Basic Model of the Stationary X-ray Induced Conductivity of Wide-Gap Semiconductors

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\textbf{Abstract} - The concept of construction and stage-by-stage development of basic model of stationary X-ray induced conductivity for wide-gap semiconductors is proposed. Within the limits of first stage, calculation of spatial distribution of the generated electrons and holes in an ideal crystal which absorbs X-ray flux is carried out and volt-ampere characteristic is calculated.

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\section{Introduction}

Nowadays semiconductor detectors of X-ray and gamma radiation are the most perspective devices for realization of radioactive control in all fields of the industry, power and a science \cite{1-4}. They have considerable advantages in comparison with scintillation and gas detectors, and also high potential of betterment in the near future. Deriving and examination of materials with wide-gap (for example, ZnSe \cite{5, 6}; SiC \cite{7, 8}) is perspective direction in the physics of semiconductors which have characteristics of dielectrics (due to $E_{\text{gap}} > 2.5 \text{ eV}$). Under condition of moderate values of an impurity concentration and flaws of a crystalline lattice such materials can be used effectively for the solution of problems of radioactive control in extreme requirements at high temperatures and considerable dose rate \cite{6}. For use wide-gap semiconductors as detectors of ionizing radiation with low intrinsic conductivity becomes actual developing a model of stationary X-ray induced conductivity of wide-band semiconductors which will allow to obtain such integral characteristics, as efficiency of gathering of a charge, volt-ampere and lux-ampere dependences. The X-rays that does not lead to formation of structural flaws in a crystal material even at the considerable levels of excitation have been considered as a base case.

\section{Model for a Kinetics of X-ray Induced Conductivity in Wide-gap Semiconductors}

First of all, the kinetic model of X-ray conductivity at first needs complete examination of a kinetics of movement of the generated free charge carriers when only one X-ray quantum absorbs in the semiconductor. Consideration of an ideal crystal is a convenient method of constructing the basic mathematical model for X-ray induced conductivity. Its subsequent complication associated with the introduction of components in the charge-transfer equation that describe the localization and recombination of electrons and holes in traps and recombination centers. Such an analysis has been conducted in several works \cite{9, 10} and allowed to determine the influence of main characteristics of the material on the kinetics of charge collection in the crystal and on the value of X-ray induced conductivity. Studies have shown that for correct calculation of the kinetics of charge collection in the crystal, firstly, it is necessary to calculate spatial distribution of electrons and holes \((N^\pm(x, y, z, t))\) whose motion is determined by the diffusion and drift in an external electric field:

\begin{equation} N^\pm(x, y, z, t) = \frac{N_0}{4\pi D^\pm t} \exp \left\{ \frac{\mu^\pm E_0 \cdot (x-x_0)}{2D^\pm} \right\} \sum_{d=\pm} \exp \left\{ -\left( \frac{\pi n_d}{d} \right)^2 D^\pm t \sin \left( \frac{\pi n_d}{d} \right) \sin \left( \frac{\pi n_d}{d} \right) \right\}, \end{equation}

where \(N_0\) - an amount of the generated electrons and holes after absorption of X-ray quantum, \(x_0\) - coordinate of absorption of a X-ray quantum in the semiconductor, \(d\) - distance between electrodes on crystal, \(E_0\) - intensity of an external electric field, \(D^\pm\) - diffusion constants of charge carriers, \(\mu^\pm\) - mobilities of charge carriers. While considering the absorption of one X-ray photon another important conclusion can be made: the value of the intrinsic electric field generated electron and hole, which quickly relaxes during 0.1-1 ns,\cite{9}, is small. This means that during the drift of electrons and holes at low excitation levels Coulomb interaction between them can be neglected. However, in many practical problems it is important to measure not only separate impulses of a current in a crystal which arise at absorption of separate quanta, but also measuring common a gamma- or X-ray induced conductivity. In this case it is necessary to consider the absorption in a crystal of a stream of quanta, to consider their spatial distribution, allocation of the localized charge carriers on traps, electric fields of the free and localized electrons and holes. The logic of building of a kinetics X-ray induced conductivity at absorption of one X-ray quantum can be used to develop the model.
Stationary X-ray induced conductivity arises during prolonged irradiation of semiconductors by the stream of quanta, when average concentrations of the generated electrons and holes reach equilibrium values. In general case it is possible to distinguish some consecutive stages in development of model stationary X-ray induced conductivity. First stage - development of a basic model of stationary X-ray induced conductivity for an ideal semiconductor, which includes the choice of the geometry of the detector, the calculation of spatial distributions of electrons and holes which drift in an external electric field, the calculation of its own electric field, the calculation of spatial distributions of electrons and holes which drift in an external electric field, the calculation of its own electric field for an ideal semiconductor, which includes the choice of the geometry of the detector, the calculation of spatial distributions of electrons and holes which drift in an external electric field, the calculation of its own electric field. Second stage of development of model stationary X-ray induced conductivity is an addition to model of the ideal semiconductor by traps for electrons and holes. Depending on quantity of time of drift of electrons and holes and time of localization on traps it is possible to consider traps shallow or deep. The third stage is an adding of recombination centers to model of the semiconductor with traps. Results obtained at the third stage of modeling may be compared with the results of experimental measurements of current-voltage and lux-ampere characteristics.

III. THE SPATIAL DISTRIBUTION OF CONCENTRATIONS OF ELECTRONS AND HOLES IN AN IDEAL SEMICONDUCTOR

Let’s choose the following scheme of measuring stationary X-ray induced conductivity. On rectangular sample \((d \times L \times H)\) directed the flow of X-rays and its direction is perpendicular to the vector of external electric field \(E_0 = U_0/d\). The model of the detector and the scheme of measuring of a current of stationary X-ray induced conductivity are shown on fig. 1. In the plane OXZ at the depth \(y\) in the layer \(dy\) per second absorbed by the following number of quanta: \(dF(y) = F_0 \cdot \kappa_X \cdot \exp(-\kappa_X \cdot y) \cdot dy\), where \(F_0\) - the flux of X-rays \((\text{cm}^{-2} \cdot \text{s}^{-1})\), \(\kappa_X\) - the absorption coefficient of X-rays at absorption of one X-ray quantum will create on the average \(N_g = \hbar v_x / 3E_g\) the electron-hole pairs, where \(\hbar v_x\) – energy of X-ray quantum, \(E_g\) – band gap of the semiconductor. Immediately after the generation some electrons and holes may recombine with each other at the centers of recombination (glow), so their number is at the stage of the drift can vary by \(\eta \leq 1\) times.

![Figure 1: Model of the detector and the scheme of measuring of a current of stationary X-ray induced conductivity (top view).](image)

The rate of the generation of electrons and holes \(N_g(y)\) at a depth \([y; y+dy]\):

\[
N_g(y) = N_{g0} \cdot e^{-\kappa_X \cdot y}, \quad N_{g0} = F_0 \left(\frac{\hbar v_x}{3E_g}\right) \cdot \kappa_X
\]  

(2)

The kinetics equations of motion of electrons and the holes in the ideal semiconductor:

\[
\frac{\partial N^\pm}{\partial t} = N_g + D \cdot \Delta N^\pm + \mu \cdot \nabla \left(\frac{E_0}{N^\pm}\right),
\]  

(3)

And boundary conditions: \(N^+(0) = N^+(d) = 0\). In the case of stationary X-ray induced conductivity \(\partial N^+/\partial t = 0\). In the equations (3) the electric field which is created by electrons and holes at drift is not considered. It superimposes restriction on the peak value of rate of generation of charge carriers: \(N_{g0} < 10^{14} \text{ cm}^{-3} \cdot \text{s}^{-1}\). The equations (3) have following approximate solutions:

\[
\begin{align*}
N^-(x,y) &\approx \frac{N_{g0} \cdot e^{-x \cdot x / \mu^+ \cdot E_0}}{\mu^- \cdot E_0} \cdot \left(1 - e^{-\frac{E_0 \cdot x}{k_B T}}\right), \\
N^+(x,y) &\approx \frac{N_{g0} \cdot e^{-x \cdot x / \mu^+ \cdot E_0}}{\mu^- \cdot E_0} \cdot \left(x - \frac{E_0 \cdot x}{k_B T} - 1\right)
\end{align*}
\]  

(4)

During calculations Einstein’s relation has been used: \(D^\pm = \mu^\pm \cdot k_B \cdot T / e\), where \(k_B\) - a Boltzmann constant, \(T\) - crystal temperature, \(e\) – elementary electronic charge. Spatial distributions of electrons and holes essentially depend on their mobility and intensity of an exterior electric field. Result of calculation of spatial distributions of electrons and holes in ZnS are shown on fig.2. at \(E_0 = 0.1 \text{ V/cm}\) and \(E_0 = 1.0 \text{ V/cm}\) (for saving of one scale the electron concentration is incremented in 23 times, since \(\mu^- / \mu^+ \approx 23\)). In the absence of an external electric field \((E_0 = 0)\) the spatial distribution of electrons and holes is determined only by diffusion and recombination at the electrodes and consequently is the symmetrical. The drift current of
non-equilibrium carriers in an external electric circuit determined by the ratio Ramo-Shockley [11]:

\[ i_x(U) = \frac{U}{d^2} \left( q^-(U) \cdot \mu^- + q^+(U) \cdot \mu^+ \right) \]  

(5)

Fig.2: Spatial distributions of electrons and holes on surface ZnSe at \( E_0 = 0.1 \) V/cm (1a - electrons, 1b - holes) and \( E_0 = 1 \) V/cm (2a - electrons, 2b - holes); \( y = 0, d = 1 \) cm, \( N_{G0} = 10^{11} \) cm\(^{-3}\)-s\(^{-1}\), \( \mu^- = 700 \) cm\(^2\) V\(^{-1}\) s\(^{-1}\), \( \mu^+ = 30 \) cm\(^2\) V\(^{-1}\) s\(^{-1}\), \( k_X = 240 \) cm\(^{-1}\).

The charge of electrons and holes that create a drift current:

\[ q^\pm(U) = e \cdot H \cdot \int_0^d \int_0^L N^\pm(x,y) \cdot dxdy, \]  

(6)

Where \( H \) - height of crystal on axis OZ. Current of stationary X-ray induced conductivity:

\[ i_x(U) = e \cdot F_0 \cdot \left( \frac{h \nu_X}{3E_g} \cdot \eta \right) \cdot d \cdot H \cdot \left( 1 - \frac{2k_g \cdot T}{e \cdot U} \right) \]  

(7)

The peak current of stationary X-ray induced conductivity in the ideal semiconductor is equal to a charge generation rate at absorption of a stream of X-ray quanta:

\[ \lim_{U \to \infty} i_x(U) = e \cdot F_0 \cdot d \cdot H \cdot \left( \frac{h \nu_X}{3E_g} \cdot \eta \right) = \frac{dQ_0}{dt} \]  

(8)

It is obvious that formula (8) can also be obtained by adding up all of the current pulses from the individual X-rays, which is true only for the ideal model of a semiconductor at low excitation levels. Thus, when the concentration of the generated electrons and holes in an ideal crystal are negligible and their electric field can be neglected, the current-voltage characteristic is described by a sublinear function, which quickly reaches saturation (at \( E_0 > 10 \) V/cm). It is worth to note that in (8) no longer dependence on the characteristics of the semiconductor disappears.

**IV. Conclusions**

The basic model of stationary X-ray induced conductivity for wide-gap semiconductors which is grounded on calculation of average spatial distributions of electrons and holes in a crystal is offered and allows to obtain such integral characteristics of the semiconductor as efficiency of collection of a charge, volt-ampere and lux-ampere characteristics at low levels of stationary excitation. The following stage of development of model is consideration of recombination centers and calculation of spatial distributions of the localized electrons and holes on the traps, calculation of their electric fields which can exceed considerably external fields at the considerable levels of excitation.

**References Références Referencias**

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