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The Integration of Certain Products of the $\overline{H}$ - function with Extended Jaboci Polynomials

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Abstract – The object of this paper is to derive a finite integral pertaining to two $\overline{H}$-functions with extended Jacobi-polynomial. In the particular cases we have discussed the integration of product of a certain class of Feynman integral with our main integral. Application of the main result have also been discussed with the Riemann-Liouville type fractional integral operator. The results derived here are basic in nature and they are likely to be useful applications into several fields notably electromagnetic theory, statistical mechanics and probability theory.

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THE INTEGRATION OF CERTAIN PRODUCTS OF THE H - FUNCTION WITH EXTENDED JABOCI POLYNOMIALS

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The Integration of Certain Products of the $\overline{H}$-function with Extended Jaboci Polynomials

V.B.L. Chaurasia [α] & Gulshan Chand [σ]

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I. Introduction

The $\overline{H}$-function introduced by Inayat-Hussain ([9], see also [1]) in terms of Mellin-Barnes type contour integral is defined as follows

$$\overline{H}^{M,N}_{P,Q}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{N+1,Q}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\Phi(\xi)z^{\xi}\,d\xi, \tag{1.1}$$

where

$$\Phi(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

which contains fractional powers of some of the Γ–functions. Here and throughout the paper a_j (j=1,…,P) and b_j(j = 1,…,Q) are complex parameters, $\alpha_j\geq 0(j=1,\ldots,P)$, $\beta_j\geq 0(j=1,\ldots,Q)$, (not all zero simultaneously) and throughout A_j (j = 1,…,N) and B_j (j=M+1,…,Q) can take on non-integer values.

The contour in (1.2) is imaginary and Re(ξ) = 0. It is suitably idented in Γ–function and to keep these singularities on appropriate side. Again, for A_j (j=1,…,N)

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Ref.

9. Hai, N.T. and Yakubovich, S.B. – The Double Mellin-Barnes Type Integrals and Their Application to Convolution Theory, World Scientific Publishing Co. Pvt. Ltd., Singapore, New Jersey, London, Hongkong, 1992.

not an integer, the poles of the Γ–functions of the numerator in (1.2) are converted to branch points. However, as long as there is no coincidence of poles from any $\Gamma(b_j-\beta_j\xi)$ (j = 1,…,M) and $\Gamma(1-a_j+\alpha_j\xi)$ (j = 1,…,N) pair, the branch cuts can be chosen so that the path of integration can be distorted in the usual manner.

For the sake of brevity

$$T=\sum_{j=1}^{M}|\beta_j|+\sum_{j=1}^{N}A_j\alpha_j-\sum_{j=M+1}^{Q}|B_j\beta_j|-\sum_{j=N+1}^{P}\alpha_j>0.$$

Notes

II. Main Integral

$$\int_a^b (x-a)^{\beta}(b-x)^{\sigma}F_n(\beta,\alpha;x)$$

$$.\ \overline{H}_{P,Q}^{M,N}\left[z(b-x)^k\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$.\ \overline{H}_{P',Q'}^{M',N'}\left[z'(b-x)^{k'}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N'},(a'_j,\alpha'_j)_{N'+1,P'}\\(b'_j,\beta'_j)_{1,M'},(b'_j,\beta'_j;B'_j)_{M'+1,Q'}\end{matrix}\right.\right]dx$$

$$=\sum_{h=1}^{M'}\sum_{r=0}^{\infty}\frac{\prod_{\substack{j=1\\j\neq h}}^{M'}\Gamma(b'_j-\beta'_j\xi_{h,r})\prod_{j=1}^{N'}\{\Gamma(1-a'_j+\alpha'_j\xi_{h,r})\}^{A'_j}}{\prod_{j=1+M'}^{Q'}\{\Gamma(1-b'_j+\beta'_j\xi_{h,r})\}^{B'_j}\prod_{j=1+N'}^{P'}\Gamma(a'_j-\alpha'_j\xi_{h,r})}$$

$$.\ \frac{(z')^{k'\xi_{h,r}}(-1)^{r+n}\lambda^n(b-a)^{\beta+\sigma+1}\Gamma(1+\beta+n)}{r!\ \ \beta_h\ \ n!}$$

$$.\ \overline{H}_{P+2,Q+2}^{M,N+2}\left[z(b-a)^k\left|\begin{matrix}(\alpha-\sigma-k'\xi_{h,r},k;1),(-\sigma-k'\xi_{h,r},k;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(\alpha+n-\sigma-k'\xi_{h,r},k;1)(-1-\sigma-\beta-n-k'\xi_{h,r},k;1)\end{matrix}\right.\right],\qquad(2.1)$$

where

(i) $k\geq 0, k'\geq 0;$

(ii) $\mathrm{Re}\left(\sigma+k\frac{b_j}{\beta_j}+k'\frac{b'_j}{\beta'_j}\right)>0; j=1,\ldots, M$

(iii) $|\arg(z)| < \frac{1}{2} T\pi, T > 0, |\arg(z')| < \frac{1}{2} T'\pi$

(iv) $F_n(\beta, \alpha; x)$ is Fujiwara polynomials [8].

(v) $\lambda = u(b-a)$.

Proof

To establish (2.1), we express the $\bar{H}$-functions in series from and contour form as in (1.2) respectively, and then interchanging the order of summations and integrations which is permissible under the conditions stated, solving the remaining integral with the help of a known result Chiney and Bhonsle ([4], p.9, eqn. (3.1)), and thus, interpreting the result in the desired form.

Special Cases

(i) Putting M = 1, N = 3 = P = Q, and replacing z by – z in (1.2), and using

$$g(\gamma,\eta,\tau,p;y) = \frac{E_{d-1}\Gamma(p+1)\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^p\, 2^{2+p}\pi^{1/2}\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}$$

$$.\ H_{3,3}^{1,3}\left[-y \left| \begin{matrix} (1-\gamma,1;1),(1-\gamma+\frac{\tau}{2},1;1),(1-\eta,1);1+p) \\ (0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p) \end{matrix} \right.\right], \tag{3.1}$$

where $E_d = \frac{2^{1-d}\pi^{-d/2}}{\Gamma(d/2)}$ ([11], p.4121,eqn. (5))

The above function is connected with a certain class of Feynman integrals. We get

$$\int_a^b (x-a)^{\beta}(b-x)^{\sigma} F_n(\beta,\alpha;x)\, g(\gamma,\eta,\tau,p;g(b-x)^k)$$

$$.\ \bar{H}_{P',Q'}^{M',N'}\left[z'(b-x)^{k'} \left| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N'},(a'_j,\alpha'_j)_{N'+1,P'} \\ (b'_j,\beta'_j)_{1,M'},(b'_j,\beta'_j;B'_j)_{M'+1,Q'} \end{matrix} \right.\right] dx$$

$$= \sum_{h=1}^{M'}\sum_{r=0}^{\infty} \frac{\prod_{j=1, j\neq h}^{M'}\Gamma(b'_j-\beta'_j\xi_{h,r})\prod_{j=1}^{N'}\{\Gamma(1-a'_j+\alpha'_j\xi_{h,r})\}^{A'_j}}{\prod_{j=1+M'}^{Q'}\{\Gamma(1-b'_j+\beta'_j\xi_{h,r})\}^{B'_j}\prod_{j=1+N'}^{P'}\Gamma(a'_j-\alpha'_j\xi_{h,r})}$$

$$.\ \frac{(z')^{k'\xi_{h,r}}(-1)^{r+n}\lambda^n(b-a)^{\beta+\sigma+1}\Gamma(1+\beta+n)E_{d-1}\Gamma(p+1)\Gamma(\frac{1}{2}+\frac{\tau}{2})}{r!\ \ \beta_h\ \ n!(-1)^p 2^{2+p}\pi^{1/2}\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}$$

Ref.

4. Chiney, S.P.and Bhonsle, B.R. – Some results involving extended Jacobi polynomials Rev. Univ. Nac. Tucumán, A, mat. fiu.teor. Tucumán, ISSN0080-2360,V-25, No 1-(1975), 7-11.

$$. \overline{H}_{5,5}^{1,5}\left[-z(b-a)^{k}\left|\begin{matrix}(\alpha-\sigma-k'\xi_{h,r},k;1),(-\sigma-k'\xi_{h,r},k;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},(1-\gamma,1;1)\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(\alpha+n-\sigma-k'\xi_{h,r},k;1),(-1-\sigma-\beta-n-k'\xi_{h,r},k;1)\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(1-\gamma+\frac{\tau}{2},1;1),(1-\eta,1;1+p)\\(0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p)\end{matrix}\right], \tag{3.2}$$

valid under the condition as surrounding (2.1).

III. Application

We shall define the Riemann-Liouville fractional derivative of function f(x) of order σ (or, alternatively, −σth order fractional integral) ([5], p.181, 11, p.49) by

$${}_{\alpha}D_{x}^{\sigma}\{f(x)\}=\begin{cases}\dfrac{1}{\Gamma(\sigma)}\displaystyle\int_0^x (x-t)^{-\sigma-1}f(t)\,dt, & Re(\sigma)\geq 0\\ \dfrac{d^q}{dx^q}\,{}_{\alpha}D_x^{\sigma-q}\{f(x)\}, & (q-1)\leq Re(\sigma)<q,\end{cases} \tag{4.1}$$

where q is a positive integer and the integral exists.

For $\alpha=0$, we have $D_x^{\sigma}\equiv {}_0D_x^{\sigma}$.

Now, replacing b by x and a = 0 in the main result, it can be rewritten as the following fractional integral formula

$$.\ D_x^{-\sigma}\left\{x^{\beta}\overline{H}_{P,Q}^{M,N}\left[z(x-t)^{k}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]\right.$$

$$\left.. \overline{H}_{P',Q'}^{M',N'}\left[z'(x-t)^{k'}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N'},(a'_j,\alpha'_j)_{N'+1,P'}\\(b'_j,\beta'_j)_{1,M'},(b'_j,\beta'_j;B'_j)_{M'+1,Q'}\end{matrix}\right.\right]F_n(\beta,\alpha;t)\right\}$$

$$=\sum_{h=1}^{M'}\sum_{r=0}^{\infty}\frac{\prod_{j=1,\,j\neq h}^{M'}\Gamma(b'_j-\beta'_j\xi_{h,r})\prod_{j=1}^{N'}\{\Gamma(1-a'_j+\alpha'_j\xi_{h,r})\}^{A'_j}}{\prod_{j=1+M'}^{Q'}\{\Gamma(1-b'_j+\beta'_j\xi_{h,r})\}^{B'_j}\prod_{j=1+N'}^{P'}\Gamma(a'_j-\alpha'_j\xi_{h,r})}$$

$$.\ \frac{(-1)^{r+n}\lambda^{n}(z')^{k'\xi_{h,r}}x^{\beta+\sigma+k'\xi_{h,r}}\Gamma(1+\beta+n)}{r!\;n!\beta_h\;\Gamma(\sigma)}$$

Ref.

5. Erdélyi, A. et al. – Tables of Integral Transforms, Vol.II, McGraw-Hill, New York, 1954.

$$\cdot \; \overline{H}^{M,N+2}_{P+2,Q+2}\left[z\,x^{k} \left| \begin{matrix} (\alpha-\sigma-k'\xi_{h,r},k;1),(-\sigma-k'\xi_{h,r},k;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(\alpha+n-\sigma-k'\xi_{h,r},k;1),(-1-\sigma-\beta-n-k'\xi_{h,r},k;1) \end{matrix} \right. \right], \qquad (4.2)$$

where $\mathrm{Re}(\sigma) > 0$ and all other conditions of validity mentioned with (2.1) are satisfied.

Notes

The results recently derived by Gupta and Soni in [6], Chaurasia and Srivastava in [2] and Chaurasia and Pandey in [3] can be obtained on giving suitable values to the parameters and arguments. The result given in (4.2) is also quite general in nature and can easily yield Riemann-Liouville fractional integrals of large number of simpler functions and polynomials merely by specializing the parameters of H and F_n appearing in it which may find applications in electromagnetic theory, statistical mechanics and probability theory.

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Notes

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