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Global Attractor of the Nonlinearfour Order Wave Equations

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GLOBAL ATTRACTOR OF THE NONLINEARFOUR ORDER WAVE EQUATIONS

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[1] Eisley JG: Nonlinear vibration of beams and rectangular plates. Z.Angew. Math. Phys. 15, 167-175 (1964)

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I. Introduction

Nonlinear four order wave equations can be applied in mechanics of elastic constructions, and have a long history([1]-[11]). But few of the studies consider attractor of the nonlinear four order wave equations. In this paper we study attractor of the following nonlinear four order wave equations

$$u_{tt}+\triangle^2 u+\alpha u_t=f(u),\quad x\in\Omega,\quad t>0, \tag{1.1}$$

$$u=\triangle u=0,\quad x\in\partial\Omega,\quad t>0, \tag{1.2}$$

$$u(x,0)=\varphi(x),\quad u_t(x,0)=\psi(x),\quad x\in\Omega, \tag{1.3}$$

where $\alpha>0$, Δ is the Laplacian operator, Ω denotes an open bounded set of $R^n(n=1,2,3)$ with smooth boundary $\partial\Omega$, and $f(s)\in C^1(R)$ satisfies the following conditions

$$F(s)=\int_\Omega f(s)dx\le c, \tag{1.4}$$

$$sf(s)-F(s)\le c, \tag{1.5}$$

and there is a $0\le r<\infty$, such that

$$\lim_{s\to\infty}\frac{f'(s)}{s^r}=0. \tag{1.6}$$

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As far as the theory of infinite-dimensional dynamical system is concerned, we refer to [12]-[22]. In the study of infinite dimensional dynamical system, the long-time behavior of the solution to equations is an important issue. The long-time behavior of the solution to equations can be shown by the global attractor with the finite-dimensional characteristics. Some authors have already studied the existence of the global attractor for some evolution equations. The global attractor strictly defined as ω-limit set of ball, which under additional assumptions is nonempty, compact, and invariant ([14],[16]). We investigate global attractor of the equations (1.1)-(1.3) in this article.

The paper is organized as follows. In Section 2 we recall preliminary results. In Section 3, we obtain global attractor of the equations.

II. Preliminaries

Let X and X_1 be two Banach spaces, $X_1 \subset X$ a compact and dense inclusion. Consider the abstract nonlinear evolution equation defined on X, given by

$$\begin{cases} \frac{du}{dt} = Lu + G(u), \\ \\ u(x,0) = u_0. \end{cases} \tag{2.1}$$

where $u(t)$ is an unknown function, $L : X_1 \to X$ a linear operator, and $G : X_1 \to X$ a nonlinear operator.

A family of operators $S(t) : X \to X (t \geq 0)$ is called a semigroup generated by (2.1) if it satisfies the following properties:

(1) $S(t) : X \to X$ is a continuous map for any $t \geq 0$,

(2) $S(0) = id : X \to X$ is the identity,

(3) $S(t+s) = S(t) \cdot S(s), \forall t, s \geq 0$. Then, the solution of (2.1) can be expressed as

$$u(t, u_0) = S(t)u_0.$$

Next, we introduce the concepts and definitions of invariant sets, global attractors, and ω-limit sets for the semigroup $S(t)$.

Definition 2.1 Let $S(t)$ be a semigroup defined on X. A set $\Sigma \subset X$ is called an invariant set of $S(t)$ if $S(t)\Sigma = \Sigma, \forall t \geq 0$. An invariant set Σ is an attractor of $S(t)$ if Σ is compact, and there exists a neighborhood $U \subset X$ of Σ such that for any $u_0 \in U$,

Ref

[12] Babin AV, Vishik MI: Attractors of Evolution Equations, North-Holland, Amsterdam, 1992.

$$\inf_{v\in\Sigma}\|S(t)u_0 - v\|_X \to 0, \quad \text{as} \quad t \to \infty.$$

In this case, we say that Σ attracts U. Especially, if Σ attracts any bounded set of X, Σ is called a global attractor of $S(t)$ in X.

For a set $D \subset X$, we define the ω-limit set of D as follows

$$\omega(D) = \bigcap_{s\geq 0}\overline{\bigcup_{t\geq s} S(t)D},$$

where the closure is taken in the X-norm. Lemma 2.2 is the classical existence theorem of global attractor by Temam [16].

Lemma 2.2 Let $S(t) : X \to X$ be the semigroup generated by (2.1). Assume the following conditions hold

(1) $S(t)$ has a bounded absorbing set $B \subset X$, i.e., for any bounded set $A \subset X$ there exists a time $t_A \geq 0$ such that $S(t)u_0 \in B, \forall u_0 \in A$ and $t > t_A$;

(2) $S(t)$ is uniformly compact, i.e., for any bounded set $U \subset X$ and some $T > 0$ sufficiently large, the set $\overline{\bigcup_{t\geq T} S(t)U}$ is compact in X.

Then the ω-limit set $\mathcal{A} = \omega(B)$ of B is a global attractor of (2.1), and $\mathcal{A}$ is connected providing B is connected.

Definition 2.3[15] We say that $S(t) : X \to X$ satisfies $C-$condition, if for any bounded set $B \subset X$ and $\varepsilon > 0$, there exist $t_B > 0$ and a finite dimensional subspace $X_1 \subset X$, such that $\{PS(t)B\}$ is bounded, and

$$\|(I - P)S(t)u\|_X < \varepsilon, \quad \forall t \geq t_B \text{ and } u \in B,$$

where $P : X \to X_1$ is a projection.

Lemma 2.4[15] Let $S(t) : X \to X(t \geq 0)$ be a dynamical systems. If the following condition are satisfied

(1) there exists a bounded absorbing set $B \subset X$,

(2) S(t) satisfies $C-$condition,

then $S(t)$ has a global attractor in X.

We introduce the spaces

$$H = L^2(\Omega), \quad V = \{u \in H^2 | u|_{\partial\Omega} = \triangle u|_{\partial\Omega} = 0\}, \quad E = H \times V.$$

Ref

[16] Temam R: Infinite-Dimensional Dynamical Systems in Mechanics and physics. Applied Mathematical Science, Vol. 68, Springer-Verlag, New York, 2nd., 1997

The denotations $(\cdot,\cdot), \|\cdot\|$ are the inner product and norm in H respectively, i.e., for any $u, v \in H$, $(u,v) = \int_\Omega uvdx$, $\|u\| = (\int_\Omega |u|^2 dx)^{\frac{1}{2}}$. The norm in V is $\|u\|_V = \|\triangle u\|$ and we have $\|\triangle u\|^2 \geq \beta_1 \|u\|^2$, where β_1 is the first eigenvalue of Δu with boundary condition (1.2).

Lemma 2.5 Assume the nonlinear term $f : R \to R$ satisfies (1.6). Then the nonlinear operator $f : V \to H$ is compact.

Proof. Let $\{u_n\}$ be a bounded sequence in V. By the Soblev embedding theorem, $\{u_n\}$ is bounded in $L^k(\Omega)$ for any $1 \leq k < \infty$, and has a convergent subsequence in $L^2(\Omega)$. Without loss of generality, we assume $\{u_n\}$ converges to u_0 in H. It is sufficient to prove that $\{f(u_n)\}$ converges to $f(u_0)$ in H.

From (1.6), for any $\eta > 0$, we have

$$|f'(s)|^2 \leq \eta |s|^{2r} + c_\eta,$$

where $c_\eta \to 0$ if $\eta \to 0$.

Then, for some $0 \leq \varepsilon \leq 1$, we obtain

$$\begin{aligned}
\int_\Omega |f(u_n) - f(u_0)|^2 dx &= \int_\Omega |f'(u_0 + (1-\varepsilon)(u_n - u_0))|^2 |u_n - u_0|^2 dx \\
&\leq \eta \int_\Omega |u_0 + (1-\varepsilon)(u_n - u_0)|^{2r} |u_n - u_0|^2 dx + c_\eta \int_\Omega |u_n - u_0|^2 dx \\
&= \eta \int_\Omega |\varepsilon u_0 + (1-\varepsilon) u_n)|^{2r} |u_n - u_0|^2 dx + c_\eta \int_\Omega |u_n - u_0|^2 dx \\
&\leq 4^r \eta \int_\Omega (|u_0|^{2r} + |u_n|^{2r}) |u_n - u_0|^2 dx + c_\eta \int_\Omega |u_n - u_0|^2 dx \\
&\leq 4^r \eta [(\int_\Omega |u_0|^{2rp} dx)^{\frac{1}{p}} + (\int_\Omega |u_n|^{2rp} dx)^{\frac{1}{p}}](\int_\Omega |u_n - u_0|^{2q} dx)^{\frac{1}{q}} + c_\eta \int_\Omega |u_n - u_0|^2 dx
\end{aligned}$$

where $p > 0, q > 0$, and $\frac{1}{p} + \frac{1}{q} = 1$.

Let $n \to \infty$ and $\eta \to 0$. We have

$$\int_\Omega |f(u_n) - f(u_0)|^2 dx = 0.$$

Then the nonlinear operator $f : V \to H$ is compact.

III. Existence of Global Attractor

Theorem 3.1 Let $\varphi \in H$, $\psi \in V$, and f satisfy the conditions (1.4) and (1.5). Suppose that the problem (1.1)-(1.3) has a unique weak solution and $S(t), t > 0$, defined by $S(t)(\varphi, \psi) =$

Notes

$(u(t), u_t(t))$, is the semigroup generated by the problem (1.1)-(1.3). Then $S(t)$ has a bounded absorbing ball.

Proof. Take the inner product of (1.1) in H with $v = u_t + \theta u$, $0 < \theta \leq \theta_0$, and θ_0 will be chosen later. we obtain

Notes

$$(u_{tt}, v) + (\triangle^2 u, v) + \alpha(u_t, v) = (f, u).$$

Using the condition (1.4), we have

$$\frac{1}{2}\frac{d}{dt}(\|v\|^2 + \|\triangle u\|^2 - 2\int_\Omega F(u)dx) + (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2$$

$$+(\theta^2 - \alpha\theta)(u, v) - \theta\int_\Omega f(u)udx = 0. \tag{3.1}$$

Using the Hölder inequality, Young inequality and the condition (1.5), we get

$$(\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 + (\theta^2 - \alpha\theta)(u, v) - \theta\int_\Omega f(u)udx$$

$$\geq (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 + (\theta^2 - \alpha\theta)\|u\|\|v\| - \theta\int_\Omega (F(u) + c)dx$$

$$\geq (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 - \frac{\alpha\theta}{\beta_1^{\frac{1}{2}}}\|\triangle u\|\|v\| - \theta\int_\Omega (F(u) + c)dx$$

$$\geq (\alpha - \theta)\|v\|^2 + \theta\|\triangle u\|^2 - \frac{\theta}{2}\|\triangle u\|^2 - \theta\frac{\alpha^2}{2\beta_1}\|v\|^2 - \theta\int_\Omega (F(u) + c)dx$$

$$\geq [\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1})]\|v\|^2 + \frac{\theta}{2}\|\triangle u\|^2 - \theta\int_\Omega (F(u) + c)dx.$$

Choose θ_0, such that $\theta_0(1 + \frac{\alpha^2}{2\beta_1}) = \frac{\alpha}{2}$. Hence, $\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1}) \geq \frac{\alpha}{2} \geq \theta$.

From (3.1), we have

$$\frac{1}{2}\frac{d}{dt}(\|v\|^2 + \|\triangle u\|^2 - 2\int_\Omega F(u)dx) + \frac{\alpha}{2}\|v\|^2 + \frac{\theta}{2}\|\triangle u\|^2 - \theta\int_\Omega (F(u) + c)dx \leq 0.$$

Then, it follows that

$$\frac{d}{dt}[\|v\|^2 + \|\triangle u\|^2 + 2\int_\Omega (c - F(u))dx] + \theta[\|v\|^2 + \|\triangle u\|^2 + 2\int_\Omega (c - F(u))dx]$$

$$\leq 4c\theta|\Omega|, \tag{3.2}$$

where $|\Omega|$ is measure of the Ω.

Let $y(t) = \|v\|^2 + \|\triangle u\|^2 + 2\int_\Omega (c - F(u))dx$. Then $y(t) \geq 0$ and (3.2) can be read as

$$\frac{dy}{dt} + \theta y \leq 4c\theta|\Omega|.$$

Global Journal of Science Frontier Research (F) Volume XIII Issue VII Version I Year 2013

Using the Gronwall inequality, we have

$$y(t) \leq y(0)e^{-\theta t} + 4c|\Omega|(1 - e^{-\theta t}), \quad t \geq 0.$$

For any bounded set B of E, $(\varphi, \psi) \in B$ and $\int_\Omega [c - F(\varphi)]dx$ is bounded. Then

$$R(B) = \sup_{(\varphi,\psi)\in B} y(0) = \sup_{(\varphi,\psi)\in B} \{\|\varphi\|_V^2 + \|\psi + \theta\varphi\| + 2\int_\Omega (c - F(\varphi))dx\} < \infty,$$

and

$$\lim_{t\to\infty} \sup_{(\varphi,\psi)\in B} y(t) \leq 4c|\Omega| = \mu_0^2.$$

Let $\mu_1 > \mu_0$ be fixed, and there is a $t_0 = t_0(R(B), \mu_1) = \frac{1}{\theta} \ln \frac{R(B)}{\mu_1^2 - \mu_0^2}$ such that for any $t \geq t_0$, we have $y(t) \leq \mu_1^2$. Then

$$\begin{aligned}
\|u(t)\|_V^2 + \|u_t(t)\|^2 &\leq \|u(t)\|_V^2 + 2\|u_t(t) + \theta u(t)\|^2 + 2\theta^2\|u(t)\|^2 \\
&\leq \|u(t)\|_V^2 + 2\|u_t(t) + \theta u(t)\|^2 + \frac{2\theta^2}{\beta_1}\|u(t)\|_V^2 \\
&\leq 2(1 + \frac{\theta^2}{\beta_1})(\|u(t)\|_V^2 + \|u_t(t) + \theta u(t)\|^2) \\
&\leq 2(1 + \frac{\theta^2}{\beta_1})y(t) \leq 2(1 + \frac{\theta^2}{\beta_1})\mu_1^2.
\end{aligned}$$

Let $\rho_0^2 = 2(1 + \frac{\theta^2}{\beta_1})\mu_1^2$. Then For all $t \geq t_0$, we have

$$\|u\|_V^2 + \|u_t\|^2 \leq \rho_0^2, \tag{3.3}$$

which implies that the ball of E, $B_0 = B_E(0, \rho_0)$, centered at 0 of radius ρ_0, is an absorbing set.

Theorem 3.2 Assume $f : R \to R$ satisfies (1.4)-(1.6). Then the semigroup $S(t), t \geq 0$ associated with problem (1.1)-(1.3) possesses a global attractor.

Proof. The eigenvalue equation

$$\begin{cases} \Delta^2 u = \beta u, \\ u|_{\partial\Omega} = \triangle u|_{\partial\Omega} = 0, \end{cases} \tag{3.4}$$

has eigenvalues $\beta_1, \beta_2, \cdots, \beta_k, \cdots$and eigenvector $\{e_k | k = 1, 2, 3, \cdots\}$, and $0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_k \geq \cdots$. If $k \to \infty$, $\beta_k \to \infty$. $\{e_k | k = 1, 2, 3, \cdots\}$ constitutes an orthogonal base of H.

For $\forall u \in H$, we have

$$u = \sum_{k=1}^{\infty} u_k e_k, \quad \|u\|_{L^2}^2 = \sum_{k=1}^{\infty} u_k^2,$$

Introduce subspace $E_1 = span\{e_1, e_2, \cdots, e_k\} \subset H$. Let E_2 be an orthogonal subspace of $E_1 \subset H$.

For $\forall (u_t, u) \in E$, we find

$$u_t = u_{1t} + u_{2t}, \quad u_{1t} = P_m u_t, \quad u_{2t} = (I - P_m)u_t,$$

and

$$u = u_1 + u_2, \quad u_1 = P_m u \quad u_2 = (I - P_m)u,$$

where $P_m : H \to E_1$ be the orthogonal projection.

Let $P_i : E \to E_i \times E_i$ be the orthogonal projection. Thanks to Definition 2.3, we will prove that for any bounded set $B \subset E$ and $\varepsilon > 0$, there exists $t_* > 0$ such that

$$\|P_1 S(t) B\|_H \leq M, \quad \forall t > t_*, \quad M \quad \text{is a constant}, \tag{3.5}$$

$$\|P_2 S(t) B\|_H \leq \varepsilon, \quad \forall t > t_*, \quad \varphi \in B. \tag{3.6}$$

From Theorem 3.1, $S(t)$ has an absorbing set B_M. Then for any bounded set $B \subset E$, there exists $t_0 > 0$ such that $S(t)B \subset B_M$, $\forall t > t_0$, which imply (3.5).

From Lemma 2.5, $f : V \to H$ is compact, then for any $\varepsilon > 0$, there exists some $m \in N$ such that

$$\|(I - P_m)f\| \leq \varepsilon.$$

Multiply (1.1) by $v_2 = u_{2t} + \theta u_2$ $(0 < \theta \leq \theta_1)$, and integrate over Ω. We obtain

$$\int_\Omega (u_{tt} + \triangle^2 u + \alpha u_t) v_2 dx = \int_\Omega f v_2 dx.$$

Then, we have

$$\frac{1}{2}\frac{d}{dt}(\|v_2\|^2 + \|\triangle u_2\|^2) + (\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2$$

$$+(\theta^2 - \alpha\theta)(u_2, v_2) \leq (f, v_2) \leq \|(I - P_m)f\|\|v_2\| \leq \varepsilon\|v_2\| \leq \frac{\alpha}{4}\|v_2\|^2 + \frac{\varepsilon^2}{\alpha}. \tag{3.7}$$

Using the Hölder inequality and Young inequality, we get

$$(\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2 + (\theta^2 - \alpha\theta)(u_2, v_2)$$

$$\geq (\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2 + (\theta^2 - \alpha\theta)\|u_2\|\|v_2\|$$

$$\geq (\alpha - \theta)\|v_2\|^2 + \theta\|\triangle u_2\|^2 - \frac{\alpha\theta}{\beta_{m+1}^{\frac{1}{2}}}\|\triangle u_2\|\|v\|$$

Notes

Global Journal of Science Frontier Research (F) Volume XIII Issue VII Version I Year 2013

$$\geq (\alpha-\theta)\|v_2\|^2+\theta\|\triangle u_2\|^2-\frac{\theta}{2}\|\triangle u_2\|^2-\theta\frac{\alpha^2}{2\beta_{m+1}}\|v_2\|^2$$

$$\geq [\alpha-\theta(1+\frac{\alpha^2}{2\beta_{m+1}})]\|v_2\|^2+\frac{\theta}{2}\|\triangle u_2\|^2.$$

Choose θ_1, such that $\theta_1(1+\frac{\alpha^2}{2\beta_1})=\frac{\alpha}{2}$. Hence, $\alpha-\theta(1+\frac{\alpha^2}{2\beta_1})\geq\frac{\alpha}{2}\geq\theta$.

From (3.7), we have

$$\frac{1}{2}\frac{d}{dt}(\|v_2\|^2+\|\triangle u_2\|^2)+\frac{\alpha}{4}\|v_2\|^2+\frac{\theta}{2}\|\triangle u_2\|^2\leq\frac{\varepsilon^2}{\alpha}.$$

Then, it follows that

$$\frac{d}{dt}(\|v_2\|^2+\|\triangle u_2\|^2)+\theta(\|v\|^2+\|\triangle u\|^2)\leq\frac{2\varepsilon^2}{\alpha}. \tag{3.8}$$

Using the Gronwall inequality, we have

$$\|v_2(t)\|^2+\|\triangle u_2(t)\|^2\leq(\|v_2(0)\|^2+\|\triangle u_2(0)\|^2)e^{-\theta t}+\frac{2\varepsilon^2}{\alpha}(1-e^{-\theta(t-t_0)}),\quad t\geq t_0.$$

where t_0 is given in the proof of Theorem 3.1. Then

$$\|v_2(0)\|^2+\|\triangle u_2(0)\|^2\leq\rho_0^2.$$

Let $t_1-t_0=\frac{1}{\theta}\ln\frac{\rho_0^2\alpha}{\varepsilon^2}$. Hence

$$\|v_2(t)\|^2+\|\triangle u_2(t)\|^2\leq\frac{3\varepsilon^2}{\alpha},\quad t\geq t_1.$$

Then

$$\|u_2(t)\|_V^2+\|u_{2t}(t)\|^2\leq\|u_2(t)\|_V^2+2\|u_{2t}(t)+\theta u_2(t)\|^2+2\theta^2\|u_2(t)\|^2$$

$$\leq\|u_2(t)\|_V^2+2\|u_{2t}(t)+\theta u_2(t)\|^2+\frac{2\theta^2}{\beta_{m+1}}\|u_2(t)\|_V^2$$

$$\leq 2(1+\frac{\theta^2}{\beta_{m+1}})(\|u_2(t)\|_V^2+\|u_{2t}(t)+\theta u_2(t)\|^2)$$

$$\leq 2(1+\frac{\theta^2}{\beta_{m+1}})(\|v_2(t)\|^2+\|\triangle u_2(t)\|^2)\leq(1+\frac{\theta^2}{\beta_{m+1}})\frac{6\varepsilon^2}{\alpha}.$$

Let $C=\frac{6}{\alpha}(1+\frac{\theta^2}{\beta_{m+1}})$. Then for all $t\geq t_1$, we have

$$\|u\|_V^2+\|u_t\|^2\leq C\varepsilon^2, \tag{3.9}$$

which implies (3.6).

From Lemma 2.4, the equations(1.1)-(1.3) possess a global attractor.

Notes

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