (h ∧ g) = μ (h) + μ (g) (iv) If h_n ∈ σ (L), n ∈ N such that $h_1 \leq h_2 \leq \ldots \leq h_n \leq \ldots$, then $\mu(\bigvee_{n=1}^{\infty} h_n) = \lim \mu(h_n)$ then μ is called a lattice measure on the countable Boolean lattice σ (L).

The following is definition given in [5]

Definition2.4. Let σ (L) be a countable Boolean lattice, H ⊆ σ (L), and restrict ∧ and ∨ to H as follows. For a, b, c ∈ H, if a ∧ b = c (dually, a ∨ b = c), then we say that in H, a ∧ b(dually a ∨ b) is defined and it equals c, if, for a, b ∈ H, a ∧ b ∉ H(dually a ∨ b ∉ H), then we say that a ∧ b(dually a ∨ b) is not defined in H. Thus (H, ∧,∨) is a set with two binary partial operations. (H, ∧,∨) is called a partial lattice, a relative sublattice of σ (L).

Observation2.1. Every subset of a countable Boolean lattice determines a partial lattice. Every sublattice of σ (L) is a partial lattice and the converse need not be true.

Definition2.5.[7] A set A is said to be measurable partial lattice, if A is in σ (L).

Example2.7. ($\Re$, σ (L)) be lattice measurable space. Then the interval (a, ∞) is a measurable partial lattice under usual ordering.

Example2.8. [0, 1) < $\Re$ is a measurable partial lattice under usual ordering.

Definition2.9. A P_σ - lattice is a poset for which sup exist for any countable collection of its partial lattices.

Example2.9. $R = \bigvee_{n=1}^{\infty} (-n, n)$ is a p_σ -lattice.

Definition2.10. A P_δ - lattice is a poset for which inf exist for any countable collection of its partial lattices.

Example2.11. (i) $\bigwedge_{n=1}^{\infty} (-n, n) = (-1, 1)$ and (ii) $\bigwedge_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) = \{0\}$ are P_δ – lattices.

Definition2.11.Countable join property (CJP): If { E_k } is monotonic increasing sequence of partial lattices of a lattice L and $E = \bigvee_{k=1}^{\infty} E_k$. Then $\mu(E) = \underset{n\to\infty}{Lt}\, \mu(E_n)$.

Definition2.12.Countable meet property (CMP): If { E_k } is a monotonic decreasing sequence of partial lattices of a lattice L and $E = \bigwedge_{k=1}^{\infty} E_k$. Then $\mu(E) = \underset{n\to\infty}{Lt}\, m(E_n)$.

Result2.1.[1]. If E is measurable lattice so is E^c.

III. P_σ-Lattice and P_δ -Lattice

Theorem3.1. Every P_σ - lattice is lattice measurable.

Proof. Let $E_1, E_2, \ldots\ldots$ are pair wise disjoint measurable partial lattices and $E = \bigvee_{k=1}^{\infty} E_k$,

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5. Gratzer. G, General Lattice Theory, Academic Press Inc., 1978.

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Evidently,

$$\mu\,(\bigvee_{k=1}^{\infty} E_k) \leq \sum_{k=1}^{\infty}\mu(E_k) \tag{1}$$

and

$$\mu\,(\bigvee_{k=1}^{\infty} E_k) \geq \mu(\bigvee_{k=1}^{n} E_k) \tag{2}$$

From definition 2.3. We have $\mu\,(E_1 \vee E_2) = \mu\,(E_1) + \mu\,(E_2)$. By the principle of mathematical induction on number of pair wise disjoint measurable partial lattices, n, we have $\mu\,(\bigvee_{k=1}^{n} E_k) = \sum_{k=1}^{n}\mu(E_k)$. As $n \to \infty$, from (2) it follows that

$$\mu\,(\bigvee_{k=1}^{\infty} E_k) \geq \sum_{k=1}^{\infty}\mu(E_k) \tag{3}$$

From (1) and (3), we have $\mu\,(\bigvee_{k=1}^{\infty} E_k) = \sum_{k=1}^{\infty}\mu(E_k)$. Now $E = \bigvee_{k=1}^{\infty} E_k = E_1 \vee (E_2 \wedge E_1^c) \vee \ldots \vee (E_k \wedge (\bigvee_{k=1}^{n-1} E_k^c) \vee \ldots$. Since $E_1, E_2 \wedge E_1^c, \ldots$ are disjoint measurable partial lattices, we have, $\bigvee_{k=1}^{\infty} E_k$ is a measurable partial lattice. Hence every P_σ - lattice is a lattice measurable.

Theorem3.2. Every P_σ - lattice satisfies CJP.

Proof. Suppose that $\{E_k\}$ is monotonic increasing sequence of partial lattices of a σ (L) and $E = \bigvee_{k=1}^{\infty} E_k$. Write $E = E_1 \vee (E_2 \wedge E_1^c) \vee \ldots \vee (E_k \wedge (\bigvee_{k=1}^{n-1} E_k^c) \vee \ldots$

So we have $E = E_1 \vee (\bigvee_{k=1}^{\infty}(E_{k+1} \wedge E_k^c)$ (a disjoint joint). By Theorem 3.1.

Now, $\mu\,(E) = \mu\,(E_1) + \sum_{k=1}^{\infty}\mu(E_{k+1} - E_k) = \mu\,(E_1) + \underset{n\to\infty}{\mathrm{Lt}} \sum_{k=1}^{n} [\mu(E_{k+1}) - \mu(E_k)] = \mu\,(E_1) + \underset{n\to\infty}{\mathrm{Lt}} [\mu(E_2) - \mu(E_1) + \ldots\ldots + \mu(E_n) - \mu(E_{n-1})] = \mu\,(E_1) + \underset{n\to\infty}{\mathrm{Lt}} [-\mu(E_1) + \mu(E_n)] = \mu\,(E_1) - \mu\,(E_1) + \underset{n\to\infty}{\mathrm{Lt}}\, \mu(E_n) = \underset{n\to\infty}{\mathrm{Lt}}\, \mu(E_n)$.

Theorem3.3. Every P_δ -lattice is lattice measurable.

Proof. Let $E_1, E_2, \ldots$ are measurable partial lattices.

By theorem 3.1. $E = \bigvee_{k=1}^{\infty} E_k$ is a measurable partial lattice. Let $G = \bigwedge_{k=1}^{\infty} E_k$.

Then $G^c = (\bigwedge_{k=1}^{\infty} E_k)^c = \bigvee_{k=1}^{\infty} E_k^c$. Given that each E_k is a measurable partial lattice.

Hence by Result 2.1., each E_k^c is a measurable partial lattice. Which implies $\bigvee_{k=1}^{\infty} E_k^c$ is a measurable partial lattice (Every P_σ - lattice is a measurable partial lattice). This leads to G^c is measurable partial lattice. Hence G is measurable partial lattice (By Result 2.1.).

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Theorem 3.4. Every P_δ -lattice satisfies CMP.

Proof. Suppose that $\{E_k\}$ is a monotonic decreasing sequence of partial lattices of $\sigma(L)$ and $E = \bigwedge_{k=1}^{\infty} E_k$. Let $E = \bigwedge_{k=1}^{\infty} E_k$. Evidently $E_1 = E \vee (E_1 \wedge E_2^c) \vee (E_2 \wedge E_3^c) \vee \ldots$

Then $\mu(E_1) = \mu(E) + \sum_{k=1}^{\infty} \mu(E_k) - \mu(E_{k+1}) = \mu(E) + \underset{n\to\infty}{Lt} \sum_{k=1}^{n} \mu(E_k) - \mu(E_{k+1})$

$= \mu(E) + \underset{n\to\infty}{Lt} [\mu(E_1) - \mu(E_2) + \ldots\ldots\ldots + \mu(E_n) - \mu(E_{n+1})] = \mu(E) + \underset{n\to\infty}{Lt} [\mu(E_1) - \mu(E_{n+1})]$

$= \mu(E) + \mu(E_1) - \underset{n\to\infty}{Lt} \mu(E_{n+1})$. Which implies $\mu(E) = \underset{n\to\infty}{Lt} \mu(E_n)$.

IV. The Injective and Projective Properties of Lattice Measurable Functions

Definition4.1. An extended real value function f defined on a lattice measurable E is said to be lattice measurable function if the set $\{x \in E / f(x) > \alpha\}$ is lattice measurable for all real numbers α.

Example4.1. Constant functions, Continuous functions and Characteristic functions are lattice measurable functions.

Result4.1. If f and g are lattice measurable functions then $f \vee g$ and $f \wedge g$ are also lattice measurable functions.

Proof. For any real number α we have $\{x \in L / (f \vee g)(x) > \alpha\} = \{x \in L / f(x) > \alpha\} \vee \{x \in L / g(x) > \alpha\}$ and $\{x \in L / (f \wedge g)(x) > \alpha\} = \{x \in L / f(x) > \alpha\} \wedge \{x \in L / g(x) > \alpha\}$. Since $\{x \in L / f(x) > \alpha\}$ and $\{x \in L / g(x) > \alpha\}$ are lattice measurable sets implies the sets of RHS are lattice measurable implies $f \vee g$ and $f \wedge g$ are lattice measurable functions.

The following interesting property can easily be verified from the works of [6] by considering lattice measurable functions f and g defined over countable Boolean lattice.

Proporty4.1. A Countable Boolean lattice A is a Retrace of Countable Boolean lattice B if there exist homomorphism $g: A \to B$ and $f: B \to A$ such that fg is the identity on A. Here g and f are necessarily a monomorphism (injection) and epimorphism(projective) respectively. That is A Countable Boolean lattice is Retrace injective if it is a Retrace of every Countable Boolean lattice that contains it

V. Conclusion

New concepts like countable join property, countable meet property, P_σ–lattice and P_δ–lattice are introduced. Characterized partial lattices of a lattice through countable join and meet properties and proved that P_σ–lattice and P_δ–lattice are measureable partial lattices. Interesting result on the injective property of the lattice measurable functions defined over Countable Boolean lattices are established.

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Numerical Approach for Solving Stiff Differential Equations: A Comparative Study

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NUMERICAL APPROACH FOR SOLVING STIFF DIFFERENTIAL EQUATIONS A COMPARATIVE STUDY

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Numerical Approach for Solving Stiff Differential Equations: A Comparative Study

Sharaban Thohura [α] & Azad Rahman [σ]

Abstract - In this paper our attention is directed towards the discussion of phenomenon of stiffness and towards general purpose procedures for the solution of stiff differential equations. Our aim is to identify the problem area and the characteristics of the stiff differential equations for which the equations are distinguishable. Most realistic stiff systems do not have analytical solutions so that a numerical procedure must be used. Computer implementation of such algorithms is widely available e.g. DIFSUB, GEAR, EPISODE etc. The most popular methods for the solution of stiff initial value problems for ordinary differential equations are the backward differentiation formulae (BDFs). In this study we focus on a particularly efficient algorithm which is named as EPISODE, based on variable coefficient backward differentiation formula. Through this study we find that though the method is very efficient it has certain problem area for a new user. All those problem area have been detected and recommended for further modification.

I. Introduction

A very important special class of differential equations taken up in the initial value problems termed as stiff differential equations result from the phenomena with widely differing time scales. There is no universally accepted definition of stiffness.Stiffness is a subtle, difficult and important concept in the numerical solution of ordinary differential equations. It depends on the differential equation, the initial condition and the interval under consideration.

The initial value problems with stiff ordinary differential equation systems occur in many fields of engineering science, particularly in the studies of electrical circuits, vibrations, chemical reactions and so on. Stiff differential equations are ubiquitous in astrochemical kinetics, many control systems and electronics, but also in many non-industrial areas like weather prediction and biology.

A set of differential equations is "stiff" when an excessively small step is needed to obtain correct integration. In other words we can say a set of differential equations is "stiff" when it contains at least two "time constants" (where time is supposed to be the joint independent variable) that differ by several orders of magnitude. A more rigorous definition of stiffness was also given by Shampine and Gear: "By a stiff problem we mean one for which no solution component is unstable (no eigenvalue of the Jacobian matrix has a real part which is at all large and positive) and at least some component is very stable (at least one eigenvalue has a real part which is large and negative). Further, we

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will not call a problem stiff unless its solution is slowly varying with respect to most negative part of the eigenvalues. Consequently a problem may be stiff for some intervals and not for others."

When solving the (vector) system of equations

$$y' = f(t, y), \qquad y(t_0) = a \text{ (given)} \tag{1}$$

we must consider the behavior of solutions near to the one we seek. This is because as we step along from $y_n = y(x_n)$ to y_{n+1} approximating $y(x_n + h)$ we make inevitable errors causing us to move from the desired integral curve to a nearby one. If we make no further errors, we follow this new curve so that the resulting error depends on the relative behavior of the two solution curves. Let us consider the example of the single equation

$$y' = A(y - p(t)) + p'(t), \qquad y(t_o) = a \tag{2}$$

where A is constant. The analytical solution is

$$y(t) = (a - p(0))\exp(At) + p(t) \tag{3}$$

If A is large and positive, the solution curves for the various a fan out and we say the problem is unstable. Such a problem, obviously, is difficult for any general numerical method, which proceeds in a step-by-step fashion. When A is small in magnitude, the curves are more or less parallel and such neutrally stable problems are easily handled by conventional means. When A is large and negative, the solution curves converge very quickly. In fact, whatever be the value of $y(t_o)$, the solution curve is virtually identical to the particular solution $p(t)$ after a short distance called an initial transient. This super-stable situation is ideal for the propagation of error in a numerical scheme. The last class of problems is called stiff.

If A is very negative and $p(t)$ is slowly varying, equation (3) represents a stiff problem after the transient e^{At} has died out (that is, e^{At} is below the error tolerance of interest) but it is not be stiff in the transient region. If (1) is linear with a constant Jacobian J (where $\boldsymbol{J} = \partial \mathbf{f} / \partial \mathbf{y}$ is the associated Jacobian matrix), it will not be stiff in the initial transient, but will be stiff after the fastest transient has died out. We see that in case of stiff differential equation problem the solution being sought is varying slowly, but there are nearby solutions that vary rapidly, so the numerical method must take small steps to obtain satisfactory results. Stiffness is an efficiency issue. If we were not concerned with how much time a computation takes, we would not be concerned about stiffness. Nonstiff methods can solve stiff problems, but take a long time to do it.

II. Numerical Solution of Stiff Differential Equation

As stiff differential equations occur in many branches of engineering and science, it is required to solve efficiently. Most realistic stiff systems do not have analytical solutions so that a numerical procedure must be used. Conventional methods such as Euler, explicit Runge-Kutta and Adams –Moulton are restricted to a very small step size in order to that the solution be stable. This means that a great deal of computer time could be required.

In a number of areas, particularly in chemical applications one often encounters systems of ordinary differential equations which, although mathematically well conditioned, are virtually impossible to solve with traditional numerical methods because

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of the severe step size constraint imposed by numerical stability. These stiff equations can be characterized by the presence of transient components which, although negligible relative to the numerical solution, constrain the step size of traditional numerical methods to be of the order of the smallest time constant of the problem.

Notes

Over the last three decades, there has been significant progress in the development of numerical stiff ODE solvers both in the areas of ODE solution algorithms and the associated linear algebra. Consequently, a wide variety of very efficient and reliable ODE solvers have been developed. In order to take full advantage of the available state-of-the-art solvers, and to handle computationally demanding various models in the different field both accurately and efficiently, a great deal of understanding is required for the formulation of the problem. The numerical solution algorithm of a standard stiff ODE solver package comprises two major components: one is the numerical solution method for the systems of ODEs and the other is for the solution of the resulting linear algebraic system that arises due to the ODEs solution technique. The structure of the resulting matrix associated with the linear system has significant computational consequence.

To better understand the advanced ODE solvers and their differences, we first need to briefly consider the solution methods underlying stiff systems of ODEs and their corresponding linear algebra. For the solution of a system of ODEs of size N of the form (1) and a given initial condition, $\mathbf{y}\ (t_0) = \mathbf{a}$, some classes of multistep methods are generally used. To advance the solution in time t from one mesh point to the next, considering a discrete time mesh $\{t_0, t_1, \ldots\ldots t_n \ldots\ldots\}$, multistep methods make use of several past values of the variable $\mathbf{y}$ and its rate of change $\mathbf{f}$ with respect to time t (i.e. the past values of the abundances and the rate equations). The general form of a k-step multistep method is

$$\sum_{i=0}^{k} \alpha_i \mathbf{y}_{n-i} = h \sum_{i=0}^{k} \beta_i \mathbf{f}_{n-i} \tag{4}$$

where α_i and β_i are constants depending on the order the method, h is the step size in time and n denotes the mesh number. The well-known Adams methods which use mostly the past values of $\mathbf{f}$,

$$\mathbf{y}_n = \mathbf{y}_{n-1} + h \sum_{i=0}^{k} \beta_i \mathbf{f}_{n-i} \tag{5}$$

are the best-known multistep methods for solving nonstiff problems. Each step requires the solution of a nonlinear system and often a simple functional iteration with an initial guess, or predictor estimate, is used to advance the integration, which is terminated by a convergence test. For stiff problems, where sudden changes in the variables can occur (i.e. there are strong dependencies of the rate equations $\mathbf{f}$ upon abundances $\mathbf{y}$ in small time intervals say), simple iteration leads to unacceptable restriction of the step size and functional iteration fails to converge. Thus, stiffness forces the use of implicit methods with infinite stability regions when there is no restriction on the step size. The backward difference formulae (BDF) methods with unbounded region of absolute stability were the first numerical methods to be proposed for solving stiff ODEs (Curtiss and Hirschfelder, 1952). The BDF used in ODE solvers, are of the general form

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$$\mathbf{y}_n = \sum_{i=0}^{k} \alpha_i \mathbf{y}_{n-i} + h\beta_0 \mathbf{f}_n \tag{6}$$

where α_i and β_0 are coefficients of kth order, k-step BDF methods. As mentioned earlier, a simple functional iteration will usually fail to converge when problems are stiff and some form of Newton iteration is usually used for the solution of the resulting nonlinear system. The Newton iteration involves the solution of an $N \times N$ matrix, $\boldsymbol{P}$,

Notes

$$\boldsymbol{P} \approx \boldsymbol{I} - h\beta_0 \boldsymbol{J}, \tag{7}$$

where $\boldsymbol{J} = \partial \mathbf{f} / \partial \mathbf{y}$ is the associated Jacobian matrix, I is an $N \times N$ identity matrix. The solution to this linear algebraic system contributes significantly to the total computational time for the solution of stiff problems, as well as affecting the accuracy of the solution (and hence, also affecting the computational time). For stiff problems the ODE solvers use a modified Newton iteration that allows time saving strategies for the computation, storage and the use of the Jacobian matrix. When solving a linear algebraic system, there are generally two classes of solution methods, direct methods and iterative methods. The most common direct method used to solve linear systems is the Gaussian elimination method based on factorization of the matrix in lower and upper triangular factors. The GEAR, LSODE and VODE solvers all use such a method for the solution of the resulting linear system. The simplest iterative scheme used to solve linear systems is the Jacobi iteration, although the more sophisticated iterative solution methods of Krylov subspace methods, based on a sequence of orthogonal vectors and matrix-vector multiplications, have been widely used in practical applications such as computational fluid dynamics (Saad, 2003a). The ODE solvers, LSODPK and VODPK implement Preconditioned Krylov iterative techniques for the solution of the resulting linear system.
Some of the more readily available methods for stiff equations include:

- Variable- order methods based on backward differentiation multistep formulas, originally analysed and implemented by Gear (1969,1971) and later modified and studied by Hindmarsh (1974) and Byrne and Hindmarsh (1975).
- Methods based on trapezoidal rule, such as those proposed by Dahlquist (1963) and subsequently studied by Lindberg (1971, 1972).
- Implicit Runge-Kutta methods suitable for stiff equations, such as those based on the formulas of Butcher (1964) and studied by Ehle (1968).
- Methods based on the use of preliminary mathematical transformations to remove stiffness and the solution of the transformed problem by traditional techniques, such as those studied and implemented by Lawson and Ehle (1972).
- Methods based on second derivative multistep formulas, such as those developed by Linger and Willoughby (1967) and Enright (1974).

Unfortunately, although a number of methods have been developed, and many more basic formulas suggested for stiff equations, until recently there has been little advice or guidance to help a practitioner choose a good method for his problem.

In case of stiff differential equations stability requirements force the solver to take a lot of small time steps; this happens when we have a system of coupled differential equations that have two or more very different scales of the independent variable over which we are integrating. Another way of thinking about this to consider what must

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happen when two different parts of the solution require very different time steps. For example, suppose our solution is the combination of two exponential decay curves, one that decays away very rapidly and one that decays away very slowly. Except for the few time steps away from the initial condition, the slowly decaying curve dominate since the rapid curve will have decayed away. But because the variable time step routine to meet stability requirements for both components, we will be locked into small time steps even though the dominant component would allow much lager time steps. This is what we mean by stiff equations, we get locked into taking very small time steps for a component of the solution that makes infinitesimally small contributions to the solution. In other words, we are forced to move slowly when we could be leaping along to a solution.

The specific methods that we assess in this study are the methods based on backward differentiation formulas, DIFSUB (Gear (1971a, 1971b)), GEAR. Rev. 3 (Hindmarsh (1974)) and EPISODE (Byrne and Hindmarsh (1975)). As general ODE packages, GEAR and EPISODE are quite useful for both Stiff and nonstiff problems. In the nonstiff case, with the nonstiff method option, they seem to perform competitively in comparison with other sophisticated nonstiff system solvers. In the stiff case, these codes allow for the use of the Jacobian matrix, and contain routines for solving the associated linear systems, in full matrix form.

EPISODE is very similar to a package called GEAR [8], which is a heavily modified form of C.W. Gear's well-Known code, DIFSUB [9]. The GEAR package is based on fixed step formulas (Adams and BDF), and achieves changes in step size (when required) by interpolating to generate the multipoint data needed at the new spacing. In contrast, EPISODE is based on formulas that are truly variable-step, and step size changes can occurring as frequently as every step, with no interpolation involved. Like Gear, Episode varies its order in a dynamic way, as well as its step size, in an effort to complete the integration with a minimum number of steps. Lida A. M. Nejad [13] have summarized the functions of various packages in the following table

Table 1 : An abridged list of general-purpose solver packages available for solving systems of ODEs

Solver	Comments
GEAR (1974) – standard (supersedes DIFSUB – Gear 1968) GEARB – for Banded Jacobian GEARS – Sparse Jacobian	For stiff and nonstiff problems; for nonstiff problems-Adams methods, for stiff problems – fixed-coefficient form of BDF methods.
LSODE(1982) – standard LSODES – Sparse Jacobian	LSODE (Livermore Solver for ODEs) Combines the capabilities of GEAR and GEARB. Fixed-coefficient formulation of BDF methods.
LSODPK – with preconditioned Krylov iteration methods	LSODPK – uses a preconditioned Krylov iteration method for the solution of the linear system.
VODE (1989) – standard (supersedes EPISODE and EPISODEB)	VODE – variable-coefficient and fixed leading coefficient form of BDF for stiff systems.
VODPK (1992) – with preconditioned Krylov iteration methods	VODPK – uses preconditioned Krylov iteration methods for the solution of the linear system.
CVODE – in ANSI standard C	CVODE – with VODE and VODPK options written in C.
PVODE (1995) – Parallel VODE in ANSI standard C with preconditioned Krylov iteration methods.	PVODE – implements functional iteration, Newton iteration with a diagonal approximate Jacobian and Newton iteration with the iterative method SPGMR (Scaled Preconditioned Generalized Minimal Residual).

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8. Gear, C.W. (1969) "*The automatic integration of stiff ordinary differential equations*". ***Information Processing 68***, A. J. H. Morrell, Ed., North Holland, Amsterdam, pp. 187-193.

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III. Solver Package Episode

The EPISODE program is a package of FORTRAN subroutines aimed at the automatic solution of problems, with a minimum effort required in the face of possible difficulties in the problem. The program implements both a generalized Adams method, well suited for nonstiff problems, and a generalized backward differentiation formula (BDF), well suited for stiff problems. Both methods are of implicit multistep type. In solving stiff problems, the package makes the heavy use of the $N \times N$ Jacobian matrix,

$$J = \frac{\partial \mathbf{f}}{\partial \mathbf{y}} = \left(\frac{\partial \mathbf{f}_i}{\partial \mathbf{y}_j} \right)_{i,j=1}^{N}$$

the $\mathbf{f}_i$ and $\mathbf{y}_j$ are the vector components of **f** and **y,** respectively.

A complete discussion of the use of EPISODE is given in [11]. However, a few basic parameter definitions are needed here, in order to present the examples. Beyond the specification of the problem itself, represented by example 1 and perhaps example 2, the most important input parameter to EPISODE is the method flag, MF. This has eight values-10, 11, 12, 13, 20, 21, 22, and 23. The first digit of MF, called METH, indicates the two basic methods to be used namely implicit Adams and BDF. The second digit, called MITER, indicates the method of iterative solution of the implicit equations arising from the chosen formula. The parameter MITER takes four different values (0, 1, 2, 3) to indicate the following respectively

- Functional (or fixed-point) iteration (no Jacobian matrix used.).
- A chord method (or generalized Newton method, or semi-stationary Newton iteration) with Jacobian given by a subroutine supplied by the user.
- A chord method with Jacobian generated internally by finite differences.
- A chord method with a diagonal approximation to the Jacobian, generated internally (at less cost in storage and computation, but with reduced effectiveness).

The EPISODE package is used by making calls to a driver subroutine, EPSODE, which in turn calls other routines in the package to solve the problem. The function **f** is communicated by way of a subroutine, DIFFUN, which the user must write. A subroutine for the Jacobian, PEDERV, must also be written. Calls to EPSODE are made repeatedly, once for each of the user's output points. A value of t at which output is desired is put in the argument TOUT to EPSODE, and when TOUT is reached, control returns to the calling program with the value of y at t =TOUT. Another argument to EPSODE, called INDEX, is used to convey whether or not the call is the first one for the problem (and thus whether to initialize various variables). It is also used as an output argument, to convey the success or failure of the package in performing the requested task. Two other input parameters. EPS and IERROR, determine the nature of the error control performed within EPISODE.

The EPISODE package consists of eight FORTRAN subroutines, to be combined with the user's calling program and Subroutines DIFFUN and PEDERV. As discussed earlier, only Subroutine EPSODE is called by the user; the others are called within the package. The functions of the eight package routines can be briefly summarized as follows:

Ref.

11. Hindmarsh, A. C. and Byrne, G. D. (1975) "Episode. An experimental package for the integration of systems of ordinary differential equations", *L.L.L. Report UCID-30112, I.I.I.* (www.netlib.org/ode/epsode.f)

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Notes

i) EPSODE sets up storage, makes calls to the core integrator, TSTEP, checks for and deals with error returns, and prints error message as needed.

ii) INTERP computes interpolated values of $\mathbf{y}(t)$ at the user specified output points, using an array of multistep history data.

iii) TSTEP performs a single step of the integration, and does the control of local error (which entails selection of the step size and order) for that step.

iv) COSET sets coefficients that are used by TSTEP, both for the basic integration step and for error control.

v) ADJUST adjusts the history array when the order is reduced.

vi) PSET sets up the matrix $p = I - h\beta_0 J$, where I is the identity matrix, h is the step size, β_0 is a scalar related to the method, and J is the Jacobian matrix. It then processes P for subsequent solution of linear algebraic system with P for subsequent solution of linear algebraic systems with P as coefficient matrix.

vii) DEC performs an LU (lower-upper triangular) decomposition of an $N \times N$ matrix.

viii) SOL solves linear algebraic systems for which the matrix was factored by DEC.

The subroutine EPSODE based on variable coefficient backward differentiation formula can be used. The nonstiff option uses an Adams-Bashforth predictor and an Adams-Moulton corrector.

$$\text{Predictor: } y_{n+1} = y_n + h\sum_{i=1}^{k} \beta_i y'_{n+1-i} \quad \& \quad \text{Corrector: } y_{n+1} = y_n + h\sum_{i=0}^{k} \beta_i y'_{n+1-i}$$

The order may vary from one to seven.

IV. Numerical Implementation

In order to illustrate how the EPISODE package can be used to solve stiff initial value problems, we give here an example, chosen from the areas of chemical kinetics. For each example problem, the appropriate FORTRAN coding for its solution, with EPISODE, is given, followed by the output generated by that coding.

Example 1: A kinetics problem: The following kinetics problem, given by Robertson, is frequently used as an illustrative example. It involves the following three nonlinear rate equations:

$$y'_1 = -.04 y_1 + 10^4 y_2 y_3 \tag{8}$$

$$y'_2 = .04 y_1 - 10^4 y_2 y_3 - 3.10^7 y_2^2 \tag{9}$$

$$y'_3 = 3.10^7 y_2^2 \tag{10}$$

The initial values at $t = 0$ are

$$y_1(0) = 1, \qquad y_2(0) = y_3(0) = 0 \tag{11}$$

Since $\sum y'_i = 0$, the solution must satisfy $\sum y_i = 1$, identically. This identity can be used as an error check.

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Here we intend to solve this problem with the BDF method and use the chord of iteration method with the user-supplied Jacobian (MITER=1). Suppose a local error bound of $\text{EPS} = 10^{-6}$, and control absolute error (IERROR=1). We choose an initial step size of $\text{H0} = 10^{-8}$. The use of MITER=1 requires that the Jacobian $J = \partial f / \partial y$ be calculated and programmed. This is given by

Notes

$$J = \begin{pmatrix} -.04 & 10^4 y_3 & 10^4 y_2 \\ .04 & -10^4 y_3 - 6.10^7 y_2 & -10^4 y_2 \\ 0 & 6.10^7 y_2 & 0 \end{pmatrix}$$

The final value of t is 40. So we consider taking output at $t = 4 \times 10^k$, where $k =$ -1, 0, 1, 2, …. These will be the values of the argument TOUT.

The following coding, together with the EPISODE package, can be used to solve this problems with the options described above. The output of the above program in tabular form is as follows:

Table 2 : MF=21, EPS=10^{-6}

T	H	Y_1	Y_2	Y_3	SUM(Y)-
0.4E+00	0.16E+00	0.98517E+00	0.33864E-04	0.14794E-01	-0.4E-15
0.4E+01	0.56E+00	0.90552E+00	0.22405E-04	0.94462E-01	-0.5E-15
0.4E+02	0.23E+01	0.71582E+00	0.91851E-05	0.28417E+00	-0.3E-16
0.4E+03	0.20E+02	0.45051E+00	0.32228E-05	0.54949E+00	-0.5E-15
0.4E+04	0.24E+03	0.18320E+00	0.89423E-06	0.81680E+00	-0.2E-15
0.4E+05	0.33E+04	0.38986E-01	0.16219E-06	0.96101E+00	0.3E-15
0.4E+07	0.38E+06	0.50319E-03	0.20660E-08	0.99948E+00	0.3E-15
0.4E+10	0.12E+10	0.54561E-06	0.37316E-11	0.10000E+01	-0.1E-13

We see that the equilibrium values are $y_1 = y_2 = 0$, $y_3 = 1$ and that the approach to equilibrium is quite slow. Here we note that the time step, H, rises steadily with time, T. We also observe that the code generated negative and thus physically incorrect answers during the last decade. This reflects instability, or a high sensitivity of the problem to numerical errors at late t, and will, if the integration is continued, lead to answers diverging to $\pm\infty$. The accuracy of the above result can be verified in the usual way- by re running the program with a smaller value of EPS=10^{-9} and nothing else changed, the output in the tabular form is as follows:

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Table 3 : MF=21, EPS=10^{-9}

T	H	Y_1	Y_2	Y_3	SUM(Y)-
0.4E+00	0.34E-01	0.985172E+00	0.338641E-04	0.147940E-01	0.2E-15
0.4E+01	0.14E+00	0.905519E+00	0.224048E-04	0.944589E-01	0.5E-15
0.4E+02	0.13E+01	0.715827E+00	0.918552E-05	0.284164E+00	0.6E-15
0.4E+03	0.82E+01	0.450519E+00	0.322290E-05	0.549478E+00	0.8E-15
0.4E+04	0.76E+02	0.183202E+00	0.894237E-06	0.816797E+00	0.1E-14
0.4E+05	0.88E+03	0.389834E-01	0.162177E-06	0.961016E+00	0.9E-15
0.4E+07	0.20E+06	0.516813E-03	0.206835E-08	0.999483E+00	0.1E-14
0.4E+10	0.67E+09	0.522363E-06	0.208942E-11	0.999999E+00	0.1E-14

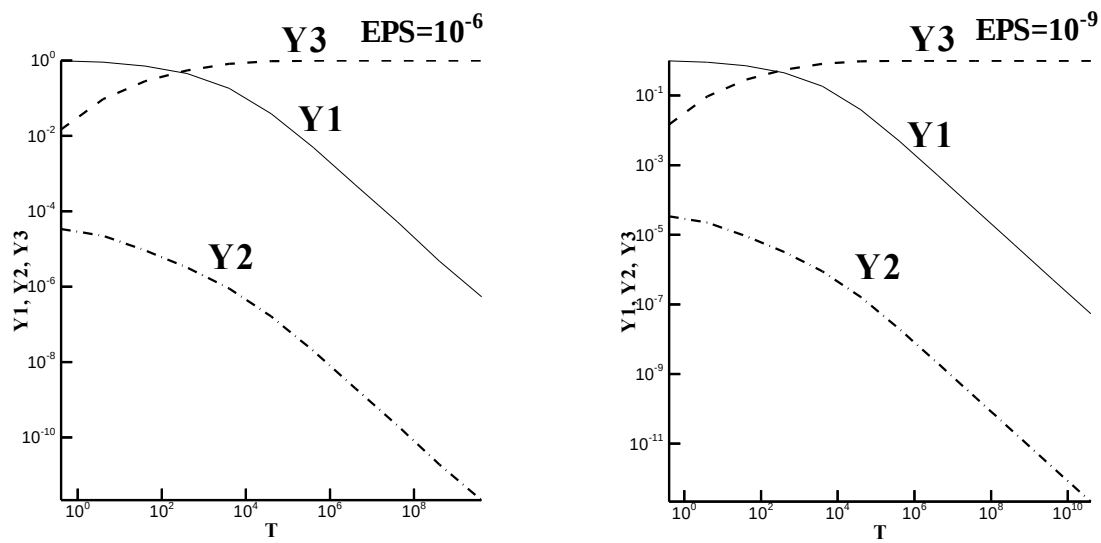

Fig. 1 : The graph of the approximated solution of Example 1 (by using log scale)

Now we consider another example of stiff system of differential equations which can be solved analytically.

Example 2: The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \qquad u_1(0) = \frac{4}{3} \tag{12}$$

$$u_2' = -24u_1 - 51u_2 - 95\cos t + \frac{1}{3}\sin t, \qquad u_1(0) = \frac{2}{3} \tag{13}$$

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has the unique solution

$$u_1(t) = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t. \tag{14}$$

$$u_2(t) = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t \tag{15}$$

Notes

The transient term e^{-39t} in the solution causes this system to be stiff. The results, obtained by EPISODE are summarized in the following table.

Table 4

t	h	Approximated value of $u_1(t)$	Approximated value of $u_2(t)$	Exact value of $u_1(t)$	Exact value of $u_2(t)$
0.0	.40E-02	1.33333333	0.666666666	1.33333333	0.666666666
0 .1	.40E-02	1.79306146	-1.03200020	1.79306300	-1.03200200
0.2	.91E-02	1.42390205	-0.87468033	1.42390200	-0.87468100
0.3	.12E-01	1.13157624	-0.72499799	1.13157700	-0.72499860
0.4	.33E-01	0.90940824	-0.60821345	0.90940860	-0.60821420
0.5	.33E-01	0.73878794	-0.51565752	0.73878780	-0.51565770
0.7	.66E-01	0.49986115	-0.37740429	0.49986030	-0.37740380
1.0	.66E-01	0.27968063	-0.22989065	0.27967490	-0.22988780

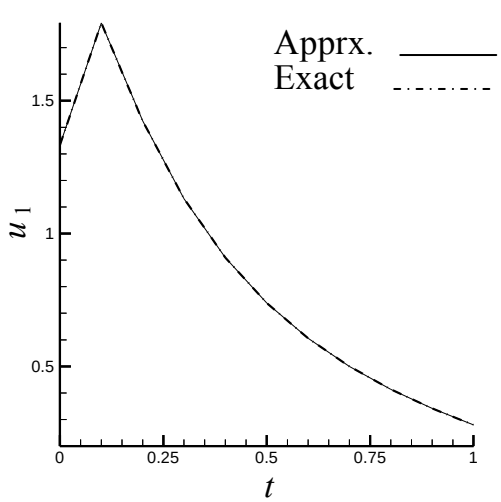

Fig. 2(a) : The graph of the solutions for u_1

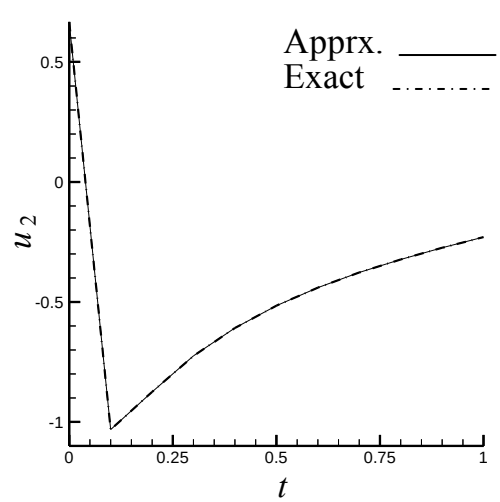

Fig. 2(b) : The graph of the solutions for u_2

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Here we observe that the graph of the approximated solution and the graph of the exact solution coincide with each other.

V. Conclusion

Notes

In this paper our aim is to study the phenomenon of stiff differential equations and the general purpose procedures for the solution of stiff differential equations. There are effective codes available to solve these problems, but it is necessary that the user may have some idea how they work in order to take full advantage of them. Although a number of methods have been developed, and many more basic formulas are suggested for stiff equations, until now there has been little advice or guidance to help a user choose a good method for his problem. In our study we focus on a particularly efficient program which is named as EPISODE. We explain the capabilities of this code and present few practical examples for which it is effective. However, this experimental package EPISODE requires some explanation. First of all, the program is relatively new and has not been used extensively, and so its position in the field of existing available ordinary differential equation software is not yet clear. Secondly we have shown that, for some types of problems, the program spends more time on the linear system of the algorithm than we feel it should. This behavior is related to the extent to which the matrix during the solution of a problem, and in this area improvement of the efficiency of the algorithm is required.

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Some Indefinite Integrals in the Light of Hypergeometric Function

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Abstract - In this paper we have evaluated some indefinite integrals associated to Hypergeometric function. The results represent here are assume to be new.

Keywords : pochhammer symbol; gaussian hypergeometric function; kampé de fériet double hypergeometric function and srivastava's triple hypergeometric function.

GJSFR-F Classification : MSC NO: 33C05,33C45,33C15,33D50,33D60

SOME INDEFINITE INTEGRALS IN THE LIGHT OF HYPERGEOMETRIC FUNCTION

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Some Indefinite Integrals in the Light of Hypergeometric Function

Salahuddin [α] & Intazar Husain [σ]

Abstract - In this paper we have evaluated some indefinite integrals associated to Hypergeometric function. The results represent here are assume to be new.

Keywords : pochhammer symbol; gaussian hypergeometric function; kampé de Fériet double hypergeometric function and srivastava's triple hypergeometric function.

I. Introduction and Preliminaries

The Pochhammer's symbol or Appell's symbol or shifted factorial or rising factorial or generalized factorial function is defined by

$$(b,\ k) = (b)_k = \frac{\Gamma(b+k)}{\Gamma(b)} = \begin{cases} b(b+1)(b+2)\cdots(b+k-1); & \text{if } k = 1,2,3,\cdots \\ 1 \qquad\qquad\qquad\qquad\qquad\qquad ; & \text{if } k = 0 \\ k! \qquad\qquad\qquad\qquad\qquad\qquad ; & \text{if } b = 1,\ k = 1,2,3,\cdots \end{cases}$$

where b is neither zero nor negative integer and the notation Γ stands for Gamma function.

a) Generalized Gaussian Hypergeometric Function

Generalized ordinary hypergeometric function of one variable is defined by

$${}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ b_1, b_2, \cdots, b_B & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!}$$

or

$${}_AF_B\left[\begin{array}{ll} (a_A) & ; \\ (b_B) & ; \end{array} z\right] \equiv {}_AF_B\left[\begin{array}{ll} (a_j)_{j=1}^{A} & ; \\ (b_j)_{j=1}^{B} & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1.1}$$

where denominator parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

b) Kampé de Fjeriet's General Double Hypergeometric Function

In 1921, Appell's four double hypergeometric functions F_1, F_2, F_3, F_4 and their confluent forms $\Phi_1, \Phi_2, \Phi_3, \Psi_1, \Psi_2, \Xi_1, \Xi_2$ were unified and generalized by Kampé de Fériet.

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We recall the definition of general double hypergeometric function of Kampé de Fériet in slightly modified notation of H.M.Srivastava and R.Panda:

$$\mathrm{F}^{A:B;D}_{E:G;H}\left[\begin{array}{ll} (a_A):(b_B);(d_D) & ; \\ (e_E):(g_G);(h_H) & ; \end{array} x,\, y\right] = \sum_{m,n=0}^{\infty} \frac{((a_A))_{m+n}\,((b_B))_m\,((d_D))_n\; x^m\, y^n}{((e_E))_{m+n}\,((g_G))_m\,((h_H))_n\; m!\, n!} \tag{1.2}$$

where for convergence

(i) $A+B<E+G+1,\ A+D<E+H+1 \quad ;|x|<\infty,\quad |y|<\infty,$ or

(ii) $A+B=E+G+1,\ A+D=E+H+1,$ and

$$\begin{cases} |x|^{\frac{1}{(A-E)}} + |y|^{\frac{1}{(A-E)}} < 1 & ,\text{if } E<A \\ \max\ \{|x|,|y|\}<1 & ,\text{if } E\geq A \end{cases}$$

c) *Srivastava's General Triple Hypergeometric Function*

In 1967, H. M. Srivastava defined a general triple hypergeometric function $F^{(3)}$ in the following form

$$F^{(3)}\left[\begin{array}{l} (a_A) :: (b_B);\ (d_D);\ (e_E) : (g_G);\ (h_H);\ (l_L); \\ (m_M) :: (n_N);\ (p_P);\ (q_Q) : (r_R);\ (s_S);\ (t_T); \end{array} x,\ y,\ z\right]$$

$$= \sum_{i,j,k=0}^{\infty} \frac{((a_A))_{i+j+k}\ ((b_B))_{i+j}\ ((d_D))_{j+k}\ ((e_E))_{k+i}\ ((g_G))_i\ ((h_H))_j\ ((l_L))_k\ x^i\ y^j\ z^k}{((m_M))_{i+j+k}\ ((n_N))_{i+j}\ ((p_P))_{j+k}\ ((q_Q))_{k+i}\ ((r_R))_i\ ((s_S))_j\ ((t_T))_k\ i!\ j!\ k!} \tag{1.3}$$

d) *Wright's Generalized Hypergeometric Function*

$${}_p\Psi_q\left[\begin{array}{ll} (\alpha_1,A_1),\cdots,(\alpha_p,A_p) & ; \\ (\lambda_1,B_1),\cdots,(\lambda_q,B_q) & ; \end{array} x\right] = \sum_{m=0}^{\infty} \frac{\Gamma(\alpha_1+mA_1)\Gamma(\alpha_2+mA_2)\cdots\Gamma(\alpha_p+mA_p)x^m}{\Gamma(\lambda_1+mB_1)\Gamma(\lambda_2+mB_2)\cdots\Gamma(\lambda_q+mA_q)m!} \tag{1.4}$$

$${}_p\Psi^*_q\left[\begin{array}{ll} (\alpha_1,A_1),\cdots,(\alpha_p,A_p) & ; \\ (\lambda_1,B_1),\cdots,(\lambda_q,B_q) & ; \end{array} x\right] = \sum_{m=0}^{\infty} \frac{(\alpha_1)_{mA_1}(\alpha_2)_{mA_2}\cdots(\alpha_p)_{mA_p}x^m}{(\lambda_1)_{mB_1}(\lambda_2)_{mB_2}\cdots(\lambda_q)_{mB_q}m!} \tag{1.5}$$

II. Main Integrals

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\sin^3 y]}} =$$

$$= -\cos y \sin^{3m+1} y\ (\sin^2 y)^{\frac{-1-3m}{2}}\ F^{1;2}_{0;1}\left[\begin{array}{ll} \frac{1}{2}\ ;\frac{1}{2},\frac{1-3m}{2} & ; \\ \text{—};\ \frac{3}{2} & ; \end{array} \frac{1+x}{2},\ \cos^2 y\right] + Constant \tag{2.1}$$

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$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\cos^3 y]}} =$$

$$= \frac{\sqrt{-\sin^2 y}\ \ \mathrm{cosec}\ y \cos^{3m+1} y}{3m+1}\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;\frac{1}{2}, \frac{3m+1}{2}\ ; \\ \text{—}; \frac{3m+3}{2}\ ; \end{matrix}\ \frac{1+x}{2}, \cos^2 y\right] + Constant \qquad (2.2)$$

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\tan^3 y]}} = \frac{\tan^{3m+1} y}{(3m+1)}\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;1, \frac{3m+1}{2}\ ; \\ \text{—}; \frac{3m+3}{2}\ ; \end{matrix}\ \frac{1+x}{2}, -\tan^2 y\right] + Constant \qquad (2.3)$$

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\cot^3 y]}} = -\frac{\cot^{3m+1} y}{(3m+1)}\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;1, \frac{3m+1}{2}\ ; \\ \text{—}; \frac{3m+3}{2}\ ; \end{matrix}\ \frac{1+x}{2}, -\cot^2 y\right] + Constant \qquad (2.4)$$

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\ \sec^3\ y]}} =$$

$$= \sin(y)\cos^2(y)^{\frac{3m+1}{2}} \sec^{3m+1}(y)\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;\frac{1}{2}, \frac{1+3m}{2}\ ; \\ \text{—}; \frac{3}{2}\ ; \end{matrix}\ \frac{1+x}{2}, \sin^2 y\right] + Constant \qquad (2.5)$$

$$\int \frac{\mathrm{d}x}{\sqrt{(1-(\frac{1+x}{2})\ \mathrm{cosech}^3\ y)}} =$$

$$= -\cos y(\sin^2(y))^{\frac{3m+1}{2}} \mathrm{cosec}^{3m+1} y\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;\frac{1}{2}, \frac{1+3m}{2}\ ; \\ \text{—}; \frac{3}{2}\ ; \end{matrix}\ \frac{1+x}{2}, \cos^2 y\right] + Constant \qquad (2.6)$$

III. Derivation of Integrals

Derivation of integral (2.1)

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\sin^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\sin^3 y\right]^{-\frac{1}{2}} \mathrm{d}y$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \sin^{3m} y\ \mathrm{dy} = \sum_{\mathrm{m=0}}^{\infty} \frac{(\frac{1}{2})_{\mathrm{m}} (\frac{1+\mathrm{x}}{2})^{\mathrm{m}}}{\mathrm{m!}} \int \sin^{3\mathrm{m}} \mathrm{y\ dy}$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} (-\cos y)\ \sin^{3m+1} y (\sin^2 y)^{\frac{1-3m}{2}}\ {}_2F_1\left[\begin{matrix} \frac{1}{2}, \frac{1-3m}{2}\ ; \\ \frac{3}{2}\ ; \end{matrix}\ \cos^2 y\right] + Constant$$

$$= -\cos y \sin^{3m+1} y\ (\sin^2 y)^{\frac{-1-3m}{2}}\ F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2}\ ;\frac{1}{2}, \frac{1-3m}{2}\ ; \\ \text{—}; \frac{3}{2}\ ; \end{matrix}\ \frac{1+x}{2}, \cos^2 y\right] + Constant \qquad (3.1)$$

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Derivation of integral (2.2)

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\cos^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\cos^3 y\right]^{-\frac{1}{2}} \mathrm{d}y$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \cos^{3m} y \ \mathrm{dy} = \sum_{\mathrm{m}=0}^{\infty} \frac{(\frac{1}{2})_{\mathrm{m}} (\frac{1+\mathrm{x}}{2})^{\mathrm{m}}}{\mathrm{m}!} \int \cos^{3\mathrm{m}} \mathrm{y} \ \mathrm{dy}$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \frac{\sqrt{-\sin^2 y}\cos^{3m+1} y \ \mathrm{cosec}\ y}{(3m+1)} {}_2F_1\left[\begin{matrix} \frac{1}{2}, \frac{3m+1}{2} \\ \frac{3m+3}{2} \end{matrix} ; \cos^2 y\right] + Constant$$

$$= \frac{\sqrt{-\sin^2 y}\ \ \mathrm{cosec}\ y \cos^{3m+1} y}{3m+1} F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2} ; \frac{1}{2}, \frac{3m+1}{2} \\ \text{—}; \frac{3m+3}{2} \end{matrix} ; \frac{1+x}{2}, \cos^2 y\right] + Constant \qquad (3.2)$$

Derivation of integral (2.3)

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\tan^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\tan^3 y\right]^{-\frac{1}{2}} \mathrm{d}y$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \tan^{3m} y \ \mathrm{dy} = \sum_{\mathrm{m}=0}^{\infty} \frac{(\frac{1}{2})_{\mathrm{m}} (\frac{1+\mathrm{x}}{2})^{\mathrm{m}}}{\mathrm{m}!} \int \tan^{3\mathrm{m}} \mathrm{y} \ \mathrm{dy}$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \frac{\tan^{3m+1} y}{(3m+1)} {}_2F_1\left[\begin{matrix} 1, \frac{3m+1}{2} \\ \frac{3m+3}{2} \end{matrix} ; -\tan^2 y\right] + Constant$$

$$= \frac{\tan^{3m+1} y}{(3m+1)} F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2} ; 1, \frac{3m+1}{2} \\ \text{—}; \frac{3m+3}{2} \end{matrix} ; \frac{1+x}{2}, -\tan^2 y\right] + Constant \qquad (3.3)$$

Derivation of integral (2.4)

$$\int \frac{\mathrm{d}y}{\sqrt{[1-(\frac{1+x}{2})\cot^3 y]}} = \int \left[1-\left(\frac{1+x}{2}\right)\cot^3 y\right]^{-\frac{1}{2}} \mathrm{d}y$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \cot^{3m} y \ \mathrm{dy} = \sum_{\mathrm{m}=0}^{\infty} \frac{(\frac{1}{2})_{\mathrm{m}} (\frac{1+\mathrm{x}}{2})^{\mathrm{m}}}{\mathrm{m}!} \int \cot^{3\mathrm{m}} \mathrm{y} \ \mathrm{dy}$$

$$= -\sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1+x}{2})^m}{m!} \frac{\cot^{3m+1} y}{(3m+1)} {}_2F_1\left[\begin{matrix} 1, \frac{3m+1}{2} \\ \frac{3m+3}{2} \end{matrix} ; -\cot^2 y\right] + Constant$$

$$= -\frac{\cot^{3m+1} y}{(3m+1)} F^{1;2}_{0;1}\left[\begin{matrix} \frac{1}{2} ; 1, \frac{3m+1}{2} \\ \text{—}; \frac{3m+3}{2} \end{matrix} ; \frac{1+x}{2}, -\cot^2 y\right] + Constant \qquad (3.4)$$

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Derivation of integral (2.5)

$$\int \frac{\mathrm{d}y}{\sqrt{(1-(\frac{1+x}{2})\ \sec^3\ y)}} = \int \left[1-\left(\frac{1+x}{2}\right)\ \sec^3\ y\right]^{-\frac{1}{2}}\mathrm{d}y$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\sec^{3m}y\ \mathrm{d}y = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\int \sec^{3m}y\ \mathrm{d}y$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\ \sin y\ \cos^2(y)^{\frac{3m+1}{2}}\sec^{3m+1}\ y\ {}_2F_1\left[\begin{matrix}\frac{1}{2}, \frac{3m+1}{2} & ; \\ \frac{3}{2} & ;\end{matrix}\ \sin^2 y\right] + Constant$$

$$= \sin(y)\cos^2(y)^{\frac{3m+1}{2}}\sec^{3m+1}(y)\ F^{1;2}_{0;1}\left[\begin{matrix}\frac{1}{2}\ ;\frac{1}{2}, \frac{1+3m}{2} & ; \\ —; \frac{3}{2} & ;\end{matrix}\ \begin{matrix}\frac{1+x}{2}, \sin^2 y\end{matrix}\right] + Constant \qquad (3.5)$$

Derivation of integral (2.6)

$$\int \frac{\mathrm{d}y}{\sqrt{(1-(\frac{1+x}{2})\ \mathrm{cosec}^3\ y)}} = \int \left[1-\left(\frac{1+x}{2}\right)\ \mathrm{cosec}^3\ y\right]^{-\frac{1}{2}}\mathrm{d}y$$

$$\int \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\mathrm{cosec}^{3m}y\ \mathrm{d}y = \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\int \mathrm{cosec}^{3m}y\ \mathrm{d}y$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m(\frac{1+x}{2})^m}{m!}\ (-\cos y)\ (\sin^2(y))^{\frac{3m+1}{2}}\mathrm{cosec}^{3m+1}\ y\ {}_2F_1\left[\begin{matrix}\frac{1}{2}, \frac{3m+1}{2} & ; \\ \frac{3}{2} & ;\end{matrix}\ \cos^2 y\right] + Constant$$

$$= -\cos y(\sin^2(y))^{\frac{3m+1}{2}}\mathrm{cosec}^{3m+1}y\ F^{1;2}_{0;1}\left[\begin{matrix}\frac{1}{2}\ ;\frac{1}{2}, \frac{1+3m}{2} & ; \\ —; \frac{3}{2} & ;\end{matrix}\ \begin{matrix}\frac{1+x}{2}, \cos^2 y\end{matrix}\right] + Constant \qquad (3.6)$$

IV. Conclusion

In our work we have established hypergeometric form of some indefinite integrals . We can only expect that the development presented in this work will stimulate further interest and research in this important area of classical special functions. Just as the mathematical properties of the Gauss hypergeometric function are already of immense and significant utility in mathematical sciences and numerous other areas of pure and applied mathematics, the elucidation and discovery of the formula of hypergeometric functions considered herein should certainly eventually prove useful to further developments in the broad areas alluded to above.

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Solution of Kinematic Wave Equation Using Finite Difference Method and Finite Element Method

By Dr. M. M. Hossain & J. Ferdous Ema
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Abstract - The Various Numerical Methods are applied to solve the spatially varied unsteady flow equations (Kinematic Wave) in predicting the discharge, depth and velocity in a river. Solutions of Kinematic Wave equations through finite difference method (Crank Nicolson) and finite element method are developed for this study. The computer program is also developed in Lahey ED Developer and for graphical representation Tecplot 7 software is used. Finally some problems are solved to understand the method.

Keywords : kinematic wave, overland flow, channel flow, finite element method, crank-nicolson method.

GJSFR-F Classification : MSC 2010: 51J15 , 81R20, 35R20

SOLUTION OF KINEMATIC WAVE EQUATION USING FINITE DIFFERENCE METHOD AND FINITE ELEMENT METHOD

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Solution of Kinematic Wave Equation Using Finite Difference Method and Finite Element Method

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Abstract - The Various Numerical Methods are applied to solve the spatially varied unsteady flow equations (Kinematic Wave) in predicting the discharge, depth and velocity in a river. Solutions of Kinematic Wave equations through finite difference method (Crank Nicolson) and finite element method are developed for this study. The computer program is also developed in Lahey ED Developer and for graphical representation Tecplot 7 software is used. Finally some problems are solved to understand the method.

Keywords : kinematic wave, overland flow, channel flow, finite element method, crank-nicolson method.

I. Introduction

Hydrology (from Greek : Υδωρ, hudōr, "water"; and λόγος, logos, "study") is the study of the movement, Distribution, and quality of water throughout the Earth and thus addresses both the hydrologic cycle and water resources. So in the broadest sense it is the study of water in all its phases and includes hydraulics, the physics and chemistry of water, meteorology, geology and biology. But the word as used by the scientists and engineers usually has a considerably narrower connotation. In this more limited sense, "Hydrology can be defined as that branch of physical geography, which is concerned with the origin. distributaries movement and properties of the waters of the Earth". The study of hydrology thus concerns itself with the occurrence and transportation of the waters through air, Over the ground and through the strata of the earth and this includes three important phases of what is known as the hydrological cycle, namely rainfall, runoff and evaporation. Hydrology is therefore, bounded above by meteorology, below by geology and at land's end by oceanology. Engineering hydrology includes those segments of hydrology pertinent to the design and operation of engineering projects for the control and use of water. Hydrology means the science of water. It is a branch of earth science. Basically it is an applied science.

Domains of hydrology include hydrometeorology, surface hydrology, hydrogeology, drainage basin management and water quality, where water plays the central role. In general sense hydrology deals with (i) Water resources estimation (ii) Acquisition of processes such as precipitation, runoff and evapo-transporation.

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II. Model Development

a) Kinemtic Wave Equations From Saint Venant Equations

The St. Venant equations characterizing the dynamic flow can be written as:

Continuity:
$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q+(i-\phi) \tag{1}$$

Momentum:
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial y_0}{\partial x}=g(s_f-s_0)-q(\frac{u-v}{A}) \tag{2}$$

The equation (1) may be rewritten in the following form for a ready reference to the various types of wave models that are recognized.

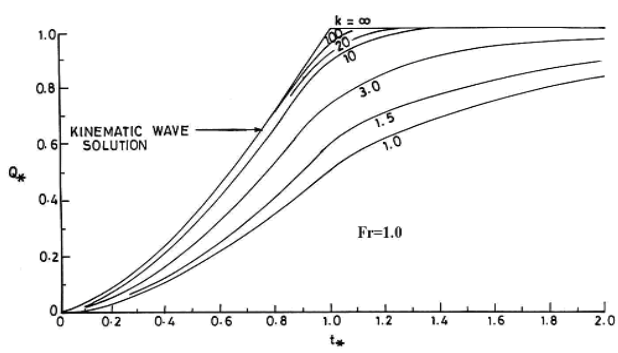
Term: I II III IV Equation of motion: $\frac{1}{g}\frac{\partial u}{\partial t}+\frac{u}{g}\frac{\partial u}{\partial x}+\frac{\partial y_0}{\partial x}+\quad(s_f-s_0)=0$

Local Convective Depth acceleration acceleration slope

Wave model and terms used to describe it are:

Kinematics wave only term	IV= 0
Diffusion wave	III+ IV=0
Steady dynamic wave	II+III+IV=0
Dynamic wave	I+II+III+IV=0
Gravity wave	I+II+III= 0

and other terms are neglected.



b) Hydrodynamic Theory And Kinematic Wave Equations

The hydrodynamic theory for incompressible fluid flows gives the following set of equations (also known as the Navier-Stokes' equations):

$$\rho(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z})=X-\frac{\partial P}{\partial x}+\mu\nabla^2 u$$

$$\rho(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z})=Y-\frac{\partial P}{\partial y}+\mu\nabla^2 v$$

$$\rho(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z})=Z-\frac{\partial P}{\partial z}+\mu\nabla^2 w$$

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and continuity equation: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

where $$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2};$$

ρ = the mass density ;
u, v and w are the velocity components in the x, y and z direction respectively;
X, Y, Z are the body forces per unit volume;
P = pressure and μ = viscosity.

c) Elements Used In Kinematics Wave Models

In this work, for computational purpose, the following two types of elements have been identified:

(i) Overland flow elements and

(ii) Channel flow elements (Fig. 3.1)

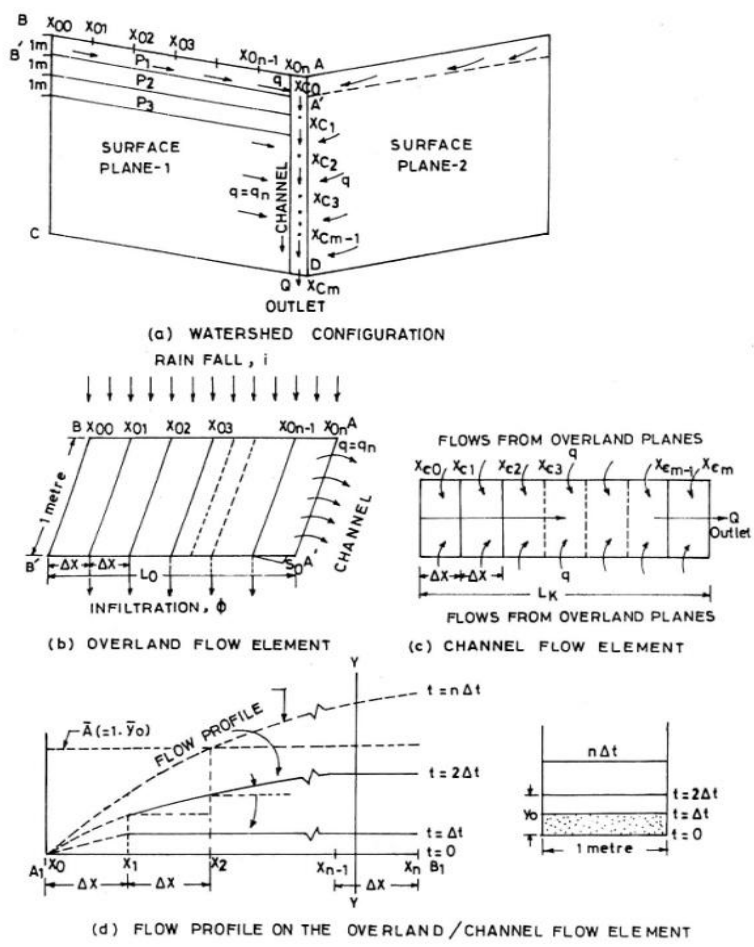

FIG. 3.1 GENERATION OF FLOW PROFILE

d) Trapezoidal Channel Cross Section

A trapezoidal cross-section is the most general type of channel cross-section. It is defined by the cannel side slope (Z), and the channel bottom width (B) (Fig.3.2).

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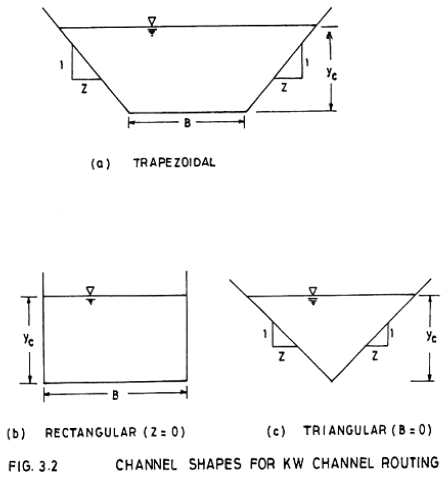

FIG. 3.2 CHANNEL SHAPES FOR KW CHANNEL ROUTING

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e) The Final Form of Kinematic Wave Equations For The Channel Flows

The unknown parameters for the channel shapes under consideration i.e. α_k and m_k being the unknown functions. The KW equation for the channel flow can be written by combining equations (3.19) and (3.20) as given below:

$$\frac{\partial A}{\partial t} + \frac{\partial(\alpha_k A^{m_k})}{\partial x} = q$$

If α_k is independent of x, then the equation becomes:

$$\frac{\partial A}{\partial t} + \alpha_k m_k \frac{\partial(A^{m_k - 1})}{\partial x} = q$$

Crank-Nicolson and other methods:

First Order one-way wave equation

The first order wave equation in one-dimensional space is as follows:

$$u_t = cu_x$$

where c is a positive constant, and u(x, t) is subject to the initial condition

$$u(x, 0) = f(x), \qquad -\infty < x < \infty.$$

The solution for $t \geq 0$ and all x is a family of characteristics, which are straight lines shifted to the left in the x, t- plane, inclined to the x-axis at an angle

$$\Theta = \tan^{-1}\left(\frac{1}{c}\right).$$

The explicit solution is

$$u(x, t) = f(x + ct).$$

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Table 1. Explicit finite difference schemes for first order 1-D wave equation

FD Scheme	Matrix Representation
Forward Euler (FEU) $u_j^{n+1} = u_j^n + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1 & \frac{1}{2}r & & & \\ -\frac{1}{2}r & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \\ & & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -\frac{1}{2}r \\ 0 \\ \vdots \\ 0 \\ \frac{1}{2}r \end{bmatrix}$
Upwind (UPW) $u_j^{n+1} = u_j^n + r\left(u_{j+1}^n - u_j^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1-r & r & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ r \end{bmatrix}$
Lax-Friedrichs (LXF) $u_j^{n+1} = \frac{1}{2}\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 0 & \frac{1-r}{2} & & & \\ \frac{1-r}{2} & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} \frac{1-r}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{1+r}{2} \end{bmatrix}$
Lax-Wendroff (LXW) $u_j^{n+1} = u_j^n + \frac{1}{2}r\left(u_{j+1}^n - u_{j-1}^n\right) + \frac{1}{2}r^2\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \begin{bmatrix} 1-r^2 & \frac{r(r+1)}{2} & & & \\ \frac{r(r-1)}{2} & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} \frac{r(r-1)}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{r(r+1)}{2} \end{bmatrix}$
Leapfrog (LFG) $u_j^{n+1} = u_j^{n-1} + r\left(u_{j+1}^n - u_{j-1}^n\right)$	$\mathbf{u}^{n+1} = \mathbf{u}^{n-1} + \begin{bmatrix} 0 & r & & & \\ -r & \ddots & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -r \\ 0 \\ \vdots \\ 0 \\ r \end{bmatrix}$
Fourth-order Leapfrog (LF4) $u_j^{n+1} = u_j^{n-1} + \frac{4}{3}r\left(u_{j+1}^n - u_{j-1}^n\right) - \frac{1}{6}r\left(u_{j+2}^n - u_{j-2}^n\right)$	$\mathbf{u}^{n+1} = \mathbf{u}^{n-1} + \begin{bmatrix} 0 & \frac{4}{3}r & -\frac{1}{6}r & & & \\ -\frac{4}{3}r & \ddots & \ddots & \ddots & & \\ \frac{1}{6}r & \ddots & \ddots & \ddots & \ddots & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & & \end{bmatrix} \mathbf{u}^n + \begin{bmatrix} -\frac{4}{3}r \\ \frac{1}{6} \\ 0 \\ \vdots \\ -\frac{1}{6}r \\ \frac{4}{3}r \end{bmatrix}$

Finite Element Formulation for Solving KW Equation: $\left.\frac{\partial h}{\partial x}\right|_{x=x_j} = \frac{h_{j+1} - h_{j-1}}{2\Delta x}$

$$h(x,t) = \sum_{j=1}^{M} \Phi_j(x)\, h_j(t)$$

Channel Discretization and Selection of Approximations Functions

The flow equations are one-dimensional. The channel is divided into small reaches called elements. Each element will be modeled with the same flow equations but with different channel geometry and hydraulic parameters. The elements equations are later assembled into global matrix equations for solution. By applying the Galerkin's principle to the continuity equation the following equation is obtained:

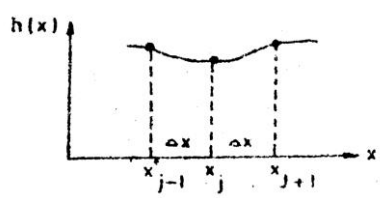

Figure (A) : Finite Difference Computational Mesh

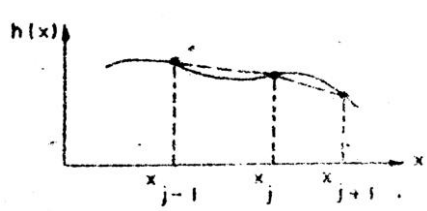

Figure (B) : Finite Element Computational Mesh

$$\sum_{i=I}^{K-1} \int_{x_K}^{x_{K+I}} N^T \left(\frac{\partial y}{\partial t} + y\frac{\partial v}{\partial x} + v\frac{\partial y}{\partial x} - q(x,t)\right) dx = 0$$

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Where $\sum_{1}^{K-1}$ is the expression for summary individual element equation from 1 to (k-1) elements; N^T transpose to the shape functions.
Using the shape functions, Equations may be written as

$$\sum_{1}^{K-1} \int_{0}^{1} N^T \frac{\partial y}{\partial t} + Y \frac{\partial v}{\partial x} + v \frac{\partial y}{\partial x} - q(x,t)\, L ds = 0$$

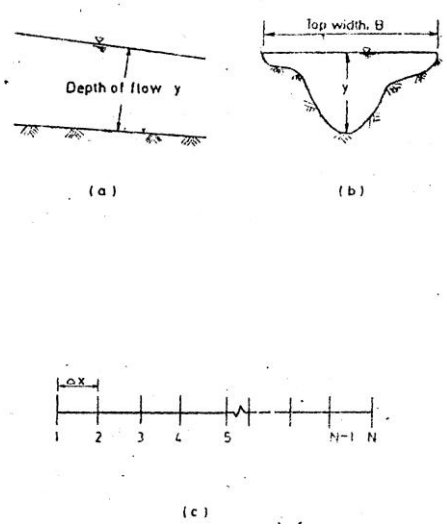

Figure : Natural Idealized Flow Sections (a) Longitudinal Profile (b) vertical Cross sectional Area Flow (c) Longitudinal Channel discretized into finite elements.

Evaluating each term of Equation (5.24) the following elements equation may be written:

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial y_1}{\partial t} \\ \frac{\partial y_2}{\partial t} \end{Bmatrix} + \frac{1}{6}\begin{bmatrix} (2y_1 + y_2)(v_2 - v_1) \\ (y_1 + 2y_2)(v_2 - v_1) \end{bmatrix} + \frac{1}{6}\begin{bmatrix} (2v_1 + v_2)(y_2 - y_1) \\ (v_1 + 2v_2)(y_2 - y_1) \end{bmatrix} - l_q \begin{Bmatrix} 3 \\ 3 \end{Bmatrix} = 0$$

Similar way the momentum Equation for an element can be derived as

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \frac{\partial v_1}{\partial t} \\ \frac{\partial v_2}{\partial t} \end{Bmatrix} + \frac{1}{12}\begin{bmatrix} -2v_1 - v_2 & -v_1 - 2v_2 \\ 2v_1 + v_2 & v_1 + 2v_2 \end{bmatrix}\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} + \frac{q}{l}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} + \frac{l_q}{2}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} (\frac{v}{y})_1 \\ (\frac{v}{y})_2 \end{Bmatrix}$$

$$+ \frac{q}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} S_{f_1} \\ S_{f_2} \end{Bmatrix} + \frac{g s_0 l}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0$$

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III. Formation of Global Matrix

The element properties originally expressed in local coordinates need to be transformed into global coordinates before a solution algorithm is initated. Based on the node to node relationship, it is possible to generate an overall element property matrix for the entire domain, a process called assembling of element equations.

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The concept of discreatization employed earlier is based on the fact that a domain with varying geometric and hydraulic properties can be treated independently as subdomains but systematically from one subdomain to another. Considering N number elements of varying lengths the assembled global matrix equations for continuity and momentum equations become:

$$\begin{bmatrix} 2l_1 & l_1 & \cdots & & & \circ \\ l_1 & 2(l_1+2) & l_2 & & & \\ & l_2 & 2(l_2+l_3) & l_3 & & \\ & \cdots & \cdots & \cdots & & \\ & \circ & & l_i & 2(l_i+l_{i+1}) & l_{i+1} \\ & & & \cdots & \cdots & \cdots \\ & & & & & l_{N-1}\ (l_{N-1}) \end{bmatrix} \begin{Bmatrix} \frac{\partial y_1}{\partial x} \\ \frac{\partial y_2}{\partial x} \\ \frac{\partial y_3}{\partial x} \\ \cdots \\ \frac{\partial y_N}{\partial x} \end{Bmatrix} + \begin{bmatrix} v_2 - 4v_1 & 2v_2+v_1 & & & \\ -v_2-2v_1 & v_3-v_1 & 2v_3-v_z & & \\ & v_3-2v_2 & v_4-v_2 & 2v_4-v_3 & \\ & \cdots & \cdots & \cdots & \\ & & v_{i+1}-2v_i \quad v_{i+2}-v_1 & 2v_{i+2}-v_{i+1} & \\ & & & -v_N-2v_{N-1} & v_N-v_{N-1} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_i \\ y_N \end{Bmatrix}$$

$$-3\begin{Bmatrix} l_1 q_1 \\ q_1 l_1 + q_2 l_2 \\ q_2 l_2 + l_3 q_3 \\ \cdots \\ l_i q_i + l_{i+1} q_{i+1} \\ \cdots \\ l_{N-1} q_{N-1} \end{Bmatrix} = 0$$

In matrix form the global continuity equation can be written as

$$[A]\left\{\frac{dy}{dt}\right\} + [B]\{y\} - \{c\} = 0$$

Where A, B are the matrices and C is the column vector, Y is the dependent variable. The global momentum equation can be formed similarly.

The Solution of time dependent global matrix Equation is sought through a semi discrete approach, This approach requires the time derivative of the dependant variable at each node to be replaced by finite difference scheme (in time domain). Such as the forward, backward, and central differences and are given below with time level k as:

Forward difference, $\frac{dy}{dt} = \frac{y^{k+1} - y^k}{\Delta t}$

Backward difference, $\frac{dy}{dt} = \frac{y^k - y^{k-1}}{\Delta t}$

Central difference, $\frac{dy}{dt} = \frac{y^{k+1} - y^{k-1}}{2\Delta t}$

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Substitution of Equation (5.29a) in Equation (5.28) yields

$$[A]\left\{\frac{y^{k+1}-y^{k}}{\Delta t}\right\}+[B]\left\{y^{k}\right\}-\{c\}=0$$

An implicit equation will be generated from this Equation with the aid of the time weighting factor in the next section.

Development of the Numerical models

The deterministic stream flow models are investigated with three distinct options: (1) the kinematic flow models comprises (a) the simplified version of momentum equation that neglects pressure and inertia terms are compared to friction and gravity terms and (b) the complete form of continuity equation; (2) the diffusion flow models comprises (a) the simplified momentum equation that accounts only for pressure, friction, and gravity terms and (b) the complete form of continuity equation; and (3) the complete flow model comprises (a) the complete form of momentum equation and (b) the complete continuity equation.

The kinematic flow model is investigated in both explicit and implicit sense. The explicit kinematic flow model leads to linear equations. They are solved using a direct method similar to the tridiagonal matrix algorithm set up by varga (1962). The solution proceeds by matrix reduction similar to Gaussian elimination. In contrast the explicit model, the implicit kinematic model yields a set of non-linear tridiagonal matrix equations which are solved by the functional Newton-Raphson iterative method.

The diffusion model as well as the complete flow model each results in a non-linear bitridiagonal matrix equation. The functional Newton-Raphson's method, along with the direct solution algorithm, triangular decomposition technique that yields a recursion algorithm (Douglas, et al, 1959, Von Rosenberg, 1969), is utilized to predict depth and velocity of flow for each option.

Finite Element Kinematic Wave Model

Explicit Model:

The non-linear continuity equation is easily converted to linear form by use of geometric and flow relations:

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}-q(x,t)=0$$

Where, A = Area of flow, L^2 ;

Q = volumetric flow rate, $\frac{L^3}{T}$

The appropriate simplified momentum equation for coupling with the continuity equation has been obtained and is presented below

$$S_f = S_0 = \frac{n_1^2\, v^2}{R^{4/3}} = \frac{v^2 R^{4/3}}{M^2}$$

$$\text{Or} \quad Q = \frac{AR^{2/3}S_0^{1/2}}{n_1} = MAR^{2/3}\, S_0^{1/2}$$

These equations are written in matrix units. For fps units first equation to be divided by 2.216 and the second equation to be multiplied by 1.486.

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Applying the Galerkin's weighted residual method results in the following liner first order ordinary differential equation.

$$\frac{l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \dfrac{\partial A_j}{\partial t} \\ \dfrac{\partial A_2}{\partial t} \end{Bmatrix} + \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} - \frac{ql}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 0$$

For total length of the stream reach the assembled matrix equation becomes:

$$\begin{bmatrix} 2l_1 & l_1 & \cdots & & \circ & \\ l_1 & 2(l_1+l_2) & l_2 & & & \\ & l_2 & 2(l_2+l_3) & l_3 & & \\ & \cdot\cdot & \cdot\cdot\cdot & \cdot\cdot & & \\ & \circ & -l_i & 2(l_i+l_{i+1}) & l_{i+1} & \\ & & & \cdots & \cdots & \cdot\cdot \\ & & & l_{N-1} & 2l_{N-1} & \end{bmatrix}\begin{Bmatrix} \frac{\partial A_1}{\partial t} \\ \frac{\partial A_2}{\partial t} \\ \frac{\partial A_3}{\partial t} \\ \cdots \\ \frac{\partial A_i}{\partial t} \\ \cdots\cdots \\ \frac{\partial AN}{\partial t} \end{Bmatrix} + \begin{Bmatrix} Q_2 - Q_1 \\ Q_3 - Q_1 \\ Q_4 - Q_2 \\ \cdots\cdots\cdots \\ Q_{i+1} - Q_{i-1} \\ \cdots \\ Q_N - Q_{N-1} \end{Bmatrix} - \frac{l}{2}\begin{Bmatrix} l_1 q_1 \\ l_1 q_1 + l_2 q_2 \\ l_2 q_2 + l_3 q_3 \\ \cdots \\ l_i q_i + l_{i+1} q_{i+1} \\ \cdots \\ l_{N-1} q_{N-1} \end{Bmatrix} = 0$$

The above Equation is expressed in a matrix form:

$$[K]\left\{\frac{dy}{dt}\right\} + [D]\{F\} = 0$$

The solution of this Equation is possible upon implementation of the forward differencing in time derivative.

$$[K]\{A\}^{N+1} = [K]\ \{A\}^{N} + \Delta t\ \{F\}^{n} - \Delta t\ \ \{D\}^{N}$$

The solution of the area of flow at various nodes proceeds forward in time with the right hand side evaluated at a previous time level, n. Thus, the Equation can be expressed in more compact form:

$$[K]\ \{A\}^{N+1} = \{X\}^{N}$$

Where X is the known column vector at previous time level. The matrix, K is a linear and tridiagonal type that easily leads to direct solution algorithm. The computer program solving Equation is facilitated by the use of the compact tridiagonal algorithms proposed by Varga (1962). The computed area of flow at current time level, n+1, is used to update cycle is repeated as new time level is reached. The coded explicit finite element scheme exhibits dynamic instability to restriction on the step. This drawback is inherent in explicit numerical schemes, is expected regardless of the finite element approach.

To solve the KW model through the above finite element method one can study the flow problem of overland flow as well as channel flow by using practical data collecting from any river in Bangladesh.

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IV. Conclusion

A hydrological model is an important tool for estimating and organizing quantitative hydrologic information. The main objectives of this thesis is to develop a suitable surface hydrological model for study the movement of overland, (i.e. through its surface runoff) as well as stream flow components of the hydrologic cycle. To achieve these objectives, various techniques and available models were studied. It was concluded that the dynamic approached are the best to account for the physical processes associated with the runoff mechanics of the watersheds. Among these approaches, the kinematic wave theory is the best suited to the prevailing condition.

A further work can be done by developing computer program using these methods to solve KW equation for channel and overland flows for various practical data set collecting from any small river in Bangladesh

Notes

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Classes of Fuzzy Real-Valued Double Sequences Related to the Space ℓ^p

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Abstract - In this article, corresponding to certain general sequence $\phi = \{\phi_{nk}\}$, we introduce the fuzzy real valued double sequence spaces ${}_2m(\phi, p)$ space where $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers, closely related to the space ℓ^p. We study their different properties. We study some algebraic and topological properties of the space ${}_2m(\phi, p)$. Also we obtained the necessary and sufficient conditions for inclusion and equality of ${}_2m(\phi, p)$ and ${}_2m(\psi, p)$.

Keywords : fuzzy real valued double sequence, normal, symmetric etc.

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CLASSES OF FUZZY REAL-VALUED DOUBLE SEQUENCES RELATED TO THE SPACE LP

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Classes of Fuzzy Real-Valued Double Sequences Related to the Space ℓ^p

Bijan Nath [α] & Santanu Roy [σ]

Abstract - In this article, corresponding to certain general sequence $\phi = \{\phi_{nk}\}$, we introduce the fuzzy real valued double sequence spaces ${}_2m(\phi, p)$ space where $p = (p_{nk})$ is a double sequence of bounded strictly positive numbers, closely related to the space ℓ^p. We study their different properties. We study some algebraic and topological properties of the space ${}_2m(\phi, p)$. Also we obtained the necessary and sufficient conditions for inclusion and equality of ${}_2m(\phi, p)$ and ${}_2m(\psi, p)$.

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I. Introduction

To overcome limitations induced by vagueness and uncertainty of real life data, neoclassical analysis [6] has been developed. It extends the scope and results of classical mathematical analysis by applying fuzzy logic to conventional mathematical objects, such as functions, sequences and series etc. Since the introduction of the concept of fuzzy sets by Zadeh [28] in 1965, fuzzy set theory has become an active area of research in science and engineering. The ideas of fuzzy set theory have been used widely not only in many engineering applications, such as, in the computer programming [10], in quantum physics [15], in population dynamics [2], in the control of chaos [9], in bifurcation of non-linear dynamical system [12], but also in various branches of mathematics, such as, theory of metric and topological spaces [8], in the theory of linear systems [17], studies of convergence of sequences of functions [5,13,27] and in approximation theory [1].

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Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. The initial works on double sequences of real or complex terms are found in Bromwich [4]. Hardy [11] introduced the notion of regular convergence for double sequences of real or complex terms. Moričz [16], Basarir and Solancan [3], Choudhary and Tripathy [7], Tripathy and Dutta [22,23], Tripathy and Sarma [24,25] are a few to be named those who studied different aspect of double sequences of fuzzy real numbers

The space $m(\phi)$ was introduced by Sargent [19]. He studied some properties of the space $m(\phi)$. Later on it was studied from sequence space point of view and some matrix classes with one member as $m(\phi)$ were characterized by Rath and Tripathy [18], Tripathy and Sen [26] and many others.

II. Definitions and Background

Throughout the thesis *N*, *R* and *C* denote the sets of **natural**, **real** and **complex** numbers respectively and w, ℓ_∞ denote the spaces of all and bounded sequences of complex terms respectively.

A fuzzy real number *X* is a fuzzy set on R, ie. a mapping $X : R \to L(=[0,1])$

associating each real number *t* with its grade of membership $X(t)$.

A fuzzy real number *X* is said to be **convex** if

$$X(t) \geq X(s) \wedge X(r) = \min [X(s) \wedge X(r)], \text{ where } s < t < r.$$

If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number *X* is said to be **normal**.

A fuzzy real number *X* is said to be **upper semi-continuous** if for each $\varepsilon > 0$, $X^{-1}([0, a+\varepsilon))$, for all $a \in L$ is open in the usual topology of *R*. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $R(L)$. Throughout the thesis, by a fuzzy real number we mean that the number belongs to $R(L)$.

The **α-level** set of a fuzzy real number *X*, $0 < \alpha \leq 1$, is denoted and defined as

$$[X]^\alpha = \{t \in R : X(t) \geq \alpha\}.$$

Ref.

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Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows :

$$\bar{r}(t) = \begin{cases} 1 & \text{if} \quad t = r \\ 0 & \text{otherwise} \end{cases}$$

The additive identity and multiplicative identity in $R(L)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

The **absolute value** $|X|$ of $X \in R(L)$, is defined as (one may refer to Kaleva and Seikkla [42]):

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if} \quad t \geq 0 \\ 0 & \text{if} \quad t < 0. \end{cases}$$

Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R.

If $Y = [Y^L, Y^R]$, then $X \leq Y$ if and only if $X^L \leq Y^L$ and $X^R \leq Y^R$.

Also let $d(X,Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$. Then (D, d) is a complete metric space.

Let $\bar{d} : R(L) \times R(L) \to R$ be defined by

$$\bar{d}(X,Y) = \sup_{0 \leq \alpha \leq 1} d([X]^\alpha, [Y]^\alpha), \text{ for } X, Y \in R(L).$$

Then $\bar{d}$ defines a metric on $R(L)$ and $(R(L), \bar{d})$ is a complete metric space.

A fuzzy real-valued double sequence is a double infinite array of fuzzy real numbers X_{nk} for all $n, k \in N$ and is denoted by (X_{nk}), where $X_{nk} \in R(L)$.

A fuzzy real-valued double sequence (X_{nk}) is said to be **convergent** in Pringsheim's sense to the fuzzy real number X_0, if for every $\varepsilon > 0$, there exists $n_0 = n_0(\varepsilon)$, $k_0 = k_0(\varepsilon) \in N$ such that $\bar{d}(X_{nk}, X_0) < \varepsilon$, for all $n \geq n_0$ and $k \geq k_0$.

A fuzzy real-valued double sequence (X_{nk}) is said to be bounded if $\sup_{n,k} \bar{d}(X_{nk}, \bar{0}) < \infty$.

A fuzzy real valued double sequence space E^F is said to be **solid** (or **normal**) if $(Y_{nk}) \in E^F$, whenever $\bar{d}(Y_{nk}, \bar{0}) \leq \bar{d}(X_{nk}, \bar{0})$ for all $n, k \in N$ and $(X_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **monotone** if E^F contains the canonical pre-image of all its step spaces.

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Throughout π denotes a permutation over $N \times N$. For $X = (X_{nk})$ a given sequence, $S(X)$ denotes the set of all permutation of the elements of (X_{nk}), that is $S(X) = \{(X_{\pi(n,k)})\}$.

A fuzzy real valued double sequence space E^F is said to be **symmetric** if $S(X) \subset E^F$, for all $X \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **sequence algebra** if $(X_{nk} \otimes Y_{nk}) \in E^F$, whenever $(X_{nk}), (Y_{nk}) \in E^F$.

A fuzzy real valued double sequence space E^F is said to be **convergence free** if $(Y_{nk}) \in E^F$, whenever $(X_{nk}) \in E^F$ and $X_{nk} = \bar{0}$ implies $Y_{nk} = \bar{0}$.

Let $\wp$ denote the set of all subsets of N. For any $s \in N$, $\wp_s$ denote the class of all $\sigma \in \wp$ such that σ does not contain more than s elements. $\phi = (\phi_{nk})$ is a non-decreasing sequence such that

$$(n+1)(k+1)\phi_{nk} \geq nk\phi_{n+1,k+1} \text{ for all } n,k \in N.$$

A BK-space is a Banach space of complex double sequences $x = (x_{nk})$ in which the co-ordinate maps are continuous, that is,

$$\left|x_{nk}^{(i)} - x_{nk}\right| \to 0, \text{ whenever } \left\|x^{(i)} - x\right\| \to 0 \text{ as } n,k \to \infty,$$

where $x^{(i)} = (x_{nk}^{(i)})$, for all $i \in N$ and $x = (x_{nk})$.

The space $m(\phi)$ introduced by Sargent [19] is defined as

$$m(\phi) = \left\{(x_k) \in w : \|x\|_{m(\phi)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \sum_{n \in \sigma} |x_k| < \infty\right\}.$$

Tripathy and Sen [26] introduce the sequence spaces $m(\phi, p)$ as follows :

For $1 \leq p < \infty$,

$$m(\phi, p) = \left\{(x_n) \in w : \|x\|_{m(\phi,p)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \left\{\sum_{n \in \sigma} |x_n|^p\right\}^{\frac{1}{p}} < \infty\right\},$$

For $0 < p < 1$,

$$m(\phi, p) = \left\{(x_n) \in w : \|x\|_{m(\phi,p)} = \sup_{s \geq 1, \sigma \in \wp_s} \frac{1}{\phi_s} \sum_{n \in \sigma} |x_n|^p < \infty\right\}.$$

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Generalizing the above sequence spaces, we now introduce the spaces ${}_2m(\phi, p)$ as follows:

$$m(\phi, p) = \left\{ X = (X_{nk}) \in {}_2w^F : \|X\|_{{}_2m(\phi,p)} = \sup_{s\geq 1, \sigma\in\wp_s} \sup_{r\geq 1, \sigma'\in\wp_r} \frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}, \bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}} < \infty \right\};$$

where $p = (p_{nk})$ is a double sequence of bounded strictly positive real numbers.

III. Main Results

Theorem 1. *The class of sequences ${}_2m(\phi, p)$ is a linear space.*

Proof. With standard techniques, we can easily prove the result.

Theorem 2. *The class of sequences ${}_2m(\phi, p)$ is complete.*

Proof. Let $(X^{(i)})$ be a Cauchy sequence in ${}_2m(\phi, p)$ where $X^{(i)} = (X_{nk}^{(i)})$.

$$\sup_{s\geq 1, \sigma\in\wp_s} \sup_{r\geq 1, \sigma'\in\wp_r} \left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}, \bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty, \quad \text{for all } n, k \in N.$$

Then for a given $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$\left\|X^{(i)} - X^{(j)}\right\| < \varepsilon, \text{ for all } i, j \geq n_0.$$

$$\Rightarrow \sup_{s\geq 1, \sigma\in\wp_s} \sup_{r\geq 1, \sigma'\in\wp_r} \left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk}^{(j)})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } n, k \in N \quad \ldots(1)$$

$$\Rightarrow \bar{d}(X_{nk}^{(i)}, X_{nk}^{(j)}) < \varepsilon\phi_1, \text{ for all } i, j \geq n_0, \text{ for all } n, k \in N.$$

Hence for each fixed $n, k \in N$, the sequence $(X_{nk}^{(i)})$ is a Cauchy sequence in $R(L)$.

Since $R(L)$ is complete, the sequence $(X_{nk}^{(i)})$ converges in $R(L)$, for each $n, k \in N$.

Let $\lim_{i\to\infty} X_{nk}^{(i)} = X_{nk}$, for all $n, k \in N$, where $X = (X_{nk})$.

We now show that (i) $X \in {}_2m(\phi, p)$ and

and (ii) $X^{(i)} \to X$.

From equation (1), we get for each fixed s and r

$$\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk}^{(j)})]^{p_{nk}} < \varepsilon^{p_{nk}}(\phi_s\phi_r)^{p_{nk}}, \text{ for all } i, j \geq n_0, \ \sigma \in \wp_s.$$

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Letting $j \to \infty$, we get

$$\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk})]^{p_{nk}} < \varepsilon^{p_{nk}}(\phi_s\phi_r)^{p_{nk}}, \text{ for all } i \geq n_0,\ \sigma \in \wp_s.$$

$$\Rightarrow \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } i \geq n_0 \quad \ldots(2)$$

$$\Rightarrow X^{(i)} - X \in {}_2m(\phi, p), \text{ for all } i \geq n_0.$$

Hence $X = X^{(i)} + (X - X^{(i)}) \in {}_2m(\phi, p)$, since ${}_2m(\phi, p)$ *is a linear space.*

Also (2) $\Rightarrow \left\|X^{(i)} - X\right\|_{{}_2m(\phi,p)} < \varepsilon$, for all $i \geq n_0$.

$$\Rightarrow X^{(i)} \to X \in {}_2m(\phi, p).$$

Hence ${}_2m(\phi, p)$ is complete.

Theorem 3. *The class of sequences* ${}_2m(\phi, p)$ *is a B K-space.*

Proof. By Theorem 2, ${}_2m(\phi, p)$ is a Banach space.

Let $\left\|X^{(i)} - X\right\|_{{}_2m(\phi,p)} \to 0$, as $i \to \infty$.

Then for a given $\varepsilon > 0$, there exists $n_0 \in N$ such that

$$\left\|X^{(i)} - X\right\|_{{}_2m(\phi,p)} < \varepsilon, \text{ for all } i \geq n_0.$$

$$\Rightarrow \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk}^{(i)}, X_{nk})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \varepsilon, \text{ for all } i \geq n_0.$$

$$\Rightarrow \bar{d}(X_{nk}^{(i)}, X_{nk}) < \varepsilon\phi_1, \text{ for all } i \geq n_0, \text{ for all } n, k \in N.$$

$$\Rightarrow \bar{d}(X_{nk}^{(i)}, X_{nk}) \to 0, \text{ as } i \to \infty.$$

Hence ${}_2m(\phi, p)$ *is a B K-space.*

This completes the proof of the theorem.

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Theorem 4. (i) *The class of sequences* ${}_2m(\phi, p)$ *is a symmetric space. If* $X \in {}_2m(\phi, p)$ and $U \in S(X)$, where $S(X)$ denotes the set of all permutation of the elements of (X_{nk}), then $\|U\|_{{}_2m(\phi,p)} = \|X\|_{{}_2m(\phi,p)}$.

(ii) *The class of sequences* ${}_2m(\phi, p)$ *is a normal space.*

Proof. The proof of the result follows from definition.

Theorem 5. ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$ if and only if $\sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Let $\sup_{s\geq 1,}\left(\frac{\phi_s}{\psi_s}\right) = K(<\infty)$.

Then $\phi_s \leq K\psi_s$.

Let $(X_{nk}) \in {}_2m(\phi, p)$. Then

$$\sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\phi_s}\frac{1}{\phi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty$$

$$\Rightarrow \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{K\psi_s}\frac{1}{K\psi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] < \infty$$

$$\Rightarrow \|X\|_{{}_2m(\psi,p)} < \infty.$$

Hence ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$.

Conversely let ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$. we have to show that $\sup_{s\geq 1}\left(\frac{\phi_s}{\psi_s}\right) = \sup_{s\geq 1}(\eta_s) < \infty$.

If possible let $\sup_{s\geq 1,}(\eta_s) = \infty$. Then there exists a subsequence (η_{s_i}) of (η_s) such that $\lim_{i\to\infty}(\eta_{s_i}) = \infty$.

Then for $(X_{nk}) \in {}_2m(\phi, p)$, we have

$$\sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\frac{1}{\psi_s}\frac{1}{\psi_r}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] \geq \sup_{s\geq 1,\sigma\in\wp_s}\ \sup_{r\geq 1,\sigma'\in\wp_r}\left[\eta_{s_i}\frac{1}{\phi_{s_i}}\eta_{r_i}\frac{1}{\phi_{r_i}}\left\{\sum_{n\in\sigma}\sum_{k\in\sigma'}[\bar{d}(X_{nk},\bar{0})]^{p_{nk}}\right\}^{\frac{1}{p_{nk}}}\right] = \infty.$$

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$\Rightarrow (X_{nk}) \in {}_2m(\psi, p)$, a contradiction.

This step concludes the proof of the theorem.

In view of the above theorem, we formulate the following result.

Corollary 1. ${}_2m(\phi, p) = {}_2m(\psi, p)$ if and only if $\sup_{s\geq 1,}(\eta_s) < \infty$ and $\sup_{s\geq 1,}(\eta_s^{-1}) < \infty$, where $\eta_s = \dfrac{\phi_s}{\psi_s}$.

Theorem 7. ${}_2m(\psi, p) = \ell^p$ if and only if $\sup_{s\geq 1,}(\phi_s) < \infty$ and $\sup_{s\geq 1,}(\phi_s^{-1}) < \infty$.

Proof. Putting $\psi_{nk} = 1$, for all $n, k \in N$, in Corollary 1, we get the result easily.

Theorem 8. *If* $0 < p_{nk} < q_{nk} \leq \sup_{n,k} q_{nk}$, then ${}_2m(\phi, p) \subset {}_2m(\phi, q)$.

Proof. Using the properties of ℓ^p spaces, we get the result easily.

Theorem 9. ${}_2m(\phi, p) \subseteq {}_2m(\psi, p)$ *if* $0 < p_{nk} < q_{nk} \leq \sup_{n,k} q_{nk}$ and $\sup_{s\geq 1,}\left(\dfrac{\phi_s}{\psi_s}\right) < \infty$.

Proof. Using the properties of ℓ^p spaces and Theorem 5, we get the result.

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Coefficient Problem for Certain Subclass of Analytic Functions Using Quasi-Subordination

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Abstract - An analytic function f is quasi-subordinate to an analytic function g, in the open unit disk if there exist analytic function φ and w, with $|\varphi(z)| \leq 1$, w (0) = 0 and $|w(z)| < 1$ such that $f(z) = \varphi(z)g(w(z))$. Certain subclasses of analytic univalent functions associated with quasi-subordination are defined and the bounds for the Fekete-Szego coefficient functional $|a_3 - \mu a_2^2|$ for functions belonging to these subclasses are derived.

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COEFFICIENT PROBLEM FOR CERTAIN SUBCLASS OF ANALYTIC FUNCTIONS USING QUASI-SUBORDINATION

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Coefficient Problem for Certain Subclass of Analytic Functions Using Quasi-Subordination

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Abstract - An analytic function f is quasi-subordinate to an analytic function g, in the open unit disk if there exist analytic function φ and w, with $|\varphi(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = \varphi(z)g(w(z))$. Certain subclasses of analytic univalent functions associated with quasi-subordination are defined and the bounds for the Fekete-Szego coefficient functional $|a_3 - \mu a_2^2|$ for functions belonging to these subclasses are derived.

I. Introduction and Motivation

Let A be the class of analytic function f in the open unit disk $D = \{z : |z| < 1\}$ normalized by $f(0) = 0$ and $f'(0) = 1$ of the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. For two analytic functions f and g, the function f is subordinate to g, written as follows:

$$f(z) \prec g(z), \tag{1.1}$$

if there exists an analytic function w, with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$. In particular, if the function g is univalent in D, then $f(z) \prec g(z)$ is equivalent to $f(0) = g(0)$ and $f(D) \subset g(D)$. For brief survey on the concept of subordination, see [1].

Ma and Minda [2] introduced the following class

$$S^*(\phi) = \left\{ f \in A : \frac{zf'(z)}{[f(z)]} \prec \phi(z) \right\}, \tag{1.2}$$

where ϕ is an analytic function with positive real part in D, $\phi(D)$ is symmetric with respect to the real axis and starlike with respect to $\phi(0) = 1$ and $\phi'(0) > 0$. A function $f \in S^*(\phi)$ is called Ma-Minda starlike (with respect to ϕ). The class $C(\phi)$ is the class of functions $f \in A$ for which

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Ref

[1] P. Duren, Subordination, in Complex Analysis, Lecture Notes in Mathematics, Springer, Berlin, Germany, 599 (1977), 22–29.

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$1 + zf''(z)/f'(z) \prec \phi(z)$. The class $S^*(\phi)$ and $C(\phi)$ include several well-known subclasses of starlike functions as special case.

In the year 1970, Robertson [3] introduced the concept of quasi-subordination. For two analytic functions f and g, the function f is quasi-subordinate to g, written as follows:

$$f(z) \prec_q g(z), \tag{1.3}$$

if there exist analytic function φ and w, with $|\varphi(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = \varphi(z)g(w(z))$. Observe that when $\varphi(z) = 1$, then $f(z) = g(w(z))$, so that $f(z) \prec g(z)$ in D. Also notice that if $w(z) = z$, then $f(z) = \varphi(z)g(z)$ and it is said that f is majorized by g and written $f(z) \ll g(z)$ in D. Hence it is obvious that quasi-subordination is a generalization of subordination as well as majorization. See [4,5,6] for works related to quasi-subordination.

Throughout this paper it is assumed that ϕ is analytic in D with $\phi(0) = 1$. Motivated by [2,3], we define the following classes.

Definition 1.1. *Let the class $R_q^*(\alpha, \phi)$ consists of functions $f \in A$ satisfying the quasi-subordination*

$$\frac{z^{1-\alpha} f'(z)}{[f(z)]^{1-\alpha}} - 1 \prec_q \phi(z) - 1, \quad \alpha \geq 0 \tag{1.4}$$

Example 1.2. *The function $f : D \to C$ defined by the following*

$$\frac{z^{1-\alpha} f'(z)}{[f(z)]^{1-\alpha}} - 1 = z[\phi(z) - 1], \quad \alpha \geq 0 \tag{1.5}$$

belongs to the class $R_q^*(\alpha, \phi)$.

It is well known (see [10]) that the nth coefficient of a univalent function $f \in A$ is bounded by n. The bounds for coefficient give information about various geometric properties of the function. Many authors have also investigated the bounds for the Fekete-Szego coefficient for various classes [11, 12, 13,14, 15,16,17,18,19,20,21,22,23,24,25]. In this paper, we obtain coefficient estimates for the functions in the above defined classes.

Let Ω be the class of analytic functions w, normalized by $w(0) = 0$, and satisfying the condition $|w(z)| < 1$. We need the following lemma to prove our results.

Lemma 1.3 (see [26]). *If $w \in \Omega$, then for any complex number f*

$$|w_2 - tw_1^2| \leq \max\{1; |t|\}. \tag{1.6}$$

The result is sharp for the functions $w(z) = z^2$ or $w(z) = z$.

Ref.

[3] M.S. Robertson, Quasi-subordination and coefficient conjectures, Bulletin of the American Mathematical Soceity, 76 (1970), 1–9.

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II. Main Results

Throughout, let $f(z) = z+a_2z^2+a_3z^3+\cdots$, $\phi(z) = 1+B_1z+B_2z^2+B_3z^3+\cdots$, $\varphi(z) = c_0 + c_1z + c_2z^2 + c_3z^3 + \cdots$, $B_1 \in R$ and $B_1 > 0$.

Theorem 2.1. *If $f \in A$ belongs to $R_q^*(\alpha, \phi)$, then*

$$|a_2| \leq \frac{B_1}{1+\alpha},$$
$$|a_3| \leq \frac{B_1}{2}\left(1 + \max\left\{1, B_1\left|\frac{1-\alpha}{1+\alpha} + \frac{\alpha}{2B_1}\right| + \left|\frac{B_2}{B_1}\right|\right\}\right) \tag{2.1}$$

and for any complex number μ,

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2}\left(1 + \max\left\{1, B_1\left|\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2} + \frac{\alpha}{2B_1}\right| + \left|\frac{B_2}{B_1}\right|\right\}\right). \tag{2.2}$$

Proof. If $f \in R_q^*(\alpha, \phi)$, then there exist analytic functions φ and w, with $|\varphi(z)| \leq 1$, $w(0) = 0$ and $|w(z)| < 1$ such that

$$\frac{z^{1-\alpha}f'(z)}{[f(z)]^{1-\alpha}} - 1 = \varphi(z)(\phi(w(z)) - 1). \tag{2.3}$$

Since

$$\phi(w(z)) - 1 = B_1w_1z + (B_1w_2 + B_2w_1^2)z^2 + \cdots$$

$$\varphi(z)(\phi(w(z)) - 1) = B_1c_0w_1z + (B_1c_1w_1 + c_0(B_1w_2 + B_2w_1^2))z^2 + \cdots \tag{2.4}$$

it follows from (2,3) that

$$a_2 = \frac{B_1c_0w_1}{(1+\alpha)}$$
$$a_3 = \frac{1}{2+\alpha}\left[\frac{\alpha}{2}B_1c_0w_1 + B_1c_1w_1 + B_1c_0w_2 + c_0\left(\left(\frac{1-\alpha}{1+\alpha}\right)B_1^2c_0 + B_2\right)w_1^2\right] \tag{2.5}$$

Since $\varphi(z)$ is analytic and bounded in D, we have [27, page 172]

$$|c_n| \leq 1 - |c_0|^2 \leq 1 \quad (n > 0). \tag{2.6}$$

By using this fact and the well-known inequality, $|w_1| \leq 1$, we get

$$|a_2| \leq \frac{B_1}{1+\alpha}. \tag{2.7}$$

Ref

[27] Z. Nehari, Conformal mapping, Dover, New York, NY, USA, 1975, Reprinting of the 1952 edition.

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Further,

$$a_3 - \mu a_2^2 = \frac{1}{2+\alpha}\left[B_1c_1w_1 + c_0(B_1w_2 + \frac{\alpha}{2}B_1w_1 + \left(B_2 + \left(\frac{1-\alpha}{1+\alpha}\right)B_1^2c_0 - \frac{2\mu}{(1+\alpha)^2}B_1^2c_0\right)w_1^2\right]. \tag{2.8}$$

Then

$$|a_3 - \mu a_2^2| \leq \frac{1}{2+\alpha}\Bigg(|B_1c_1w_1| + \left|B_1c_0\left(w_2 - \left(\frac{2\mu}{(1+\alpha)^2}B_1c_0 - \left(\frac{1-\alpha}{1+\alpha}\right)B_1c_0 + \frac{\alpha}{2}\frac{w_1}{c_0} - \frac{B_2}{B_1}\right)w_1^2\right)\right|\Bigg). \tag{2.9}$$

Again applying $|c_n| \leq 1$ and $|w_1| \leq 1$, we have

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2+\alpha}\left(1 + \left|w_2 - \left(\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right)B_1c_0 - \frac{B_2}{B_1}\right)w_1^2\right|\right). \tag{2.10}$$

Applying Lemma 1.3 to

$$\left|w_2 - \left(\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right)B_1c_0 - \frac{B_2}{B_1}\right)w_1^2\right| \tag{2.11}$$

yields

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2+\alpha}\left(1 + \max\left\{1, \left|\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right)B_1c_0 - \frac{B_2}{B_1}\right|\right\}\right). \tag{2.12}$$

Observe that

$$\left|\frac{\alpha}{2} - \left(\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2}\right)B_1c_0 - \frac{B_2}{B_1}\right| \leq B_1|c_0|\left|\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2} + \frac{\alpha}{2B_1}\right| + \left|\frac{B_2}{B_1}\right|, \tag{2.13}$$

and hence we can conclude that

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{2}\left(1 + \max\left\{1, B_1\left|\frac{1-\alpha}{1+\alpha} - \frac{2\mu}{(1+\alpha)^2} + \frac{\alpha}{2B_1}\right| + \left|\frac{B_2}{B_1}\right|\right\}\right). \tag{2.14}$$

For $\mu = 0$, the above will reduce to estimate of $|a_3|$.

Theorem 2.2. *If $f \in A$ satisfies*

$$\frac{z^{1-\alpha}f'(z)}{[f(z)]^{1-\alpha}} - 1 \ll \phi(z) - 1, \tag{2.15}$$

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then the following inequalities hold:

$$|a_2| \leq \frac{B_1}{1+\alpha},$$

$$|a_3| \leq \frac{1}{2+\alpha}(B_1 + B_1^2 + |B_2|), \tag{2.16}$$

and, for any complex number μ,

$$|a_3 - \mu a_2^2| \leq \frac{1}{(2+\alpha)(1+\alpha)^2}((1+\alpha)^2 B_1 + |(1+\alpha)^2 - (2+\alpha)\mu| B_1^2 + (1+\alpha)^2 |B_2|). \tag{2.17}$$

Proof. The result follows by taking $w(z) = z$ in the proof of Theorem 2.1.

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- Never confuse figures with tables - there is a difference.

Approach

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- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

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References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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