Online ISSN: 2249-4626
Print ISSN: 0975-5896

Global Journal

of Science Frontier Research: F

Mathematics and Decision Sciences



Highlights

Results Involving Certain Generalized Fermat Points

Regular Recursion Trees Fuzzy Multiset Approach

Discovering Thoughts, Inventing Future

VOLUME 13 ISSUE 10 VERSION 1.0

© 2001-2013 by Global Journal of Science Frontier Research , USA

Volume 13 Issue 47 was originally issued as F10

Global Journal of Science Frontier Research: F
Mathematics & Decision Sciences

Volume 13 Issue 47
Originally issued as F10

Global Journal of Science Frontier Research: F

Mathematics & Decision Sciences

Volume 13 Issue 10 (Ver. 1.0)

Open Association of Research Society

© Global Journal of Science Frontier Research .2013.

All rights reserved.

This is a special issue published in version 1.0 of "Global Journal of Science Frontier Research." By Global Journals Inc.

All articles are open access articles distributed under "Global Journal of Science Frontier Research"

Reading License, which permits restricted use. Entire contents are copyright by of "Global Journal of Science Frontier Research" unless otherwise noted on specific articles.

No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage and retrieval system, without written permission.

The opinions and statements made in this book are those of the authors concerned. Ultraculture has not verified and neither confirms nor denies any of the foregoing and no warranty or fitness is implied.

Engage with the contents herein at your own risk.

The use of this journal, and the terms and conditions for our providing information, is governed by our Disclaimer, Terms and Conditions and Privacy Policy given on our website http://globaljournals.us/terms-and-condition/menu-id-1463/

By referring / using / reading / any type of association / referencing this journal, this signifies and you acknowledge that you have read them and that you accept and will be bound by the terms thereof.

All information, journals, this journal, activities undertaken, materials, services and our website, terms and conditions, privacy policy, and this journal is subject to change anytime without any prior notice.

Incorporation No.: 0423089
License No.: 42125/022010/1186
Registration No.: 430374
Import-Export Code: 1109007027
Employer Identification Number (EIN):
USA Tax ID: 98-0673427

Global Journals Inc.

*(A Delaware USA Incorporation with "Good Standing"; **Reg. Number: 0423089**)*

Sponsors: Open Association of Research Society
Open Scientific Standards

Publisher's Headquarters office

Global Journals Headquarters
301st Edgewater Place Suite, 100 Edgewater Dr.-Pl,
Wakefield MASSACHUSETTS, Pin: 01880,
United States of America
USA Toll Free: +001-888-839-7392
USA Toll Free Fax: +001-888-839-7392

Offset Typesetting

Global Journals Incorporated
2nd, Lansdowne, Lansdowne Rd., Croydon-Surrey,
Pin: CR9 2ER, United Kingdom

Packaging & Continental Dispatching

Global Journals
E-3130 Sudama Nagar, Near Gopur Square,
Indore, M.P., Pin:452009, India

Find a correspondence nodal officer near you

To find nodal officer of your country, please email us at *local@globaljournals.org*

eContacts

Press Inquiries: *press@globaljournals.org*
Investor Inquiries: *investers@globaljournals.org*
Technical Support: *technology@globaljournals.org*
Media & Releases: *media@globaljournals.org*

Pricing (Including by Air Parcel Charges):

For Authors:
22 USD (B/W) & 50 USD (Color)
Yearly Subscription (Personal & Institutional):
200 USD (B/W) & 250 USD (Color)

Integrated Editorial Board
(Computer Science, Engineering, Medical, Management, Natural Science, Social Science)

John A. Hamilton,"Drew" Jr.,
Ph.D., Professor, Management
Computer Science and Software Engineering
Director, Information Assurance Laboratory
Auburn University

Dr. Henry Hexmoor
IEEE senior member since 2004
Ph.D. Computer Science, University at Buffalo
Department of Computer Science
Southern Illinois University at Carbondale

Dr. Osman Balci, Professor
Department of Computer Science
Virginia Tech, Virginia University
Ph.D.and M.S.Syracuse University, Syracuse, New York
M.S. and B.S. Bogazici University, Istanbul, Turkey

Yogita Bajpai
M.Sc. (Computer Science), FICCT
U.S.A.Email:
yogita@computerresearch.org

Dr. T. David A. Forbes
Associate Professor and Range Nutritionist
Ph.D. Edinburgh University - Animal Nutrition
M.S. Aberdeen University - Animal Nutrition
B.A. University of Dublin- Zoology

Dr. Wenying Feng
Professor, Department of Computing & Information Systems
Department of Mathematics
Trent University, Peterborough,
ON Canada K9J 7B8

Dr. Thomas Wischgoll
Computer Science and Engineering,
Wright State University, Dayton, Ohio
B.S., M.S., Ph.D.
(University of Kaiserslautern)

Dr. Abdurrahman Arslanyilmaz
Computer Science & Information Systems Department
Youngstown State University
Ph.D., Texas A&M University
University of Missouri, Columbia
Gazi University, Turkey

Dr. Xiaohong He
Professor of International Business
University of Quinnipiac
BS, Jilin Institute of Technology; MA, MS, PhD,. (University of Texas-Dallas)

Burcin Becerik-Gerber
University of Southern California
Ph.D. in Civil Engineering
DDes from Harvard University
M.S. from University of California, Berkeley & Istanbul University

Dr. Bart Lambrecht
Director of Research in Accounting and FinanceProfessor of Finance
Lancaster University Management School
BA (Antwerp); MPhil, MA, PhD (Cambridge)

Dr. Carlos García Pont
Associate Professor of Marketing
IESE Business School, University of Navarra
Doctor of Philosophy (Management), Massachusetts Institute of Technology (MIT)
Master in Business Administration, IESE, University of Navarra
Degree in Industrial Engineering, Universitat Politècnica de Catalunya

Dr. Fotini Labropulu
Mathematics - Luther College
University of ReginaPh.D., M.Sc. in Mathematics
B.A. (Honors) in Mathematics
University of Windso

Dr. Lynn Lim
Reader in Business and Marketing
Roehampton University, London
BCom, PGDip, MBA (Distinction), PhD, FHEA

Dr. Mihaly Mezei
ASSOCIATE PROFESSOR
Department of Structural and Chemical Biology, Mount Sinai School of Medical Center
Ph.D., Etvs Lornd University
Postdoctoral Training,
New York University

Dr. Söhnke M. Bartram
Department of Accounting and FinanceLancaster University Management SchoolPh.D. (WHU Koblenz)
MBA/BBA (University of Saarbrücken)

Dr. Miguel Angel Ariño
Professor of Decision Sciences
IESE Business School
Barcelona, Spain (Universidad de Navarra)
CEIBS (China Europe International Business School).
Beijing, Shanghai and Shenzhen
Ph.D. in Mathematics
University of Barcelona
BA in Mathematics (Licenciatura)
University of Barcelona

Philip G. Moscoso
Technology and Operations Management
IESE Business School, University of Navarra
Ph.D in Industrial Engineering and Management, ETH Zurich
M.Sc. in Chemical Engineering, ETH Zurich

Dr. Sanjay Dixit, M.D.
Director, EP Laboratories, Philadelphia VA Medical Center
Cardiovascular Medicine - Cardiac Arrhythmia
Univ of Penn School of Medicine

Dr. Han-Xiang Deng
MD., Ph.D
Associate Professor and Research
Department Division of Neuromuscular Medicine
Davee Department of Neurology and Clinical NeuroscienceNorthwestern University
Feinberg School of Medicine

Dr. Pina C. Sanelli
Associate Professor of Public Health
Weill Cornell Medical College
Associate Attending Radiologist
NewYork-Presbyterian Hospital
MRI, MRA, CT, and CTA
Neuroradiology and Diagnostic Radiology
M.D., State University of New York at Buffalo,School of Medicine and Biomedical Sciences

Dr. Roberto Sanchez
Associate Professor
Department of Structural and Chemical Biology
Mount Sinai School of Medicine
Ph.D., The Rockefeller University

Dr. Wen-Yih Sun
Professor of Earth and Atmospheric SciencesPurdue University Director
National Center for Typhoon and Flooding Research, Taiwan
University Chair Professor
Department of Atmospheric Sciences, National Central University, Chung-Li, TaiwanUniversity Chair Professor
Institute of Environmental Engineering, National Chiao Tung University, Hsin-chu, Taiwan.Ph.D., MS The University of Chicago, Geophysical Sciences
BS National Taiwan University, Atmospheric Sciences
Associate Professor of Radiology

Dr. Michael R. Rudnick
M.D., FACP
Associate Professor of Medicine
Chief, Renal Electrolyte and Hypertension Division (PMC)
Penn Medicine, University of Pennsylvania
Presbyterian Medical Center, Philadelphia
Nephrology and Internal Medicine
Certified by the American Board of Internal Medicine

Dr. Bassey Benjamin Esu
B.Sc. Marketing; MBA Marketing; Ph.D Marketing
Lecturer, Department of Marketing, University of Calabar
Tourism Consultant, Cross River State Tourism Development Department
Co-ordinator , Sustainable Tourism Initiative, Calabar, Nigeria

Dr. Aziz M. Barbar, Ph.D.
IEEE Senior Member
Chairperson, Department of Computer Science
AUST - American University of Science & Technology
Alfred Naccash Avenue – Ashrafieh

President Editor (HON.)

Dr. George Perry, (Neuroscientist)
Dean and Professor, College of Sciences
Denham Harman Research Award (American Aging Association)
ISI Highly Cited Researcher, Iberoamerican Molecular Biology Organization
AAAS Fellow, Correspondent Member of Spanish Royal Academy of Sciences
University of Texas at San Antonio
Postdoctoral Fellow (Department of Cell Biology)
Baylor College of Medicine
Houston, Texas, United States

Chief Author (HON.)

Dr. R.K. Dixit
M.Sc., Ph.D., FICCT
Chief Author, India
Email: authorind@computerresearch.org

Dean & Editor-in-Chief (HON.)

Vivek Dubey(HON.)
MS (Industrial Engineering),
MS (Mechanical Engineering)
University of Wisconsin, FICCT
Editor-in-Chief, USA
editorusa@computerresearch.org

Sangita Dixit
M.Sc., FICCT
Dean & Chancellor (Asia Pacific)
deanind@computerresearch.org

Suyash Dixit
(B.E., Computer Science Engineering), FICCTT
President, Web Administration and
Development , CEO at IOSRD
COO at GAOR & OSS

Er. Suyog Dixit
(M. Tech), BE (HONS. in CSE), FICCT
SAP Certified Consultant
CEO at IOSRD, GAOR & OSS
Technical Dean, Global Journals Inc. (US)
Website: www.suyogdixit.com
Email:suyog@suyogdixit.com

Pritesh Rajvaidya
(MS) Computer Science Department
California State University
BE (Computer Science), FICCT
Technical Dean, USA
Email: pritesh@computerresearch.org

Luis Galárraga
J!Research Project Leader
Saarbrücken, Germany

Contents of the Volume

i. Copyright Notice
ii. Editorial Board Members
iii. Chief Author and Dean
iv. Table of Contents
v. From the Chief Editor's Desk
vi. Research and Review Papers

1. Regular Recursion Trees: Description and Theoretical Analysis. *1-11*
2. Certain New Formulae Involving Modified Bessel Function of First Kind. *13-19*
3. Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach. *21-28*
4. Computation of Some Wonderful Results Involving Certain Polynomials. *29-38*
5. The Metric Characterization of the Generalized Fermat Points. *39-50*

vii. Auxiliary Memberships
viii. Process of Submission of Research Paper
ix. Preferred Author Guidelines
x. Index

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 10 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Regular Recursion Trees: Description and Theoretical Analysis

By Ulyanov M. V. & Goloveshkin V. A.

The National Research University

Abstract- Recursive algorithms analysis by generated trees node counting refers to detailed study of their structures. In this connection the paper presents the specific description of peculiar to a number of recursive algorithms regular trees. We have developed the method which is based on introduced regular description and provides an analytical solution for a number of generated nodes at such regular tree each level. The results obtained make possible a theoretical time complexity analysis of recursive algorithms generating regular recursion trees.

GJSFR-F Classification : *MSC 2010: 03D65, 03D60*



Strictly as per the compliance and regulations of :

© 2013. Ulyanov M. V. & Goloveshkin V. A. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

Regular Recursion Trees: Description and Theoretical Analysis

Ulyanov M. V. [α] & Goloveshkin V. A. [σ]

Abstract - Recursive algorithms analysis by generated trees node counting refers to detailed study of their structures. In this connection the paper presents the specific description of peculiar to a number of recursive algorithms regular trees. We have developed the method which is based on introduced regular description and provides an analytical solution for a number of generated nodes at such regular tree each level. The results obtained make possible a theoretical time complexity analysis of recursive algorithms generating regular recursion trees.

I. Introduction

The rational algorithm selection problem solution in software development as a rule is based on one of the main procedure quality characteristics called the time complexity. The time complexity is understood to be a number of assumed computing model elementary operations specified at the concrete input by the algorithm [1]. The estimate of higher value is a time complexity function with the size of algorithm input as the function argument. Meaningfully the time complexity function is considered in the best, worst and average cases [2]. Existing methods of algorithms analysis produce algorithm computational complexity – the time complexity function asymptotical estimate in the worst case and in some cases – the time complexity true function to solve the rational choice problem in real input sizes ranges. The time complexity true function derivation is a highly topical problem both for iterative and recursive algorithm solutions in which terms of theoretical study several methods are suggested at the present time. The methods produce analytical expressions of time complexity functions [1, 2, 3]. The generated recursion tree analysis technique is one of advanced methods in this direction. The most complicated step of it is obtaining of analytic expressions for number of generated recursion tree nodes at each tree level [1].

The main problem of recursion tree theoretical study is that in common case a number of generated nodes is the function of both the tree level number and a number of the generating node at this level. Moreover, in some cases such a function is not given analytically and the tree structure is markedly caused by specifics of concrete algorithm input, so the problem becomes significantly complicated. As an example, we consider a recursive algorithm of the solution of the classical problem of optimal packing by the dynamic programming method for which a generated tree is defined by both packing size

Author α: Professor, D.Sc. in Engineering (Russian Scientific Degree is a Doctor of Technical Sciences (д.т.н.)) The National Research University "Higher School of Economics", Moscow State University of Printing Arts.

Author σ: Professor, D.Sc. in Engineering (Russian Scientific Degree is a Doctor of Technical Sciences (д.т.н.)) Moscow State University of Instrument Engineering and Informatics.

Ref

1. Goloveshkin V. A., Ulyanov M. V. Recursion theory for programmers. — M.: FIZMATLIT. 2006. 296.

© 2013 Global Journals Inc. (US)

and cargo types sizes [1]. Only solutions for complete m-ary trees are trivial to this problem.

Thus in the recursive algorithms study aspect the problem of generated recursion trees theoretical study in nontrivial cases is to be of evident interest.

II. Problem Setting Up

According to generated recursion tree nodes counting based time complexity analysis method [1, p. 181-185.] we will use the following notation:

— n is the depth of the recursion tree, i.e. the number of its levels, in addition the base of the tree is not numbered and has the specific index Root and the further numbering starts with zero;

— S is the formal description of generated recursion tree structure;

— $R_A(n,S)$ is the total node number for all n-deep levels of S structured recursion tree;

— $R_V(n,S)$ is the tree internal nodes total number;

— $R_L(n,S)$ is the terminal node number, i.e. the node number at the level n.

The article has two objects in view:

1. To offer the formal description S of the recursion tree having the regular structure that is typical of generating recursive algorithms, including the algorithms that implement dynamic programming method.
2. To develop a solution procedure that produces the analytical solution of the tree nodes number at the level $R(j,S)$ from the present tree structure formal description S and the given level number j. Thus if the current algorithm input produces the tree with the depth n and the structure S then the main tree characteristics which are necessary for the recursive algorithm time complexity analysis are calculated as

$$R_A(n,S)=\sum_{j=0}^{n}R(j,S)+1;$$

$$R_V(n,S)=\sum_{j=0}^{n-1}R(j,S)+1; \qquad (1)$$

$$R_L(n,S)=R(n,S).$$

Note that the vertex of the recursion tree root is accounted for the complementary unity.

III. Formal Description of the Regular Recursion Tree Structure

Let us start generating of the description of the structure from consideration of complete m-ary tree. It is plain, that in this case each node at each level (besides leaf level) produces evenly m new tops and the tree is completely self-similar starting from any node. Because features of some recursive algorithms are that the tops of the same level produce different amount of the next level nodes that is however less then some

© 2013 Global Journals Inc (US)

prescribed value we propose the following regular tree formal description as ordered m-ary tuple (S line) with non-ascending sorted unit values

$$S = (k_1; k_2; \ldots; k_j; \ldots; k_m), j = \overline{1,m};$$
$$k_1 \geq k_2 \geq \ldots \geq k_j \geq \ldots \geq k_m. \quad (2)$$

Notes

The S line structured tree is generated by the following algorithm using the specific node numeration within the level and the concept of node type. At the tree root level the **Root** node generates m tops of the level $n = 0$ using $1, 2, \ldots, m$ numeration. Let us assign the characteristic to each node – a number from the interval $[1, \ldots, m]$ – and call it the node type. We will say further that if a node has the type "j" then it generates k_j tops at the tree next level. At the level $n = 0$ the nodes generated by the root get the same of their number type, so the j-numbered node has the type "j" and thus generates k_j tops at the level 1. Next, at the next levels the generated nodes are numbered relative to the parent node starting from the number m from right to left in the order of numbers decreasing, and get the types that correspond to assigned numbers. Thus the nodes generated directly from the "j"-typed top (there are k_j of such nodes) are described by the following line

$$(0; 0; \ldots; k_l; k_{l+1}; \ldots; k_m), l = m - k_j + 1. \quad (3)$$

The proposed formal description as the line S (2) specifies the regular tree, i.e. the tree conforming to single at all levels regularity of nodes generating defined by the node type according to node numeration and generating way given in the formula (3). We note that in case of $k_1 = m$ the root has the type "1" and the line S is given by $S = (m; k_2; \ldots; k_j, \ldots; k_m)$ and describes a regular incomplete (truncated) m-ary tree.

For example, the formal description $S = (3; 3; 3)$ specifies a complete ternary tree and the description $S = (3; 3; 2)$ specifies the regular incomplete ternary tree illustrated in fig. 1. The tree nodes are marked by their numbers coinciding to their types in accordance to described numeration principle. Thus all "3"-typed nodes generate two tops at the next level because the formal description line $S = (3; 3; 2)$ includes $k_3 = 2$. Also let us remark here that the tree root level has the **Root** index and is not numbered and the next tree levels have numbers 0, 1 etc. We note that the node type introduced above corresponds to the top number in the formal description line $S = (k_1; k_2; \ldots; k_j; \ldots; k_m)$ and the "j"-typed node generates k_j tops which description line is specified in the formula (3).

© 2013 Global Journals Inc. (US)

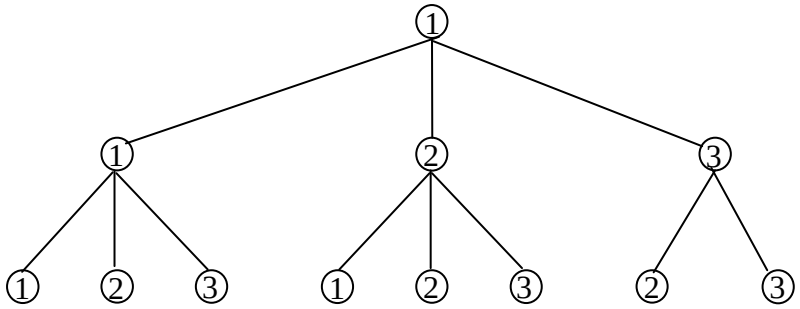


Fig. 1 : The fragment of the tree specified by the description $S=(3;3;2)$ with node types marked.

IV. Method of the Number of Regular Tree Nodes Problem Analytical Solving

Without losing generality we define the nodes number counting problem as the problem of the n-deep tree leaf number counting where the tree is generated by the description $S=(k_1;k_2;\ldots;k_j;\ldots;k_m)$, i.e. the counting of the regular tree nodes at the level n – the tree leaf level. With the preceding notation we solve the problem of determining $R(j,S)$ at $j=n$, i.e. the problem of obtaining the analytical solution for $R(n,S)=R_L(n,S)$.

Let us introduce the "j"-typed node characteristic $F(j)$. Meaningfully $F(j)$ is the type number of nodes generating the type"j" by the description S. $F(j)$ value is calculated from the formula

$$F(j)=\max_{1\le i\le m}\{\,i:m-k_i+1\le j\,\}. \tag{4}$$

Let $w_{n,j}$ is the number of "j"-typed nodes at the tree level n. Let us introduce the vector $W_n=\{w_{n,1};w_{n,2};\ldots;w_{n,m}\}$ with dimension m. Thus the vector components W_n contain the number of each type nodes at the level n of recursion tree. Then with consideration of the characteristic $F(j)$ introduced above we obtain the following system of linear recurrence relations to calculate the $w_{n,j}$ values

$$\begin{cases} w_{\text{Root},1}=1,\quad w_{\text{Root},j}=0,\ j=\overline{2,m}, & n=\text{Root};\\ w_{0,j}=1,\ j=\overline{1,m}, & n=0;\\ w_{n+1,j}=\sum\limits_{i=1}^{F(j)}w_{n,i}, & n\ge 1. \end{cases} \tag{5}$$

We note that in accordance with the (5) $W_0=\{1;1;\ldots;1\}$ and if $k_1=m$ then $W_{\text{Root}}=\{1;0;\ldots;0\}$

© 2013 Global Journals Inc (US)

From the known method of linear recurrence relations solving [1, chapter 1], we will try solutions of the relation (5) in the following form

$$\begin{cases} \lambda \cdot h_{0,j} = \sum_{i=1}^{F(j)} h_{0,i}, \quad \forall j = \overline{1,m}. \end{cases} \tag{7}$$

Our problem required solution – the vector W_n – is given by the linear combination of the computed solutions (6) with consideration of initial conditions from (5).

Let us formulate the suggested method stages for the recurrence relation (5) solving in the case when the number of eigenvectors H_0 is equal to m.

1. Evaluation of the node type characteristics $F(j)$ by the formula (4) from the offered formal description of the regular recursion tree in the form of the line S (2).
2. Formation of the linear recurrence relations system (5).
3. The derivation of the solutions in the form $H_n = \lambda^n H_0$.
   - 3.1. Construction of the equations set (7).
   - 3.2. Defining λ values by the condition of equality of zero and the system determinant (7).
   - 3.3. Finding of eigenvectors H_0 values for every value of λ.
4. Construction of the initial system (5) solutions W_n as a linear combination of the obtained solutions for H_0 from defining unknown coefficients of the linear combination by the values of initial conditions defined by the vector W_0.
5. Counting the number of nodes at the level n of the recursion tree under study

$$R(n,S) = \sum_{j=1}^{m} w_{n,j}. \tag{8}$$

6. Since obtained analytical solution for W_n holds for every value of n the formula (8) gives the solution also for $R(j,S)\ \forall j = \overline{0,n}$ what enables us to define the total number of generated nodes at all n-deep tree levels and the number of the internal nodes according to the formula (1).

We particularly note that in case of the number of eigenvectors H_0 is less than m the known special solving methods [1, 3, 4] is necessarily to be used.

V. Examples of Regular Recursion Tree Analysis

Later we will give the examples of regular trees analysis for both cases: when the number of eigenvectors H_0 is equal to or less than m.

a) At first we consider the example of the tree specified by the description $S = (3;3;2)$ which form fig. 1 shows. We point out in advance that the number of eigenvectors H_0 is equal to m in this case. We illustrate the method in details pointing

Ref

3. R. Graham, D. Knuth, Patashnik O. Concrete Mathematics. Foundation of computer science. Translated from English. — M.: Mir. 1998. 703.

© 2013 Global Journals Inc. (US)

directly numbers and content of the stages of suggested solving method.

1. Evaluation of $F(j)$ by the description $S=(3;3;2)$:

$$F(1)=2,\ F(2)=3,\ F(3)=3.$$

2. Formation of the linear recurrence relations system (5):

$$\begin{cases} w_{0,j}=1,\ j=\overline{1,3}; \\ w_{n+1,1}=w_{n,1}+w_{n,2}; \\ w_{n+1,2}=w_{n,1}+w_{n,2}+w_{n,3}; \\ w_{n+1,3}=w_{n,1}+w_{n,2}+w_{n,3}. \end{cases}$$

3.1. Construction of the equations set (7):

$$\begin{cases} \lambda h_{0,1}=h_{0,1}+h_{0,2}; \\ \lambda h_{0,2}=h_{0,1}+h_{0,2}+h_{0,3}; \\ \lambda h_{0,3}=h_{0,1}+h_{0,2}+h_{0,3}. \end{cases} \quad (9)$$

3.2. Defining λ values by the condition of equality of zero and the system determinant:

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}=0.$$

Computing the determinant we obtain the equation $\lambda\cdot(\lambda^2-3\lambda+1)=0$ which roots are:

$$\lambda_1=0;\quad \lambda_2=\frac{3-\sqrt{5}}{2};\quad \lambda_3=\frac{3+\sqrt{5}}{2}.$$

3.3. Finding of eigenvectors H_0 values for every value of λ.

Plugging in (9) the roots obtained and solving obtained systems we get eigenvectors H_0 which corresponds to the secular equation roots by the numeration:

$$H_0^{(1)}=\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix};\quad H_0^{(2)}=\begin{pmatrix} 1 \\ \frac{1-\sqrt{5}}{2} \\ \frac{1-\sqrt{5}}{2} \end{pmatrix};\quad H_0^{(3)}=\begin{pmatrix} 1 \\ \frac{1+\sqrt{5}}{2} \\ \frac{1+\sqrt{5}}{2} \end{pmatrix}.$$

4. Construction of the initial system (5) solutions W_n as a linear combination of the obtained solutions for H_0 with initial conditions W_0:

$$W_n=A\cdot 0^n\cdot H_0^{(1)}+B\cdot\left(\frac{3-\sqrt{5}}{2}\right)^n\cdot H_0^{(2)}+C\cdot\left(\frac{3+\sqrt{5}}{2}\right)^n\cdot H_0^{(3)},$$

Notes

© 2013 Global Journals Inc (US)

where by definition $0^0 = 1$ and A, B, C are linear combination unknown coefficients defined by the initial conditions W_0:

$$\begin{cases} A + B + C = 1; \\ -A + \dfrac{1-\sqrt{5}}{2} B + \dfrac{1+\sqrt{5}}{2} C = 1; \\ \dfrac{1-\sqrt{5}}{2} B + \dfrac{1+\sqrt{5}}{2} C = 1. \end{cases}$$

The solution of obtained system is of the form:

$$A = 0; \quad B = \frac{\sqrt{5}-1}{2\sqrt{5}}; \quad C = \frac{\sqrt{5}+1}{2\sqrt{5}},$$

that gives the terminal solution for the number of nodes of the regular tree under review

$$W_n = \frac{\sqrt{5}-1}{2\sqrt{5}} \cdot \left(\frac{3-\sqrt{5}}{2} \right)^n \cdot H_0^{(2)} + \frac{\sqrt{5}+1}{2\sqrt{5}} \cdot \left(\frac{3+\sqrt{5}}{2} \right)^n \cdot H_0^{(3)}.$$

5. Counting the number of nodes at the level n of the recursion tree under study
According to (8) the total nodes number at the level n is the sum of each type nodes:

$$R(n, S) = \sum_{j=1}^{3} w_{n,j}.$$

Making the necessary transformations we obtain the desired solution for $S = (3; 3; 2)$:

$$R(n, S) = \frac{7+3\sqrt{5}}{2\sqrt{5}} \cdot \left(\frac{3+\sqrt{5}}{2} \right)^n + \frac{3\sqrt{5}-7}{2\sqrt{5}} \cdot \left(\frac{3-\sqrt{5}}{2} \right)^n.$$

It may be verified by direct substitution that $R(1, S) = 8$, $R(2, S) = 21$.

б) The second example where the regular tree is given by the description $S = (3; 3; 1)$ also gives three different eigenvectors. In this case we will give the main results only. The values of λ obtained by the condition of equality of zero and the system determinant are

$$\lambda_1 = 0; \quad \lambda_2 = 1; \quad \lambda_3 = 2.$$

that gives the following solution for corresponding typed nodes number

$$w_{n,1} = 2^n; \quad w_{n,2} = 2^n; \quad w_{n,3} = 2 \cdot 2^n - 1.$$

Summing over all types we obtain the solution for the tree level

$$R(n, S) = 2^{n+2} - 1.$$

© 2013 Global Journals Inc. (US)

в) The next example produces the case that needs the specific solving method. Let us consider the description $S=(m; m-1; m-2; \ldots; 1)$. Fig. 2 shows the tree that corresponds to this description at $m=3$ – $S=(3; 2; 1)$.

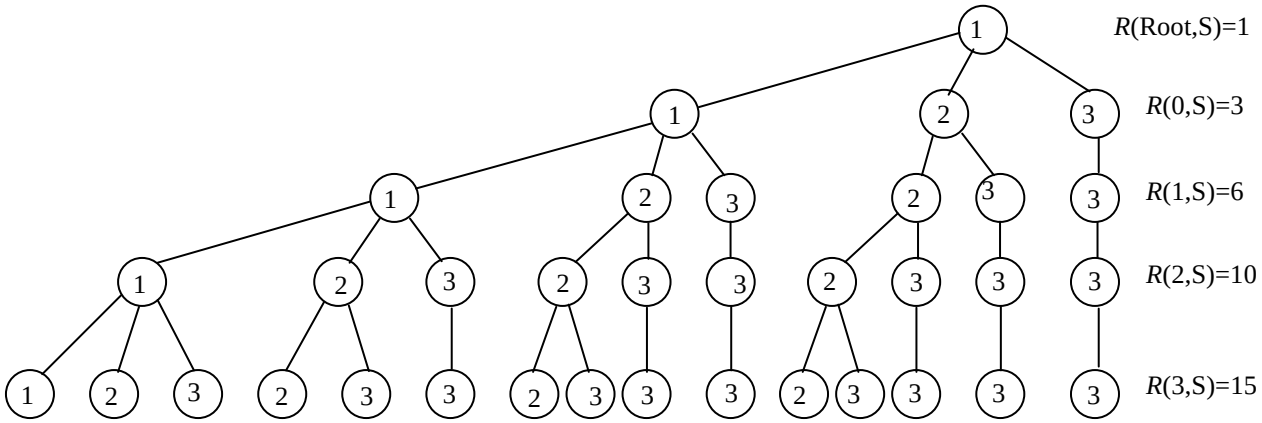


Fig. 2 : The fragment of the tree specified by the description $S=(3; 2; 1)$ with node types marked.

The initial stages of this case analytical solution are the same as for the method described above:

1. Evaluation of $F(j)$ by the description $S=(m; m-1; m-2; \ldots; 1)$:

$$F(1)=1,\ F(2)=2,\ F(3)=3,\ \ldots,F(m)=m.$$

2. Formation of the linear recurrence relations system (5):

$$\begin{cases} w_{0,j}=1,\ j=\overline{1,3}; \\ w_{n+1,1}=w_{n,1}; \\ w_{n+1,2}=w_{n,1}+w_{n,2}; \\ \ldots \\ w_{n+1,j}=w_{n,1}+w_{n,2}+\ldots+w_{n,j}; \\ \ldots \\ w_{n+1,m}=w_{n,1}+w_{n,2}+w_{n,3}+\ldots+w_{n,m}; \end{cases} \quad (10)$$

3.1. Construction of the equations set (7):

$$\begin{cases} \lambda h_{0,1}=h_{0,1}; \\ \lambda h_{0,2}=h_{0,1}+h_{0,2}; \\ \lambda h_{0,3}=h_{0,1}+h_{0,2}+h_{0,3}; \\ \ldots \\ \lambda h_{0,m}=h_{0,1}+h_{0,2}+h_{0,3}+\ldots+h_{0,m}. \end{cases} \quad (11)$$

© 2013 Global Journals Inc (US)

3.2. Defining λ values by the condition of equality of zero and the system determinant:

Calculating the determinant of the system (11) we get the equation $(1-\lambda)^m = 0$ and thus the multiplicity of the root $\lambda = 1$ is m. In this problem the number of eigenvectors is less then m and we use the specific method that provides for the direct solving of the system (10).

Notes

4. The direct solving of the system (10).

4.1. Let us solve the recurrence relation for $j = 1$:

$$\begin{cases} w_{0,1} = 1; \\ w_{n+1,1} = w_{n,1}. \end{cases} \Rightarrow \forall n \; w_{n,1} = 1.$$

4.2. From the solution obtained we get the recurrence relation for $j = 2$:

$$\begin{cases} w_{0,2} = 1; \\ w_{n+1,2} = w_{n,2} + 1. \end{cases} \Rightarrow \forall n \; w_{n,2} = n + 1.$$

4.3. With consideration of this solution the recurrence relation for $j = 3$ is of the form

$$\begin{cases} w_{0,3} = 1; \\ w_{n+1,3} = w_{n,3} + (n+1) + 1. \end{cases} \tag{12}$$

We seek the specific solution in the form $w^*_{n,3} = An^2 + Bn + C$ and plugging it in (12) we get

$$2An + A + B = n + 2, \quad \Rightarrow \quad A = \frac{1}{2}, \; B = \frac{3}{2},$$

the initial condition gives the solution $C = 1$ and finally

$$w_{n,3} = \frac{1}{2}n^2 + \frac{3}{2}n + 1 = \frac{(n+1)(n+2)}{2}.$$

4.4. We note that $w_{n,1} = 1 = C_n^0$, $w_{n,2} = n + 1 = C_{n+1}^1$, $w_{n,3} = C_{n+2}^2$. Let us prove by induction that for an arbitrary value m

$$w_{n,m} = C_{n+m-1}^{m-1}. \tag{13}$$

The relation (13) is true for $m = 1, m = 2, m = 3$. By the induction hypothesis, the relation $w_{n,k} = C_{n+k-1}^{k-1}$ is true for all $k \le m$. Then by proceeding the linear recurrence relations system (10) for $m+1$ we obtain the recurrence relation

$$\begin{aligned} w_{n+1,m+1} &= w_{n,1} + w_{n,2} + w_{n,3} + \ldots + w_{n,m} + w_{n,m+1} = \\ &= C_n^0 + C_{n+1}^1 + C_{n+2}^2 + \ldots + C_{n+m-1}^{m-1} + w_{n,m+1}. \end{aligned} \tag{14}$$

© 2013 Global Journals Inc. (US)

We note that $C_n^0 = C_{n+1}^0, C_n^k + C_n^{k-1} = C_{n+1}^k$ [3], then

$$C_n^0 + C_{n+1}^1 + C_{n+2}^2 + \ldots + C_{n+m-1}^{m-1} = C_{n+2}^1 + C_{n+2}^2 + \ldots + C_{n+m-1}^{m-1} = C_{n+m}^{m-1}.$$

We show that

$$w_{n,m+1} = C_{n+m}^m$$

is the solution of the recurrence to n relation (14). Plugging this putative solution in (14) we get

$$w_{n+1,m+1} = C_{n+m}^{m-1} + C_{n+m}^m = C_{n+m+1}^m,$$

that corresponds to (13) with substituting for corresponding indexes. Thus the solution of the system (10) $\forall m \geq 1, \forall n \geq 1$ is of the form

$$w_{n,m} = C_{n+m-1}^{m-1},$$

that proves the inductive hypothesis (13).

From the solutions obtained we define the total nodes number at the tree level numbered n by summing the number of all type nodes at this level. We introduce the third argument m into the function of nodes number since we are concerned with generic description $S = (m; m-1; m-2; \ldots; 1)$ and using the known relations for the binomial coefficients [3] we get:

$$R(n, m, S) = \sum_{j=1}^{m} w_{n,j} = \sum_{j=1}^{m} C_{n+j-1}^{j-1} = C_{n+m}^{m-1}. \quad (15)$$

From (15) let us define the total nodes number of the tree having description $S = (m; m-1; m-2; \ldots; 1)$ by summing the number of nodes at all levels including the tree root

$$R_A(n, m, S) = 1 + \sum_{i=0}^{n} R(i, m, S) = \sum_{i=0}^{n} C_{i+m}^{m-1} = C_{n+m+1}^m. \quad (16)$$

Referring to the tree fragment shown in fig. 2 and having the description $S = (3; 2; 1)$, i.e. $m = 3$ we obtain the total nodes number at the level $n = 3$ by using the formula (15) – $R(3, 3, S) = C_{3+3}^{3-1} = C_6^2 = 15$ – and define the nodes sum at all levels including the level $n = 3$ from the formula (16) – $R_A(3, 3, S) = C_{3+3+1}^3 = C_7^3 = 35$. We note that the solution obtained by us is the same as the results of the generated recursion tree study of the algorithm of the solution of the optimal one-dimensional packing problem by dynamic programming method at the problem regular parametrization [1]. Notwithstanding the result contained in [1] is particular to this paper results because it gives the solution only for the trees having the description $S = (m; m-1; m-2; \ldots; 1)$.

VI. Conclusion

For the sake of recursive algorithm analysis and study this paper suggests the specific description of regular recursion trees typical of some recursive algorithms. Such

© 2013 Global Journals Inc (US)

trees are generated for example by the algorithms that realize recursively the dynamic programming method with the special parametrization which regularizes the trees generated by recursive algorithms. Under this description the solution procedure is suggested that gives the number of generated nodes at each level of a such regular tree. The method devised by us produces the analytical solution for the number of generated nodes over every level of the recursion tree and thus the total number of nodes as the function of the tree depth.

Notes

The results obtained can be used for detailed theoretical analysis of recursive algorithms generating regular recursion trees and for getting true functions of time complexity and capacitory efficiency.

Bibliography

1. Goloveshkin V. A., Ulyanov M. V. Recursion theory for programmers. — M.: FIZMATLIT. 2006. 296.
2. T. Corman, C. Leiserson, R. Rivest, K. Stein. Algorithms: construction and analysis, Second Edition: Translated from English. — Moscow: Publishing house "Williams", 2005. — 1296.
3. R. Graham, D. Knuth, Patashnik O. Concrete Mathematics. Foundation of computer science. Translated from English. — M.: Mir. 1998. 703.
4. Sachkov V. N. Introduction to Combinatorial methods of discrete mathematics. — Second Edition, Rev. and add. — Moscow: MCCME, 2004. — 424.

© 2013 Global Journals Inc. (US)

This page is intentionally left blank

© 2013 Global Journals Inc (US)

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 10 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Certain New Formulae Involving Modified Bessel Function of First Kind

By Salahuddin & Intazar Husain
P.D.M College of Engineering, India

Abstract- In this paper we have developed certain new results involving Hypergeometric function, Modified Bessel function of first kind and exponential function. The results represent here are assume to be new.

Keywords: *hypergeometric function, bessel function, modified bessel function of first kind, exponential function.*

GJSFR-F Classification : *MSC 2010: 33C10*



Strictly as per the compliance and regulations of :

© 2013. Salahuddin & Intazar Husain. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

Certain New Formulae Involving Modified Bessel Function of First Kind

Salahuddin [α] & Intazar Husain [σ]

Abstract- In this paper we have developed certain new results involving Hypergeometric function, Modified Bessel function of first kind and exponential function. The results represent here are assume to be new.

Keywords: *hypergeometric function, bessel function, modified bessel function of first kind, exponential function.*

I. Introduction

a) Bessel Function

Bessel functions, first defined by the mathematician Daniel Bernoulli and generalized by Friedrich Bessel, are the canonical solutions y(x) of Bessel's differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0 \tag{1.1}$$

for an arbitrary complex number α (the order of the Bessel function). The most important cases are for α an integer or half-integer.

b) Bessel Function of First Kind

Bessel functions of the first kind, denoted as $J_\alpha(x)$, are solutions of Bessel's differential equation that are finite at the origin ($x = 0$) for integer or positive α, and diverge as x approaches zero for negative non-integer α. It is possible to define the function by its Taylor series expansion around $x = 0$.

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha} \tag{1.2}$$

where Γz is the gamma function, a shifted generalization of the factorial function to non-integer values. The Bessel function of the first kind is an entire function if α is an integer. The graphs of Bessel functions look roughly like oscillating sine or cosine functions that decay proportionally to $\frac{1}{\sqrt{x}}$.

c) Modified Bessel Function of First Kind

The Bessel functions are valid even for complex arguments x, and an important special case is that of a purely imaginary argument. In this case, the solutions to the Bessel equation are called

Author α : P.D.M College of Engineering, Bahadurgarh, Haryana, India. e-mails: vsludn@gmail.com, sludn@yahoo.com
Author σ : Department of Applied Sciences and Humanities, Faculty of Engineering and Technology, Jamia Millia Islamia, New Delhi, India.

© 2013 Global Journals Inc. (US)

the modified Bessel functions (or occasionally the hyperbolic Bessel functions) of the first and second kind. The first kind of Bessel function is defined as

$$I_\alpha(x) = \iota^{-\alpha} J_\alpha(\iota x) = \sum_{m=0}^{\infty} \frac{1}{m!\,\Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha} \tag{1.3}$$

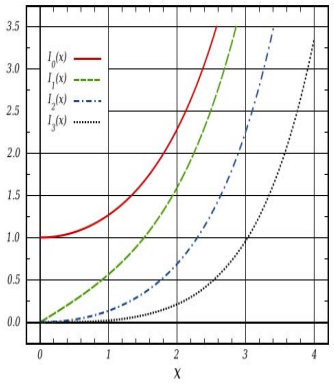


Figure 1 : Modified Bessel Function of First Kind

d) Exponential Function

In mathematics, the exponential function is the function e^x, where e is the number (approximately 2.718281828) such that the function e^x is its own derivative. The exponential function is used to model a relationship in which a constant change in the independent variable gives the same proportional change (i.e. percentage increase or decrease) in the dependent variable. The function is often written as $exp(x)$, especially when it is impractical to write the independent variable as a superscript. The exponential function is widely used in physics, chemistry, engineering, mathematical biology, economics and mathematics.
In particular the exponential function may be defined as

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... \tag{1.4}$$

e) Generalized Hypergeometric Functions

A generalized hypergeometric function ${}_pF_q(a_1, ...a_p; b_1, ...b_q; z)$ is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)...(k+a_p)}{(k+b_1)(K+b_2)...(k+b_q)(k+1)}\, z. \tag{1.5}$$

Where $k+1$ in the denominator is present for historical reasons of notation, and the resulting generalized hypergeometric function is written

$${}_pF_q \left[\begin{matrix} a_1, a_2, \cdots, a_p \\ b_1, b_2, \cdots, b_q \end{matrix} ; z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_p)_k z^k}{(b_1)_k (b_2)_k \cdots (b_q)_k k!} \tag{1.6}$$

Notes

© 2013 Global Journals Inc (US)

or

$$ {}_pF_q\left[\begin{array}{ccc}(a_p) & ; & \\ & & z \\ (b_q) & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_p))_k z^k}{((b_q))_k k!} \tag{1.7} $$

where the parameters $b_1, b_2, \cdots, b_q$ are neither zero nor negative integers and p, q are non-negative integers.

Notes

The ${}_pF_q$ series converges for all finite z if $p \leq q$, converges for $\mid z \mid < 1$ if $p \neq q+1$, diverges for all z , $z \neq 0$ if $p > q+1$.

The ${}_pF_q$ series absolutely converges for $\mid z \mid = 1$ if $R(\zeta) < 0$, conditionally converges for $\mid z \mid = 1, z \neq 0$ if $0 \leq R(\zeta) < 1$, diverges for $\mid z \mid = 1$, if $1 \leq R(\zeta)$, $\zeta = \sum_{i=1}^{p} a_i - \sum_{i=0}^{q} b_i$.

The function ${}_2F_1(a, b; c; z)$ corresponding to $p = 2, q = 1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function (Gauss 1812, Barnes 1908). To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.

The hypergeometric functions are solutions of Gaussian hypergeometric linear differential equation of second order

$$ z(1-z)y'' + [c - (a+b+1)z]y' - aby = 0 \tag{1.8} $$

The solution of this equation is

$$ y = A_0\Big[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\Big] \tag{1.9} $$

This is the so-called regular solution,denoted

$$ {}_2F_1(a, b; c; z) = \Big[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\Big] = \sum_{k=0}^{\infty} \frac{(a)_k\ (b)_k z^k}{(c)_k k!} \tag{1.10} $$

which converges if c is not a negative integer for all of $\mid z \mid < 1$ and on the unit circle $\mid z \mid = 1$ if $R(c-a-b) > 0$.

It is known as Gauss hypergeometric function in terms of Pochhammer symbol $(a)_k$ or generalized factorial function.

f) Whittaker Function

In mathematics, a Whittaker function is a special solution of Whittaker's equation, a modified form of the confluent hypergeometric equation introduced by Whittaker (1904) to make the formulas involving the solutions more symmetric. More generally, Jacquet (1966, 1967) introduced Whittaker functions of reductive groups over local fields, where the functions studied by Whittaker are essentially the case where the local field is the real numbers and the group is $SL_2(R)$. Whittaker's equation is

$$ \frac{d^2w}{dz^2} + \Big(-\frac{1}{4} + \frac{k}{z} + \frac{\frac{1}{4} - \mu^2}{z^2}\Big)w = 0 \tag{1.11} $$

© 2013 Global Journals Inc. (US)

It has a regular singular point at 0 and an irregular singular point at ∞. Two solutions are given by the Whittaker functions $M_{k,\mu}(z), W_{k,\mu}(z)$, defined in terms of Kummer's confluent hypergeometric functions M and U by

$$M_{k,\mu}(z) = exp(-\frac{z}{2})z^{\mu+\frac{1}{2}}M\Big(\mu-k+\frac{1}{2}, 1+2\mu; z\Big) \tag{1.12}$$

$$W_{k,\mu}(z) = exp(-\frac{z}{2})z^{\mu+\frac{1}{2}}U\Big(\mu-k+\frac{1}{2}, 1+2\mu; z\Big) \tag{1.13}$$

Whittaker functions appear as coefficients of certain representations of the group $SL_2(R)$, called Whittaker models.

g) Associated Laguerre Polynomials

The Rodrigues representation for the associated Laguerre polynomials is

$$L_n^k(x) = \frac{e^x x^{-k}}{n!}\frac{d^n}{dx^n}(e^{-x}x^{n+k}) \tag{1.14}$$

$$= \sum_{m=0}^{n}(-1)^m\frac{(n+k)!}{(n-m)!(k+m)!m!}x^m \tag{1.15}$$

The first few associated Laguerre polynomials are

$$\begin{cases} L_0^k(x) = 1 \\ L_1^k(x) = -x+k+1 \\ L_2^k(x) = \frac{1}{2}[x^2-2(k+2)x+(k+1)(k+2)] \end{cases} \tag{1.16}$$

II. Main Results

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;x) = 2^{2a-1}x^{\frac{1}{2}-a}\,\Gamma\Big(a+\frac{1}{2}\Big)\,I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big) - \\ -2^{2a-1}x^{\frac{1}{2}-a}\,\Gamma\Big(a+\frac{1}{2}\Big)\,I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big) \tag{2.1}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;x) = 2^{2a-3}(4a+x-2)x^{\frac{1}{2}-a}\,\Gamma\Big(a-\frac{1}{2}\Big)\,I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big) + \\ +2^{2a-3}x^{\frac{3}{2}-a}\,\Gamma\Big(a-\frac{1}{2}\Big)\,I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big) \tag{2.2}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+2;x) = \frac{2^{2a}x^{\frac{1}{2}-a}\Gamma(a+\frac{3}{2})I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)}{a+1} - \\ -\frac{2^{2a}x^{-\frac{1}{2}-a}(2a+x)\Gamma(a+\frac{3}{2})I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)}{a+1} \tag{2.3}$$

Notes

© 2013 Global Journals Inc (US)

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-2;x)=\frac{2^{2a-4}(8a^2+4ax-12a+x^2-2x+4)x^{\frac{1}{2}-a}\Gamma(a-\frac{1}{2})I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)}{a-1}+$$
$$+\frac{2^{2a-4}(2a+x-2)x^{-\frac{3}{2}-a}\Gamma(a-\frac{1}{2})I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)}{a-1} \tag{2.4}$$

Notes

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+3;x)=2^{2a+1}x^{-\frac{1}{2}-a}(a+x)\ \Gamma\Big(a+\frac{3}{2}\Big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)-$$
$$-\frac{2^{2a+1}x^{-\frac{3}{2}-a}(4a^2+3ax+2a+x^2)\ \Gamma\Big(a+\frac{3}{2}\Big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)}{a+2} \tag{2.5}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-3;x)+$$
$$=\frac{2^{2a-5}(4a^2+3ax-10a+x^2-3x+6)x^{\frac{3}{2}-a}\ \Gamma\Big(a-\frac{3}{2}\Big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)}{a-1}+$$
$$+\frac{2^{2a-5}(16a^3+12a^2x-48a^2+5ax^2-18ax+44a+x^3-3x^2+6x-12)}{a-1}\times$$
$$\times\ x^{\frac{1}{2}-a}\ \Gamma\Big(a-\frac{3}{2}\Big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big) \tag{2.6}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+4;x)=\frac{2^{2a+2}x^{-\frac{3}{2}-a}(2a^2+2ax+2a+x^2)\ \Gamma\Big(a+\frac{5}{2}\Big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)}{(a+2)(a+3)}-$$
$$-\frac{2^{2a+2}x^{-\frac{5}{2}-a}(8a^3+8a^2x+12a^2+4ax^2+4ax+4a+x^3)\ \Gamma\Big(a+\frac{5}{2}\Big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)}{(a+2)(a+3)} \tag{2.7}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-4;x)=$$
$$=\frac{2^{2a-6}x^{\frac{3}{2}-a}(8a^3+8a^2x-36a^2+4ax^2-20ax+52a+x^3-4x^2+12x-24)}{(a-2)(a-1)}\times$$
$$\times\ \Gamma\Big(a-\frac{3}{2}\Big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)+2^{2a-6}x^{\frac{1}{2}-a}\Gamma\Big(a-\frac{3}{2}\Big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)\times$$
$$\times\Bigg[\frac{(32a^4+32a^3x-160a^3+18a^2x^2-96a^2x+280a^2)}{(a-2)(a-1)}+$$
$$+\frac{(6ax^3-30ax^2+88ax-200a+x^4-4x^3+12x^2-24x+48)}{(a-2)(a-1)}\Bigg] \tag{2.8}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+5;x)=$$
$$=\frac{2^{2a+3}x^{-\frac{5}{2}-a}(4a^3+5a^2x+10a^2+3ax^2+5ax+6a+x^3)\ \Gamma\Big(a+\frac{5}{2}\Big)\ I_{\frac{1}{2}(2a-1)}\Big(\frac{x}{2}\Big)}{(a+3)(a+4)}-$$
$$-2^{2a+3}x^{-\frac{7}{2}-a}\ \Gamma\Big(a+\frac{5}{2}\Big)\ I_{\frac{1}{2}(2a+1)}\Big(\frac{x}{2}\Big)\Bigg[\frac{(16a^4+20a^3x+48a_1^33a^2x^2+30a^2x+44a^2)}{(a+3)(a+4)}+$$
$$+\frac{(5ax^3+7ax^2+10ax+12a+x^4)}{(a+3)(a+4)}\Bigg] \tag{2.9}$$

© 2013 Global Journals Inc. (US)

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-5;x) =$$
$$= 2^{2a-7}x^{\frac{3}{2}-a}\ \Gamma\left(a-\frac{5}{2}\right)\ I_{\frac{1}{2}(2a+1)}\left(\frac{x}{2}\right)\left[\frac{(16a^4+20a^3x-112a^3+13a^2x^2-90a^2x+284a^2)}{(a-2)(a-1)}+\right.$$
$$\left.+\frac{(5ax^3-33ax^2+130ax-308a+x^4-5x^3+20x^2-60x+120)}{(a-2)(a-1)}\right]+$$
$$+2^{2a-7}x^{\frac{1}{2}-a}\ \Gamma\left(a-\frac{5}{2}\right)\ I_{\frac{1}{2}(2a-1)}\left(\frac{x}{2}\right)\left[\frac{(64a^5+80a^4x-480a^4+56a^3x^2-400a^3x+1360a^3)}{(a-2)(a-1)}+\right.$$
$$+\frac{(25a^2x^3-180a^2x^2+700a^2x-1800a^2+7ax^4-45ax^3+184ax^2-500ax)}{(a-2)(a-1)}+$$
$$\left.+\frac{(1096a+x^5-5x^4+20x^3-60x^2+120x-240)}{(a-2)(a-1)}\right] \tag{2.10}$$

III. Special Cases

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;0) = e^{-\frac{x}{2}}\frac{\Gamma(1-a)\left(\Gamma(1+2a)L_{-a}^{2a}(0)\right)}{\Gamma(1+a)} \tag{3.1}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;1) =) = e^{-\frac{x}{2}}\frac{\Gamma(1-a)\left(\Gamma(1+2a)L_{-a}^{2a}(1)\right)}{\Gamma(1+a)} \tag{3.2}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a+1;2) =) = e^{-\frac{x}{2}}\frac{e^1M_{\frac{1}{2},a}(2)}{2^{\frac{1}{2}(1+2a)}} \tag{3.3}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;1) =) = e^{-\frac{x}{2}}\frac{\sqrt{e}\ M_{-\frac{1}{2},\frac{1}{2}(-2+2a)}(1)}{1^{\frac{1}{2}(-1+2a)}} \tag{3.4}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;2) =) = e^{-\frac{x}{2}}\frac{e^1\ M_{-\frac{1}{2},\frac{1}{2}(-2+2a)}(2)}{2^{\frac{1}{2}(-1+2a)}} \tag{3.5}$$

$$e^{-\frac{x}{2}}{}_1F_1(a;2a-1;3) =) = e^{-\frac{x}{2}}\frac{e^{\frac{3}{2}}\ M_{-\frac{1}{2},\frac{1}{2}(-2+2a)}(3)}{3^{\frac{1}{2}(-1+2a)}} \tag{3.6}$$

$$e^{-\frac{x}{2}}{}_1F_1(1;1;-1) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-x)^{k_2}}{2^{k_2}\ k_1!k_2!} \tag{3.7}$$

$$e^{-\frac{x}{2}}{}_1F_1(1;3;-1) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}(-x)^{k_1}(1)_{k_2}}{2^{k_1}\ k_1!k_2!(3)_{k_2}} \tag{3.8}$$

IV. Applications

The results are applied in all branches of applied sciences.These are used for solving scientific problems such as diffusion problems on a lattice, solving for patterns of acoustical radiation,Modes of vibration of a thin circular (or annular) artificial membrane (such as a drum or other membranophone),Heat conduction in a cylindrical object.

Notes

© 2013 Global Journals Inc (US)

References Références Referencias

1. Abramowitz, Milton; Stegun, Irene A.(1965), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover.
2. Ahlfors, Lars (1979), *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable* (3rd ed.). New York;London: McGraw-Hill.
3. Andrews, L.C.(1992) , *Special Function of mathematics for Engineers,Second Edition*, McGraw-Hill Co Inc., New York.
4. Bateman, Harry (1953), *Higher transcendental functions*,McGraw-Hill.
5. Bells, Richard, Wong, Roderick(2010), *Special Functions , A Graduate Text*, Cambridge Studies in Advanced Mathematics.
6. Bessel Function (2013, October 31). Retrieved from *http : //en.wikipedia.org/wiki /Bessel function.*
7. Slater, Lucy Joan (1960), *Confluent hypergeometric functions*, Cambridge University Press.
8. Goldstein, Lay, Schneider, Asmar(2006), *Brief calculus and its applications*, Prentice-Hall.
9. Whittaker, Edmund T. (1904), "An expression of certain known functions as generalized hypergeometric functions", *Bulletin of the A. M. S.* (Providence, R.I.: American Mathematical Society) 10: 125134.
10. Watson, G.N.(1995), *A Treatise on the Theory of Bessel Functions*, Cambridge University Press.

Notes

© 2013 Global Journals Inc. (US)

This page is intentionally left blank

© 2013 Global Journals Inc (US)

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 10 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach

By Shinoj T K, Sunil Jacob John
National Institute of Technology, India

Abstract- In this paper, an application of Intuitionistic Fuzzy Multiset in Robotics is discussed. The basic operations on Intuitionistic Fuzzy Multisets such as union, intersection, addition, multiplication etc. are discussed. Accuracy of Collaborative Robots using the concept of Intuitionistic Fuzzy Multiset is discussed.

Keywords: *intuitionistic fuzzy set, multiset, intuitionistic fuzzy multiset, robotics.*

GJSFR-F Classification : *MSC 2010: 68T40*



Strictly as per the compliance and regulations of :

© 2013. Shinoj T K, Sunil Jacob John. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

Global Journal of Science Frontier Research (F) Volume XIII Issue X Version I Year 2013
21

Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach

Shinoj T K, Sunil Jacob John

Abstract - In this paper, an application of Intuitionistic Fuzzy Multiset in Robotics is discussed. The basic operations on Intuitionistic Fuzzy Multisets such as union, intersection, addition, multiplication etc. are discussed. Accuracy of Collaborative Robots using the concept of Intuitionistic Fuzzy Multiset is discussed.

Keywords: intuitionistic fuzzy set, multiset, intuitionistic fuzzy multiset, robotics.

I. Introduction

Modern set theory formulated by George Cantor is fundamental for the whole Mathematics. One issue associated with the notion of a set is the concept of vagueness. Mathematics requires that all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a problem for a long time for philosophers, logicians and mathematicians. However, recently it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situations like this, many tools were suggested. They include Fuzzy sets, Multi sets, Rough sets, Soft sets and many more.

Considering the uncertainty factor, Lofti Zadeh [1] introduced Fuzzy sets in 1965, in which a membership function assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. In 1983, Krassimir. T. Atanassov [2,3] introduced the concept of Intuitionistic Fuzzy sets (IFS) by introducing a non-membership fuction together with the membership function of the fuzzy set. Among the various notions of higher-order Fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainity and vagueness. IFS reflect better the aspects of human behavior.

A human being who expresses the degree of belongingness of a given element to a set, does not often expresses the corresponding degree of non-belongingness as the complement. This psychological fact states that linguistic negation does not always coincides with logical negation. This idea of Intuitionistic fuzzy sets, which is a natural generalization of a standard Fuzzy set, seems to be useful in modelling many real life situations, like negotiation processes, psychological investigations, reasoning etc. The relation between Intuitionistic Fuzzy sets and other theories modeling imprecision can be seen in [4,5].

Many fields of modern mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. Set is a well-defined collection of distinct objects, that is, the elements of a set are pair wise

Author : Department of Mathematics, National Institute of Technology Calicut Calicut-673 601, Kerala, India.
e-mails: shinojthrissur@gmail.com, sunil@nitc.ac.in

Ref
1. L. A. Zadeh (1965) Fuzzy sets, *Inform. and Control* 8 338-353.

© 2013 Global Journals Inc. (US)

different. If we relax this restriction and allow repeated occurrences of any element, then we can get a mathematical structure that is known as Multisets or Bags. For example, the prime factorization of an integer n>0 is a Multiset whose elements are primes. The number 120 has the prime factorization $120 = 2^3 3^1 5^1$ which gives the Multiset {2, 2, 2, 3, 5}. A complete account of the development of multiset theory can be seen in [6,7]. As a generalization of multiset, Yager [8] introduced the concept of Fuzzy Multiset (FMS). An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values.

This paper explains how the concept of Intuitionist Fuzzy Multisets can be applied in the field of Robotics. Robots are machines which reduces human effort. Robots can be given intelligence to perform tasks that humans can and cannot do. They can be programmed for doing a task monotonously or they can work intelligently or dynamically according to the situations around them. Some of the applications of a mobile Robot include mine detection, surveillance, bomb detection, remote surgery, welding, cleaning small pipes, window panes and glass doors of buildings using snake-like Robots etc.

A Robot mainly contains: sensors, actuators and a controller. An accelerometer sensor is used for detecting shock/vibration, a temperature sensor can detect the temperature variations, an ultrasonic sensor/Infra-Red sensor/PIR sensor is used to detect obstacles, bump sensor senses a bump (collision), cliff sensor senses the presence of a cliff and so on. With the help of these sensors a Robot moves easily through its programmed path. iRobot Create is one such mobile robot as shown in Figure 1.

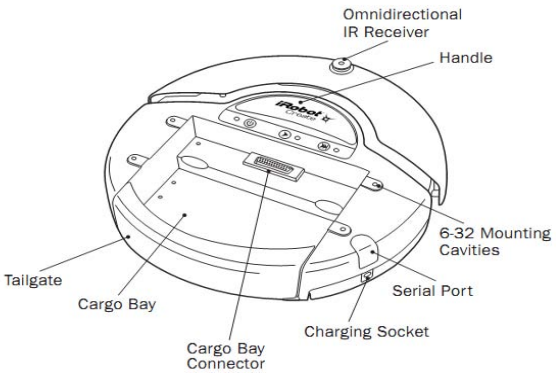


Figure 1 : iRobot Create.

When multiple Robots are used for completing a task, the system is called a multi-Robot system. Task accomplished by multiple Robots saves time and cost. To explain the concept of IFMS, a multi Robot scenario is considered consisting of a central server and a group of mobile Robots patrolling a given area for surveillance application.

Using the distance function, the sensor readings were properly interpreted for proper identification of the problem faced by the Robot.

II. Preliminaries

2.1 Definition [1] Let X be a nonempty set. A *Fuzzy set* A drawn from X is defined as $A = \{< x : \mu_A(x) > : x \in X\}$.Where : $X \rightarrow [0,1]$ is the membership function of the Fuzzy Set A.

2.2. Definition [8] Let X be a nonempty set. A *Fuzzy Multiset* (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A : X \rightarrow Q$ where Q is the set of all crisp multisets drawn from the unit interval

© 2013 Global Journals Inc (US)

[0,1]. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from [0,1]. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu^1_A(x), \mu^2_A(x),..., \mu^P_A(x))$ where $\mu^1_A(x) \geq \mu^2_A(x) \geq,..., \geq \mu^P_A(x)$.

A complete account of the applications of Fuzzy Multisets in various fields can be seen in [9].

Notes

2.3 Definition [3] Let X be a nonempty set. An *Intuitionistic Fuzzy Set* (IFS) A is an object having the form $A = \{< x : \mu_A(x), \nu_A(x) > : x \in X\}$, where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ define respectively the degree of membership and the degree of non membership of the element $x \in X$ to the set A with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

2.4 Remark Every Fuzzy set A on a nonempty set X is obviously an IFS having the form

$$A = \{ < x : \mu_A(x), 1 - \mu_A(x) > : x \in X \}$$

Using the definition of FMS and IFS, a new generalized concept called Intuitionistic Fuzzy Multiset (IFMS) is defined in [10].

III. Intuitionistic Fuzzy Multiset

3.1 Definition Let X be a nonempty set. An *Intuitionistic Fuzzy Multiset* A denoted by IFMS drawn from X is characterized by two functions: 'count membership' of A (CM_A) and 'count non membership' of A (CN_A) given respectively by $CM_A : X \to Q$ and $CN_A : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval [0, 1] such that for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CMA(x)$ which is denoted by $(\mu^1_A(x), \mu^2_A(x),..., \mu^P_A(x))$ where $(\mu^1_A(x) \geq \mu^2_A(x) \geq,... \geq, \mu^P_A(x)$ and the corresponding non membership sequence will be denoted by $(\nu^1_A(x), \nu^2_A(x),...,\nu^P_A(x))$ such that $0 \leq \mu^i_A(x) + \nu^i_A(x) \leq 1$ for every $x \in X$ and $i = 1,2,...,p$.

An IFMS A is denoted by

$$A = \{ < x: (\mu^1_A(x), \mu^2_A(x),..., \mu^P_A(x)), (\nu^1_A(x), \nu^2_A(x), ... ,\nu^P_A(x)) > : x \in X \}$$

3.2. Remark We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

3.3. Definition Length of an element x in an IFMS A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu^j_A(x) + \nu^j_A(x) \leq 1$ and it is denoted by $L(x: A)$. That is

$$L(x:A) = |CM_A(x)| = |CN_A(x)|$$

3.4 Definition If A and B are IFMSs drawn from X then $L(x:A,B) = Max\{L(x : A), L(x : B)\}$. Alternatively we use $L(x)$ for $L(x :A, B)$.

33.5.Example Consider the set $X = \{x, y, x, w\}$ with A= {< x : (0.3, 0.2), (0.4, 0.5) >, < y : (1,0.5,0.5), (0.0.5,0.2) >, < z : (0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7)>}, B= {< x : (0.4), (0.2) >, < y : (1,0.3, 0.2), (0,0.4, 0.5) >, < w : (0.2, 0.1), (0.7, 0.8) >}.

© 2013 Global Journals Inc. (US)

Now we define basic operations on IFMS. Note that we can make L(x : A) = L(x : B) by appending sufficient number of 0's and 1's with the membership and non membership values respectively.

3.6 Definition Let A and B be two IFMS. The distance function is defined as

$$d(A,B) = (\frac{1}{2}\sum_i((\mu^i_A(x) - \mu^i_B(x))^2 + (\nu^i_A(x) - \nu^i_B(x))^2 + (\Pi^i_A(x) - \Pi^i_B(x))^2))^{\frac{1}{2}}$$

where $\Pi^i_A = 1 - \nu^i_A(x) - \nu^i_A(x)$ called the IFMS index or hesitation margin.

3.7 Definition For any two IFMSs A and B drawn from a set X, the following operations and relations will hold. Let $A = \{< x : (\mu^1_A(x), \mu^2_A(x),...,\mu^P_A(x)), (\nu^1_A(x), \nu^2_A(x),...,\nu^P_A(x)) > : x \in X\}$ and $B = \{< x : (\mu^1_B(x), \mu^2_B(x),...,\mu^P_B(x)), (\nu^1_B(x), \nu^2_B(x),...,\nu^P_B(x)) > : x \in X\}$ then

1. Inclusion

$A \subset B \Leftrightarrow \mu^j_A(x) \leq \mu^j_A(x)$ and $\nu^j_A(x) \geq \nu^j_B(x)$;

$j = 1,2,..., L(x), x \in X$

$A = B \Leftrightarrow A \subset B$ and $B \subset A$

2. Complement

$\neg A = \{< x : (\nu^1_A(x),...,\nu^P_A(x)), (\mu^1_A(x),..... ..., \mu^P_A(x)) > : x \in X\}$

3. Union $(A \cup B)$

In $A \cup B$ the membership and non membership values are obtained as follows.

$\mu^j_{A\cup B}(x) = \mu^j_A(x) \vee \mu^j_B(x)$

$\nu^j_{A\cup B}(x) = \nu^j_A(x) \wedge \nu^j_B(x)$

$j = 1, 2,...,L(x), x \in X.$

4. Intersection $(A \cap B)$

In $A \cap B$ the membership and non membership values are obtained as follows.

$\mu^j_{A\cap B}(x) = \mu^j_A(x) \wedge \mu^j_B(x)$

$\nu^j_{A\cap B}(x) = \nu^j_A(x) \vee \nu^j_B(x)$

$j = 1, 2,...,L(x), x \in X.$

5. Addition $(A \oplus B)$

In $A \oplus B$ the membership and non membership values are obtained as follows.

$\mu^j_{A\oplus B}(x) = \mu^j_A(x) + \mu^j_B(x) - \mu^j_A(x).\ \mu^j_B(x)$

$\nu^j_{A\oplus B}(x) = \nu^j_A(x).\ \nu^j_B(x)$

Notes

© 2013 Global Journals Inc (US)

$j = 1, 2,...,L(x), x \in X.$

6. Multiplication (A ⊗ B)

In $A \otimes B$ the membership and nonmembership values are obtained as follows.

$$\mu^{j}_{A\otimes B}(x) = \mu^{j}_{A}(x).\ \mu^{j}_{B}(x)$$

$$\nu^{j}_{A\otimes B}(x) = \nu^{j}_{A}(x) + \nu^{j}_{B}(x) - \nu^{j}_{A}(x).\ \nu^{j}_{B}(x)$$

$j = 1, 2,...,L(x), x \in X.$

here $\vee$, $\wedge$, . , +, - denotes maximum, minimum, multiplication, addition, subtraction of real numbers respectively.

IV. IFMS Theory for Multi Robot System

Most of human reasoning involves the use of variables whose values are fuzzy sets. This is the basis for the concept of a linguistic variable, that is, a variable whose values are words rather than numbers. But in some situations like decision making problems (such as Medical diagnosis, Sales analysis, Marketing etc.) the description by a linguistic variable in terms of membership function only is not adequate. There is chance of existing a non-null complement. IFS can be used in this context as a proper tool for representing both membership and non-membership of an element to a set. Such situations are explained in [11]. But there are situations that each element has different membership values. In such situations IFMS is more adequate. Here we present IFMS as a tool for reasoning such a situation.

An example of a multi Robot system is presented. The multi Robot system [12] considered consists of a central controller and four patrolling Robots in a large area. The total area is divided into four equal parts and assigned to each Robot. The Robot patrols in its assigned area. Each Robot is equipped with ultrasonic sensor, accelerometer sensor, cliff sensor, bump sensor and temperature sensor and is wirelessly controlled by the controller. The controller makes decisions depending upon the sensor readings. For example, if the cliff sensor value in Robot1 indicates the presence of a cliff, the controller can change the commands that are sent to the Robot1; that is, the controller can direct the Robot1 towards the right, left or backward directions. Similar is the case with every other sensor reading.

Let R = {R1, R2, R3, R4} be a set of four Robots, C = {Fire, Obstacle, Bump, Cliff, Vibration} be a set of situations or conditions and S = {Temperature sensor, Ultrasonic sensor, Bump sensor, Cliff sensor, Accelerometer sensor} be a set of sensors deployed on each Robot. A single Robot can be assigned different membership and non membership values for the five different sensor readings. This is where IFMS comes into picture.

Ref

11. De S.K., Biswas R. and Roy A.R. (2001) An Application of Intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets and Systems*, Vol. 117, No. 2, pp. 209-213.

© 2013 Global Journals Inc. (US)

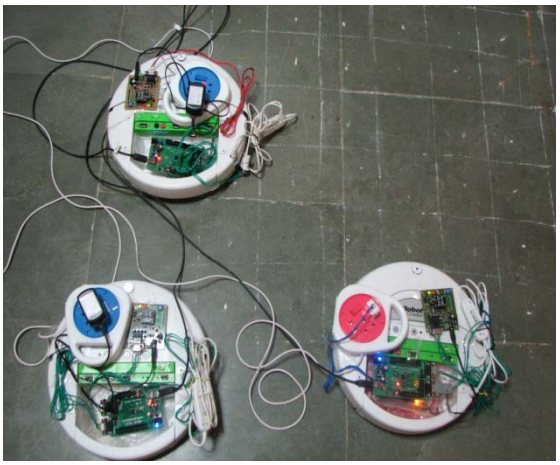


Figure 2 : A multi robot system with three patrolling robots

Whether from a single reading can we conclude what are the situations faced by the Robots? The sensor readings from the Robots have to be monitored for a particular time, say for three minutes. If for example, the ultrasonic sensor in Robot1 indicates an obstacle, it sends a message to the controller so that the corrective measure could be taken. The controller has to make sure whether the Robot1 is really faced with an obstacle or not. For that purpose, the controller monitors the ultrasonic sensor reading for three minutes. Depending upon the consistency of the readings, the controller identifies the situation.

To understand IFMS theory, let us consider the situation where the Robot1 faces an obstacle, Robot2 experiences a shock/vibration, Robot3 faces a bump and Robot4 detects a cliff. Thus whenever the ultrasonic sensor detects an obstacle and the accelerometer sensor detects a vibration, alert is sent to the controller and the controller monitors the situation for 3 minutes.

In Table-I each sensor reading is described by three numbers: Membership μ, non-membership ν and hesitation margin Π.

Table II shows the Robots and the corresponding membership functions to the sensor values.

Table 1

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
Temperature sensor	(0.8,0.,1,0.1)	(0.2,0.7,0.1)	(0.1,0.7,0.2)	(0.2,0.5,0.3)	(0.5,0.2,0.3)
Ultrasonic sensor	(0.2,0.,7,0.1)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	(0.2,0.,7,0.1)	(0.1,0.7,0.2)
Bump sensor	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.9,0.1,0.0)	(0.1,0.7,0.2)	(0.2,0.5,0.3)
Cliff sensor	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.7,0.1,0.2)	(0.1,0.7,0.2)
Accelerometer sensor	(0.1,0.7,0.2)	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.8,0.2,0.0)

The objective is to make a proper decision for each Robot. Hence the readings are monitored for a particular interval time (3 minutes).

© 2013 Global Journals Inc (US)

Notes

Table II

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Accelerometer sensor
R_1	(0.8,0.1, 0.1)	(0.8, 0.1, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.8,0.0)	(0.3, 0.6, 0.1)
R_2	(0.4, 0.5, 0.1)	(0.3, 0.7,0.0)	(0.1, 0.7, 0.2)	(0.2, 0.6, 0.2)	(0.8, 0.1, 0.1)
R_3	(0.1, 0.8, 0.1)	(0.6, 0.4,0.0)	(0.8, 0.1, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.7, 0.1)
R_4	(0.1, 0.7, 0.2)	(0.3, 0.6, 0.1)	(0.2, 0.7, 0.1)	(0.7, 0.2, 0.1)	(0.1, 0.7, 0.2)

Table III shows the sensor readings monitored for 3 minutes, one reading per minute. Table IV shows the distances of each Robot to the situation considered. Thus, using this distance function, IFMS theory is able to make out the correct situation of each Robot.

Table III

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Accelerometer sensor
R_1	(0.8,0.7, 0.9) (0.1, 0.2, 0.0) (0.1, 0.1, 0.1)	(0.8, 0.8, 0.9) (0.1, 0.1, 0.1) (0.1, 0.1, 0.0)	(0.1, 0.2, 0.0) (0.9, 0.7, 0.8) (0.0, 0.1, 0.2)	(0.2, 0.1, 0.0) (0.8, 0.6, 0.7) (0.0, 0.3, 0.3)	(0.3, 0.3, 0.4) (0.6, 0.4, 0.4) (0.1, 0.3, 0.2)
R_2	(0.4, 0.3, 0.3) (0.5, 0.4, 0.6) (0.1, 0.3, 0.1)	(0.3, 0.2, 0.3) (0.7, 0.6, 0.1) (0, 0.2, 0.7)	(0.1, 0.2, 0.4) (0.7, 0.6, 0.4) (0.2, 0.2, 0.2)	(0.2, 0.5, 0.2) (0.6, 0.4, 0.7) (0.2, 0.1, 0.1)	(0.8, 0.7, 0.6) (0.1, 0.2, 0.3) (0.1, 0.1, 0.1)
R_3	(0.1, 0.2, 0.1) (0.8, 0.6, 0.9) (0.1, 0.2, 0.0)	(0.6, 0.2, 0.1) (0.4, 0.0, 0.7) (0, 0.8, 0.2)	(0.8, 0.7, 0.8) (0.1, 0.1, 0.1) (0.1, 0.2, 0.1)	(0.1, 0.2, 0.2) (0.9, 0.7, 0.6) (0.0, 0.1, 0.2)	(0.2, 0.3, 0.2) (0.7, 0.7, 0.7) (0.1, 0.0, 0.1)
R_4	(0.1, 0.4, 0.5) (0.7, 0.4, 0.3) (0.2, 0.2, 0.2)	(0.3, 0.3, 0.4) (0.6, 0.3, 0.5) (0.1, 0.4, 0.1)	(0.2, 0.1, 0.0) (0.7, 0.6, 0.7) (0.1, 0.3, 0.3)	(0.8, 0.6, 0.9) (0.2, 0.3, 0.0) (0, 0.1, 0.1)	(0.1, 0.5, 0.4) (0.7, 0.4, 0.3) (0.2, 0.1, 0.3)

Table IV

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
R_1	0.72	**0.65**	1.07	1.06	0.90
R_2	0.84	0.79	0.97	0.83	**0.52**
R_3	1.07	0.89	**0.50**	1.03	1.02
R_4	0.79	0.84	1.07	**0.46**	0.87

© 2013 Global Journals Inc. (US)

In the above table the lowest distance point gives the accuracy of the Robot. Robot R_1 is near an obstacle, R2 experiences a vibration, R3 is bumped and R4 is near a cliff.

V. Conclusions

In this paper, we have discussed the various basic operations of Intuitionistic Fuzzy Multiset and its application in Robotics. In the proposed method, we measured the distances of each Robot from each situation by considering the sensor readings. The concept of multiness is incorporated by taking the samples from the same Robot for a particular time.

VI. Acknowledgements

The first author acknowledges the financial support given by the University Grants Commission (UGC), Government of India throughout the preparation of this paper.

References Références Referencias

1. L. A. Zadeh (1965) Fuzzy sets, *Inform. and Control* 8 338-353.
2. K. T. Atanassov (1983) Intuitionistic Fuzzy sets, *VII ITKRs Session, Sofia* (deposed in Central Science-Technical Library of Bulgarian Academy of Science, 1697/84) (in Bulgarian).
3. K. T. Atanassov (1986) Intuitionistic Fuzzy sets, *Fuzzy sets and Systems* 20, 87-96.
4. G. Takeuti, S. Titani, (1984) Intuitionistic fuzzy Logic and intuitionistic Fuzzy set theory, *J. Symbolic Logic* 49, 851-866.
5. Glad Deschrijver, Etienne E. Kerre, (2007) on the position of intuitionistic Fuzzy set theory in the framework of theories modelling imprecision, *Information Sciences* 177, 1860-1866.
6. W. D. Blizard (1993) Dedekind multisets and function shells, *Theoretical Computer Science*, 110, 79-98.
7. W. D. Blizard (1989) Multiset Theory, *Notre Dame Journal of Formal Logic*, Vol.30, No.1, 36-66.
8. R. R Yager (1986) on the theory of bags, *Int. J. of General Systems* 13 23-37.
9. Cristian S. Calude, Gheorghe Paun, Grzegorz Rozenberg, Arto Salomaa (2001) *Multiset Processing*, Springer-Verlag, Germany.
10. Shinoj T K, Sunil Jacob John(2012), Intuitionistic Fuzzy Multiset and its application in Medical Diagnosis, *International Journal of Mathematical and Computational Sciences,* WASET 6, 2012, 34-38
11. De S.K., Biswas R. and Roy A.R. (2001) An Application of Intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets and Systems*, Vol. 117, No. 2, pp. 209-213.
12. Mazda Ahmadi and Peter Stone, "A Multi-Robot System for Continuous Area Sweeping Tasks", Proceedings of the 2006 IEEE International Conference on Robotics and Automation, May 2006, pp: 1724-1729.

© 2013 Global Journals Inc (US)

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 10 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Computation of Some Wonderful Results Involving Certain Polynomials

By Salahuddin & R.K. Khola
Mewar University, India

Abstract- In this paper we have developed some indefinite integrals involving certain polynomials in the form of Hypergeometric function. The results represent here are assume to be new.

Keywords: *hypergeometric function, lucas polynomials, gegenbaur polynomials, harmonic number, bernoulli polynomial, hermite polynomials.*

GJSFR-F Classification : *MSC 2010 11B39, 11B68, 33C05, 33C45, 33D50, 33D60*



Strictly as per the compliance and regulations of :

© 2013. Salahuddin & R.K. Khola. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

Computation of Some Wonderful Results Involving Certain Polynomials

Salahuddin[α] & R.K. Khola[σ]

Abstract- In this paper we have developed some indefinite integrals involving certain polynomials in the form of Hypergeometric function. The results represent here are assume to be new.

Keywords: *hypergeometric function, lucas polynomials, gegenbaur polynomials, harmonic number, bernoulli polynomial, hermite polynomials.*

I. Introduction

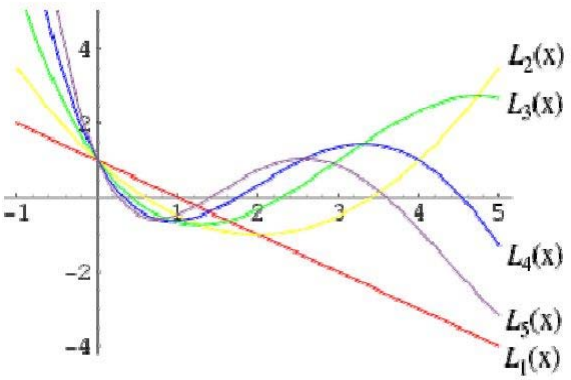
a) We have the generalized Gaussian hypergeometric function of one variable

$$ {}_AF_B(a_1,a_2,\ldots,a_A;b_1,b_2,\ldots b_B;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \ldots\ldots (a_A)_k \, z}{(b_1)_k (b_2)_k \ldots\ldots (b_B)_k \, k!} \qquad (1) $$

where the parameters b_1 , b_2 , ...,b_B are neither zero nor negative integers and A , B are non negative integers.

The series converges for all finite z if A ≤ B, converges for $|z|<1$ if $A=B+1$, diverges for all z, $z \neq 0$ if A > B+1.

b) Lucas Polynomials



The Lucas polynomials are the w-polynomials obtained by setting p(x) = x and q(x) = 1 in the Lucas polynomials sequence. It is given explicitly by

$$ L_n(x) = 2^{-n}[(x-\sqrt{x^2+4}\ )^n + (x+\sqrt{x^2+4}\ )^n] \qquad (2) $$

Authors α σ : Mewar University, Gangrar, Chittorgarh (Rajasthan), India. e-mails: vsludn@gmail.com, sludn@yahoo.com

© 2013 Global Journals Inc. (US)

The first few are

$$\left.\begin{aligned} L_1(x) &= x \\ L_2(x) &= x^2 + 2 \\ L_3(x) &= x^3 + 3x \\ L_4(x) &= x^4 + 4x^2 + 2 \end{aligned}\right\} \tag{3}$$

c) Generalized Harmonic Number

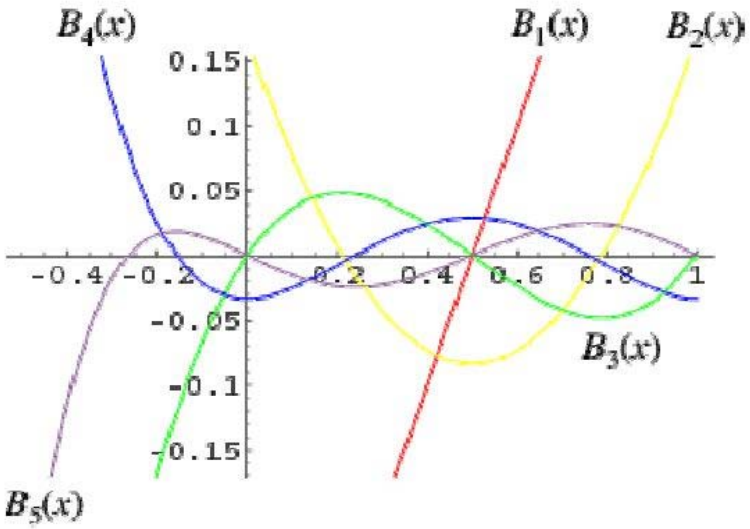
The generalized harmonic number of order *n* of *m* is given by

$$H_n^{(m)} = \sum_{k=1}^{n} \frac{1}{k^m} \tag{4}$$

In the limit of $n \to \infty$ the generalized harmonic number converges to the Riemann zeta function

$$\lim_{n\to\infty} H_n^{(m)} = \varsigma(m) \tag{5}$$

d) Bernoulli Polynomial



In mathematics, the Bernoulli polynomials occur in the study of many special functions and in particular the Riemann zeta function and Hurwitz zeta function. This is in large part because they are an Appell sequence, i.e. a Sheffer sequence for the ordinary derivative operator. Unlike orthogonal polynomials, the Bernoulli polynomials are remarkable in that the number of crossing of the *x*-axis in the unit interval does not go up as the degree of the polynomials goes up. In the limit of large degree, the Bernoulli polynomials, appropriately scaled, approach the sine and cosine functions.
Explicit formula of Bernoulli polynomials is

$B_n(x) = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} b_{n-k} x^k$, for n≥0, where b_k are the Bernoulli numbers.

The generating function for the Bernoulli polynomials is

Notes

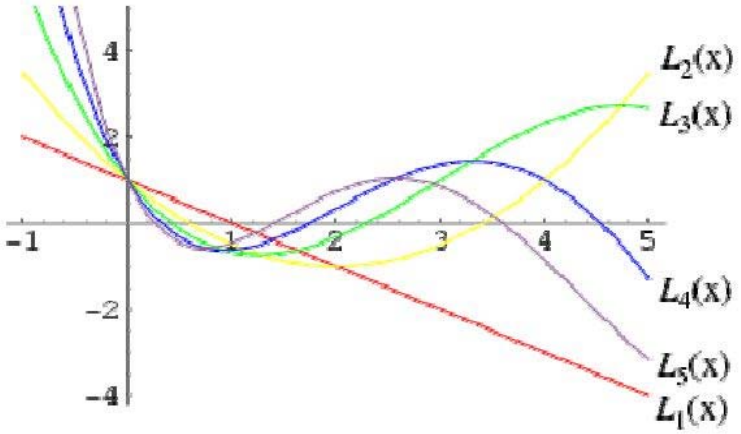
© 2013 Global Journals Inc (US)

e) Gegenbauer polynomials

In Mathematics, Gegenbauer polynomials or ultraspherical polynomials $C_n^{(\alpha)}(x)$ are orthogonal polynomials on the interval [-1,1] with respect to the weight function $(1-x^2)^{\alpha-\frac{1}{2}}$. They generalize Legendre polynomials and Chebyshev polynomials, and are special cases of Jacobi polynomials. They are named after Leopold Gegenbauer.
Explicitly ,

$$C_n^{(\alpha)}(x) = \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k \frac{\Gamma(n-k+\alpha)}{\Gamma(\alpha)k!(n-2k)!}(2z)^{(n-2k)} \tag{7}$$

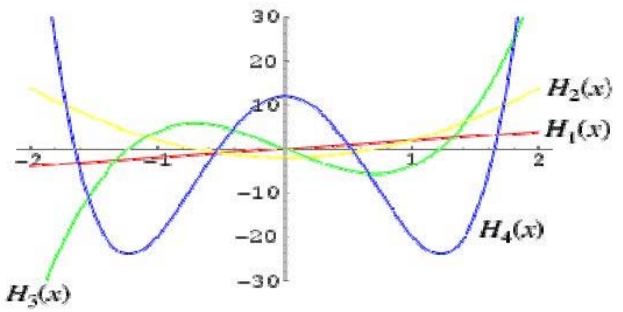
f) Laguerre polynomials



The Laguerre polynomials are solutions $L_n(x)$ to the Laguerre differential equation
$xy''+(1-x)y'+\lambda y=0$, which is a special case of the more general associated Laguerre differential equation, defined by
$xy''+(\nu+1-x)y'+\lambda y=0$, where λ and ν are real numbers with ν=0.
The Laguerre polynomials are given by the sum

$$L_n(x) = \sum_{k=0}^{n} \frac{(-1)^k}{k!} \frac{n!}{k!(n-k)!} x^k \tag{8}$$

g) Hermite polynomials



© 2013 Global Journals Inc. (US)

The Hermite polynomials $H_n(x)$ are set of orthogonal polynomials over the domain (-∞,∞) with weighting function e^{-x^2}.

The Hermite polynomials $H_n(x)$ can be defined by the contour integral

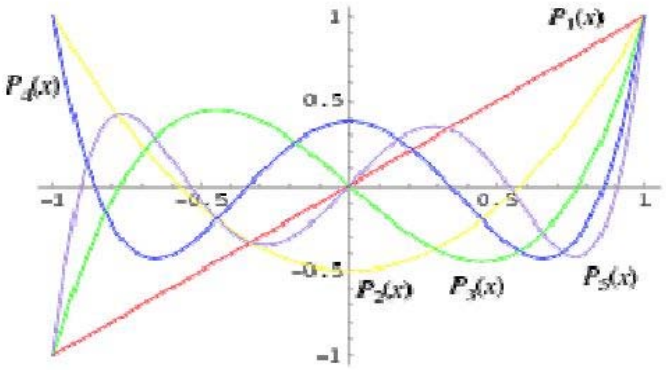
$$H_n(z) = \frac{n!}{2\pi i} \oint e^{-r^2+2rz} t^{-n-1} dt ,$$

Where the contour incloses the origin and is traversed in a counterclockwise direction (Arfken 1985, p. 416).

The first few Hermite polynomials are

$$\left.\begin{aligned} H_0(x) &= 1 \\ H_1(x) &= 2x \\ H_2(x) &= 4x^2 - 2 \\ H_3(x) &= 8x^3 - 12x \\ H_4(x) &= 16x^4 - 48x^2 + 12 \end{aligned}\right\} \qquad (9)$$

h) Legendre function of the first kind



The Legendre polynomials, sometimes called Legendre functions of the first kind, Legendre coefficients, or zonal harmonics (Whittaker and Watson 1990, p. 302), are solutions to the Legendre differential equation. If *l* is an integer, they are polynomials. The Legendre polynomials $P_n(x)$ are illustrated above for $x \in [-1, 1]$ and *n*=1, 2, ..., 5.

The Legendre polynomials $P_n(x)$ can be defined by the contour integral

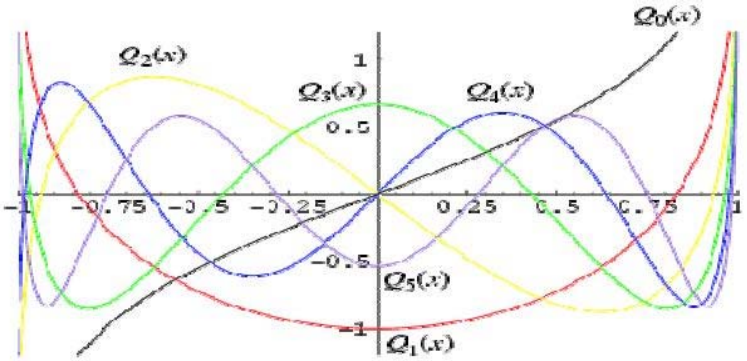
$$P_n(z) = \frac{1}{2\pi i} \oint (1 - 2tz + t^2)^{-\frac{1}{2}} t^{-n-1} dt , \qquad (10)$$

where the contour encloses the origin and is traversed in a counterclockwise direction (Arfken 1985, p. 416).

© 2013 Global Journals Inc (US)

Notes

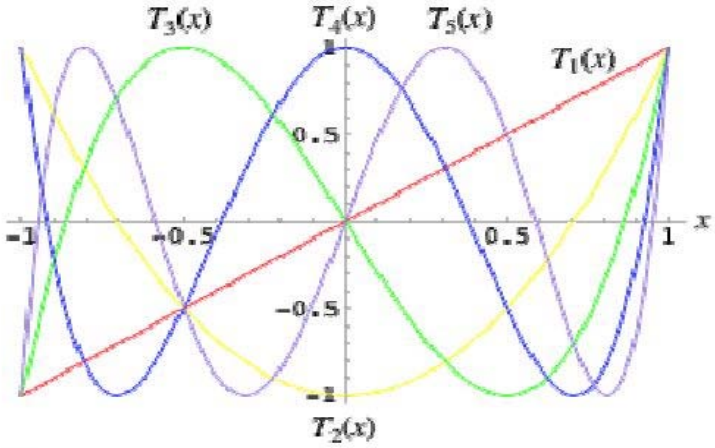
i) Legendre function of the second kind



The second solution $Q_1(x)$ to the Legendre differential equation. The Legendre functions of the second kind satisfy the same recurrence relation as the Legendre polynomials.
The first few are

$$\left.\begin{aligned} Q_0(x) &= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \\ Q_1(x) &= \frac{x}{2}\ln\left(\frac{1+x}{1-x}\right)\text{-}1 \\ Q_2(x) &= \frac{3x^2-1}{4}\ln\left(\frac{1+x}{1-x}\right)-\frac{3x}{2} \end{aligned}\right\} \tag{11}$$

j) Chebyshev polynomial of the first kind



The Chebyshev polynomials of the first kind are a set of orthogonal polynomials defined as the solutions to the Chebyshev differential equation and denoted $T_n(x)$. They are used as an approximation to a least squares fit, and are a special case of the Gegenbauer polynomial with $\alpha=0$. They are also intimately connected with trigonometric multiple-angle formulas.

The Chebyshev polynomial of the first kind $T_n(z)$ can be defined by the contour integral

$$T_n(z) = \frac{1}{4\pi i}\oint \frac{(1-t^2)t^{-n-1}}{1-2tz+t^2}dt\ , \tag{12}$$

© 2013 Global Journals Inc. (US)

where the contour encloses the origin and is traversed in a counterclockwise direction (Arfken 1985, p. 416).

The first few Chebyshev polynomials of the first kind are

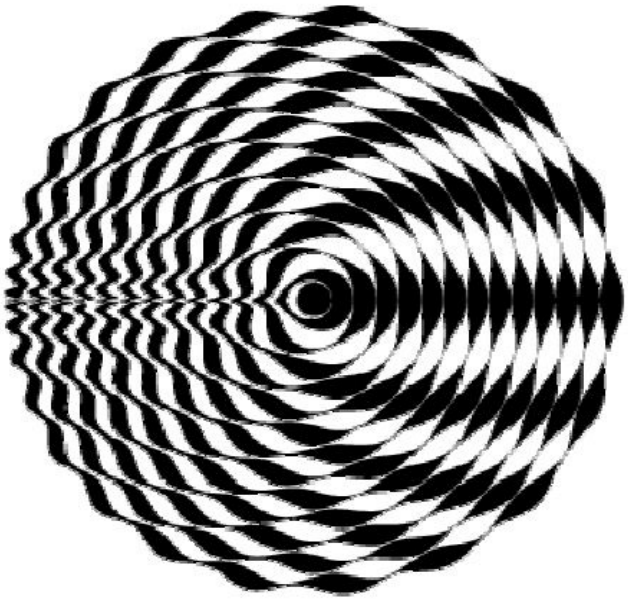
$T_0(x) = 1$

$T_1(x) = x$

$T_2(x) = 2x^2 - 1$

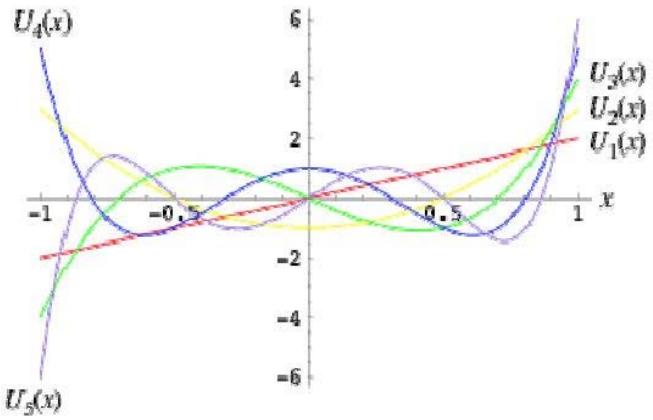
$T_3(x) = 4x^3 - 3x$

A beautiful plot can be obtained by plotting $T_n(x)$ radially, increasing the radius for each value of n, and filling in the areas between the curves (Trott 1999, pp. 10 and 84).



The Chebyshev polynomials of the first kind are defined through the identity $T_n(\cos\theta)=\cos n\theta$.

k) Chebyshev polynomial of the second kind



A modified set of Chebyshev polynomials defined by a slightly different generating function. They arise in the development of four-dimensionalspherical harmonics in angular momentum

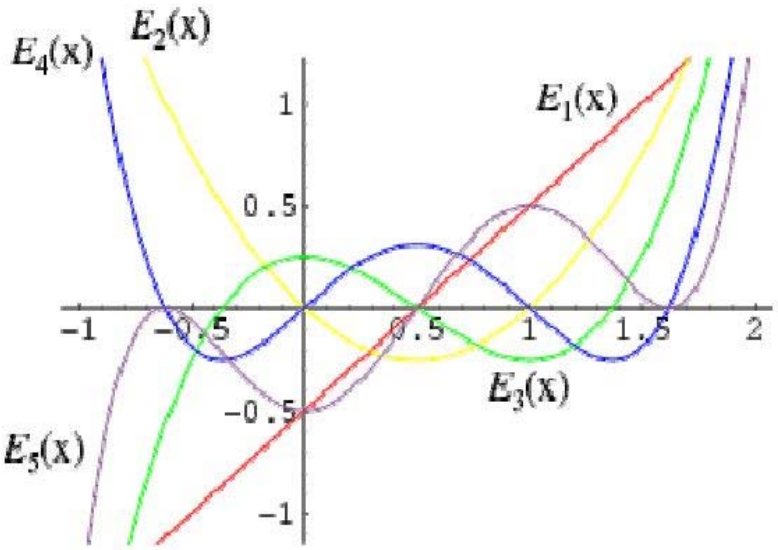
© 2013 Global Journals Inc (US)

theory. They are a special case of the Gegenbauer polynomial with $\alpha=1$. They are also intimately connected with trigonometric multiple-angle formulas.

The first few Chebyshev polynomials of the second kind are

$$\left.\begin{aligned} U_0(x) &= 1 \\ U_1(x) &= 2x \\ U_2(x) &= 4x^2 - 1 \\ U_3(x) &= 8x^3 - 4x \end{aligned}\right\} \tag{13}$$

1) Euler polynomial



The Euler polynomial $E_n(x)$ is given by the Appell sequence with

$$g(t) = \frac{1}{2}(e^t + 1) \ ,$$

giving the generating function

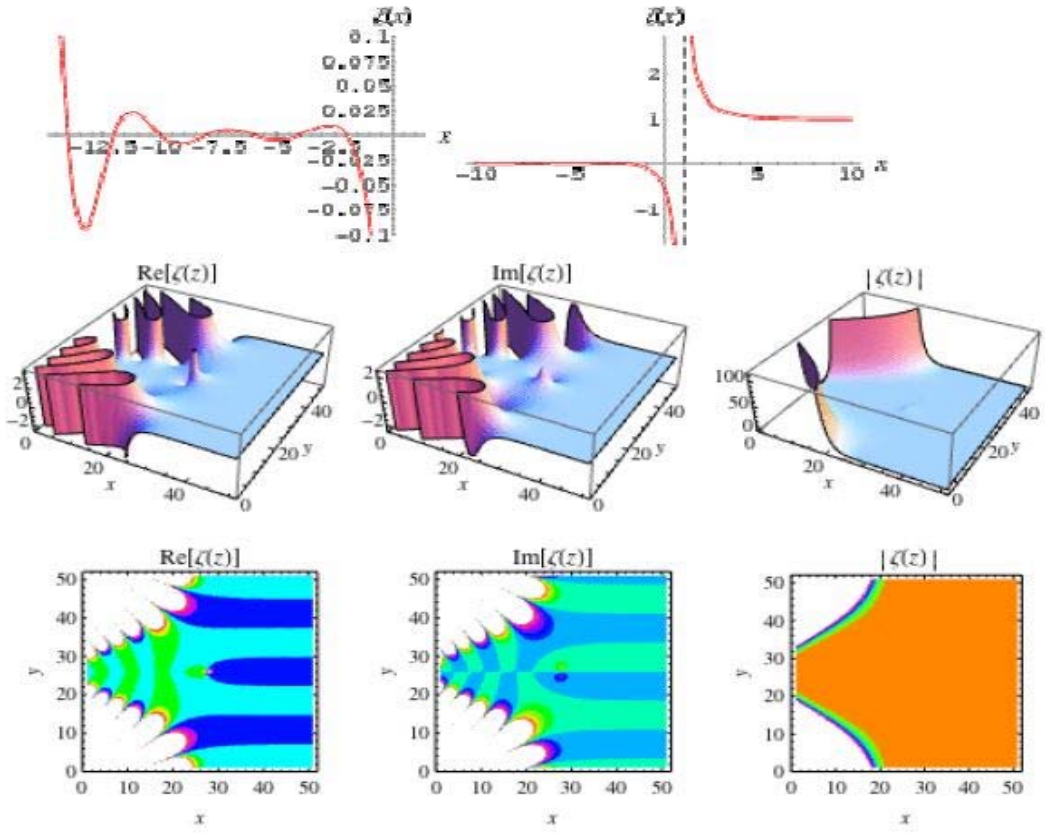
$$\frac{2e^{xt}}{e^t + 1} \equiv \sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} \cdot \tag{14}$$

The first few Euler polynomials are

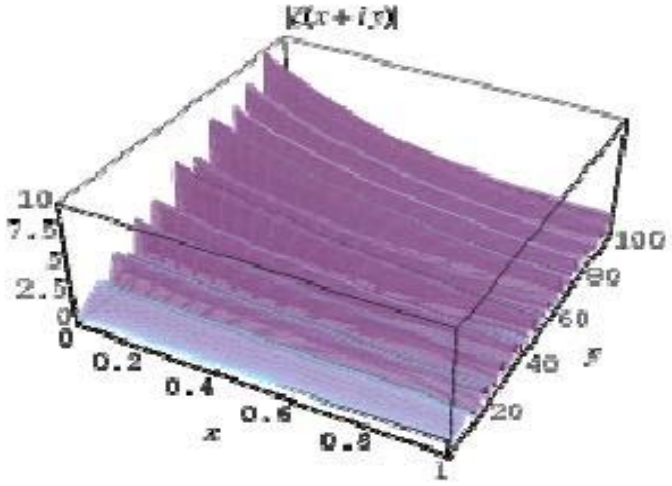
$$\left.\begin{aligned} E_0(x) &= 1 \\ E_1(x) &= x - \frac{1}{2} \\ E_2(x) &= x^2 - x \\ E_3(x) &= x^3 - \frac{3}{2}x^2 + \frac{1}{4} \end{aligned}\right\} \tag{15}$$

© 2013 Global Journals Inc. (US)

m) Generalized Riemann zeta function



The Riemann zeta function is an extremely important special function of mathematics and physics that arises in definite integration and is intimately related with very deep results surrounding the prime number theorem. While many of the properties of this function have been investigated, there remain important fundamental conjectures (most notably the Riemann hypothesis) that remain unproved to this day. The Riemann zeta function $\zeta(s)$ is defined over the complex plane for one complex variable, which is conventionally denoted s (instead of the usual z) in deference to the notation used by Riemann in his 1859 paper that founded the study of this function (Riemann 1859).



Notes

© 2013 Global Journals Inc (US)

The plot above shows the "ridges" of $|\zeta(x+iy)|$ for $0<x<1$ and $1<y<100$. The fact that the ridges appear to decrease monotonically for $0\le x\le 1/2$ is not a coincidence since it turns out that monotonic decrease implies the Riemann hypothesis (Zvengrowski and Saidak 2003; Borwein and Bailey 2003, pp. 95-96).

On the real line with $x>1$, the Riemann zeta function can be defined by the integral

$$\zeta(x)\equiv\frac{1}{\Gamma(x)}\int_0^\infty\frac{u^{x-1}}{e^u-1}du$$, where $\Gamma(x)$ is the gamma function.

Notes

n) Complex infinity

Complex infinity is an infinite number in the complex plane whose complex argument is unknown or undefined. Complex infinity may be returned by*Mathematica*, where it is represented symbolically by ComplexInfinity. The Wolfram Functions Site uses the notation $\tilde{\infty}$ to represent complex infinity.

II. Main Results

$$\int\frac{L_1(x)}{\sqrt{1-x^n}}dx=\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)+C \tag{17}$$

$$\int\frac{H_1(x)}{\sqrt{1-x^n}}dx=x\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};\frac{n+1}{n};x^n\right)+C \tag{18}$$

$$\int\frac{B_1(x)}{\sqrt{1-x^n}}dx=\frac{1}{2}x\left[x\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)-{}_2F_1\left(\frac{1}{2},\frac{1}{n};\frac{n+1}{n};x^n\right)\right]+C \tag{19}$$

$$\int\frac{C_1(x)}{\sqrt{1-x^n}}dx=x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)+C \tag{20}$$

$$\int\frac{F_1(x)}{\sqrt{1-x^n}}dx=\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)+C \tag{21}$$

$$\int\frac{L_1[x]}{\sqrt{1-x^n}}dx=x\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};\frac{n+1}{n};x^n\right)-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)+C \tag{22}$$

$$\int\frac{H_1(x)}{\sqrt{1-x^n}}dx=x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)+C \tag{23}$$

$$\int\frac{P_1(x)}{\sqrt{1-x^n}}dx=\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right)+C \tag{24}$$

© 2013 Global Journals Inc. (US)

$$\int \frac{T_1(x)}{\sqrt{1-x^n}}\,dx = \frac{1}{2}\,x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right) + C \tag{25}$$

$$\int \frac{U_1(x)}{\sqrt{1-x^n}}\,dx = x^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right) + C \tag{26}$$

$$\int \frac{E_1(x)}{\sqrt{1-x^n}}\,dx = \frac{1}{2}\,x\left[x\,{}_2F_1\left(\frac{1}{2},\frac{2}{n};\frac{n+2}{n};x^n\right) - {}_2F_1\left(\frac{1}{2},\frac{1}{n};\frac{n+1}{n};x^n\right)\right] + C \tag{27}$$

$$\int \frac{\zeta(1,x)}{\sqrt{1-x^n}} = \tilde{\infty} + C \tag{28}$$

Notes

References Références Referencias

1. Abramowitz, Milton; Stegun, Irene A.(1965), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, New York: Dover.
2. Andrews, L.C.(1992) ; *Special Function of mathematics for Engineers,Second Edition*, McGraw-Hill Co Inc., New York.
3. Arfken, G. ; *Mathematical Methods for Physicists*, Orlando, FL: Academic Press.
4. Bells, Richard, Wong, Roderick ; *Special Functions , A Graduate Tex* t, Cambridge Studies in Advanced Mathematics, 2010.
5. Prudnikov, A. P., Marichev, O. I., and Brychkov, Yu. A. ; *Integrals and Series, Vol. 3: More Special Functions.*, NJ:Gordon and Breach, 1990.
6. Rivlin, T. J. ;*Chebyshev Polynomials.,* New York: Wiley, 1990.
7. Wikipedia(2013,October), Bernoulli Polynomials. Retrived from website: http://en.wikipedia.org/wiki/Bernoulli_polynomials.
8. Wikipedia(2013,October), Gegenbaur Polynomials. Retrived from website: http://en.wikipedia.org/wiki/Gegenbauer_polynomials.
9. WolframMathworld(2013,October), Riemann Zeta function. Retrived from website: http://mathworld.wolfram.com/RiemannZetaFunction.html.
10. WolframMathworld(2013,October), Harmonic Number . Retrived from website: http://mathworld.wolfram.com/HarmonicNumber.html.
11. WolframMathworld(2013,October), LaguerrePolynomials. Retrived from website: http://mathworld.wolfram.com/LaguerrePolynomial.html.
12. WolframMathworld(2013,October), Hermite Polynomials. Retrived from website: http://mathworld.wolfram.com/HermitePolynomial.html.
13. WolframMathworld(2013,October), Legendre Polynomial. Retrived from website: http://mathworld.wolfram.com/LegendrePolynomial.html.
14. WolframMathworld(2013,October), Chebyshev Polynomial of the first kind. Retrived from website: http://mathworld.wolfram.com/ChebyshevPolynomialoftheFirstKind.html.
15. WolframMathworld(2013,October), Lucas polynomials. Retrived from website: http://mathworld.wolfram.com/LucasPolynomials.html.

© 2013 Global Journals Inc (US)

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 10 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Metric Characterization of the Generalized Fermat Points

By Sándor N. Kiss

Abstract- We build on the sides of a triangle ABC, outwards and inwards, similar triangles with one another. We prove some relations referring to the generalized Fermat points. We deduce these results based on trigonometrical identities.

GJSFR-F Classification : *MSC 2010: 11J83, 11D41*



Strictly as per the compliance and regulations of :

© 2013. Sándor N. Kiss. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

eJournal
physical existance Print Journal
epaper
open standard
3d virtual journal

[2] H. Mowaffaq, An Advanced Calculus Approach to Finding the Fermat Point, *Math. Mag.* **67**, 29-34, 1994.

The Metric Characterization of the Generalized Fermat Points

Sándor N. Kiss

Abstract- We build on the sides of a triangle ABC, outwards and inwards, similar triangles with one another. We prove some relations referring to the generalized Fermat points. We deduce these results based on trigonometrical identities.

I. Introduction

If we build outwards on the sides BC, CA, AB of an arbitrary triangle ABC three equilateral triangles BCX_1, CAY_1, ABZ_1 then

$$AX_1 = BY_1 = CZ_1 = \pm AF_1 \pm BF_1 \pm CF_1, \tag{1}$$

where $F_1 = AX_1 \cap BY_1 \cap CZ_1$ is *the first (outward) Fermat point of triangle* ABC (Figure 1). A minus sign being taken if the angle of triangle ABC at that vertex exceeds 120°.

If we build inwards on the sides BC, CA, AB of an arbitrary triangle ABC three equilateral triangles BCX_2, CAY_2, ABZ_2 then

$$AX_2 = BY_2 = CZ_2 = \pm AF_2 \pm BF_2 \pm CF_2, \tag{2}$$

where $F_2 = AX_2 \cap BY_2 \cap CZ_2$ is *the second (inward) Fermat point of triangle* ABC (Figure 2). A minus sign being taken at each vertex where the angle of triangle ABC is larger than 60°.

In this paper we generalize these results: on the sides of triangle ABC we build, outwards resp. inwards, three similar triangles.

We mention but in this article we don't discuss the Fermat-Steiner problem:

Given a triangle ABC, how can we find a point P in its plane for which PA+PB+PC is minimal?

The solution and some generalization of this problem was a deal for many mathematicians (see [2], [4], [5]). The historical survey of the Fermat-Steiner problem could be found in [3].

Let be given a triangle ABC with the usual notations: A, B, C the angles, a, b, c the sides, R the circumradius, Δ the area of the triangle ABC, S the twice of the area of the triangle $(S = 2\Delta)$.

Sándor N. Kiss: "Constantin Brâncuşi" Technology Lyceum, Satu Mare, Romania. e-mail: d.sandor.kiss@gmail.com

© 2013 Global Journals Inc. (US)

II. The First Generalized Fermat Point of the Triangle

We build on the sides of the triangle ABC outwards the similar triangles BCD, CAE and ABF in that way, that $BAF\angle = \alpha = CAE\angle$, $CBD\angle = \beta = ABF\angle$, $ACE\angle = \gamma = BCD\angle$ (Figure 3). Since $A+B+C = \pi = \alpha+\beta+\gamma$, therefore $BDC\angle = \alpha$, $CEA\angle = \beta$, $AFB\angle = \gamma$.

We work with barycentric coordinates with reference to the triangle ABC. According to the Conway's formulas, the barycentric coordinates of the points D, E, F are: $D = \left(-a^2 : S_\gamma + S_C : S_\beta + S_B\right)$, $E = \left(S_\gamma + S_C : -b^2 : S_\alpha + S_A\right)$, $F = \left(S_\beta + S_B : S_\alpha + S_A : -c^2\right)$, where $S_\theta = S \cdot ctg\theta$. Consequently $S_A = bc\cos A$, $S_B = ca\cos B$, $S_C = ab\cos C$.

The equations of the lines AD, BE, CF are: $-\left(S_\beta + S_B\right)y + \left(S_\gamma + S_C\right)z = 0$,

$$\left(S_\alpha + S_A\right)x - \left(S_\gamma + S_C\right)z = 0,$$

$$-\left(S_\alpha + S_A\right)x + \left(S_\beta + S_B\right)y = 0.$$

Swmming random by two of the above equations, we get the third equation, the lines AD, BE and CF meet each other in a point $K = AD \cap BE \cap CF$. This point K will be called **the first generalized Fermat point of the triangle**.

The barycentric coordinates of the point K are:

$$K = \left(\left(S_\beta + S_B\right)\left(S_\gamma + S_C\right) : \left(S_\gamma + S_C\right)\left(S_\alpha + S_A\right) : \left(S_\alpha + S_A\right)\left(S_\beta + S_B\right)\right)$$

$$= \left(\frac{1}{S_\alpha + S_A} : \frac{1}{S_\beta + S_B} : \frac{1}{S_\gamma + S_C}\right).$$

We introduce the following notations:

$$\lambda = \lambda\left(A, B, C, \alpha, \beta, \gamma\right) =$$

$$= \sin A \sin B \sin C\left(ctgA\sin^2\alpha + ctgB\sin^2\beta + ctgC\sin^2\gamma + 2\sin\alpha\sin\beta\sin\gamma\right)$$

$$= \frac{1}{4R^2}\left(S_A\sin^2\alpha + S_B\sin^2\beta + S_C\sin^2\gamma + 2S\sin\alpha\sin\beta\sin\gamma\right).$$

$$\lambda_0 = \lambda\left(\alpha, \beta, \gamma, A, B, C\right) =$$

$$= \sin\alpha\sin\beta\sin\gamma\left(ctg\alpha\sin^2 A + ctg\beta\sin^2 B + ctg\gamma\sin^2 C + 2\sin A\sin B\sin C\right)$$

$$= \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2}\left(a^2 ctg\alpha + b^2 ctg\beta + c^2 ctg\gamma + 2S\right)$$

$$= \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2 S}\left(a^2 S_\alpha + b^2 S_\beta + c^2 S_\gamma + 2S^2\right).$$

We will proove that $\lambda_0 = \lambda$:

$$\sin\alpha\sin\beta\sin\gamma\left(ctg\alpha\sin^2 A + ctg\beta\sin^2 B + ctg\gamma\sin^2 C + 2\sin A\sin B\sin C\right) =$$

$$= \sin A\sin B\sin C\left(ctgA\sin^2\alpha + ctgB\sin^2\beta + ctgC\sin^2\gamma + 2\sin\alpha\sin\beta\sin\gamma\right) \Leftrightarrow$$

Notes

© 2013 Global Journals Inc (US)

$$\Leftrightarrow \cos\alpha\sin\beta\sin\gamma\sin^2 A+\sin\alpha\cos\beta\sin\gamma\sin^2 B+\sin\alpha\sin\beta\cos\gamma\sin^2 C=$$
$$=\cos A\sin B\sin C\sin^2\alpha+\sin A\cos B\sin C\sin^2\beta+\sin A\sin B\cos C\sin^2\gamma\Leftrightarrow$$
$$\Leftrightarrow\cos\alpha\left(\cos\alpha+\cos\beta\cos\gamma\right)\sin^2 A+\cos\beta\left(\cos\beta+\cos\alpha\cos\gamma\right)\sin^2 B+$$
$$+\cos\gamma\left(\cos\gamma+\cos\alpha\cos\beta\right)\sin^2 C=\cos A\left(\cos A+\cos B\cos C\right)\sin^2\alpha+$$
$$+\cos B\left(\cos B+\cos A\cos C\right)\sin^2\beta+\cos C\left(\cos C+\cos A\cos B\right)\sin^2\gamma\Leftrightarrow$$
$$\Leftrightarrow\sin^2 A\cos^2\alpha+\sin^2 B\cos^2\beta+\sin^2 C\cos^2\gamma+\cos\alpha\cos\beta\cos\gamma\left(\sin^2 A+\sin^2 B+\sin^2 C\right)=$$
$$=\sin^2\alpha\cos^2 A+\sin^2\beta\cos^2 B+\sin^2\gamma\cos^2 C+\cos A\cos B\cos C\left(\sin^2\alpha+\sin^2\beta+\sin^2\gamma\right)\Leftrightarrow$$
$$\Leftrightarrow\cos^2\alpha+\cos^2\beta+\cos^2\gamma-\left(\cos^2 A+\cos^2 B+\cos^2 C\right)+$$
$$+2\cos\alpha\cos\beta\cos\gamma\left(1+\cos A\cos B\cos C\right)-2\cos A\cos B\cos C\left(1+\cos\alpha\cos\beta\cos\gamma\right)=0\Leftrightarrow$$
$\Leftrightarrow 1-2\cos\alpha\cos\beta\cos\gamma-1+2\cos A\cos B\cos C+2\cos\alpha\cos\beta\cos\gamma-2\cos A\cos B\cos C=0$, and this is true.

a) The length of the segments AD, BE and CF

In absolute barycentric coordinates the distance between two points $P=(x,y,z)$ and $P'=(x',y',z')$ is given by $|PP'|=\sqrt{(x-x')^2 S_A+(y-y')^2 S_B+(z-z')^2 S_C}$.

In order to calculate the lenght of the segments *AD, BE* and *CF* we introduce the following notations:

$$t_\alpha=S_\beta+S_\gamma=Sctg\beta+Sctg\gamma=S\left(\frac{\cos\beta}{\sin\beta}+\frac{\cos\gamma}{\sin\gamma}\right)=\frac{S}{\sin\beta\sin\gamma}\sin(\beta+\gamma)=\frac{S\sin\alpha}{\sin\beta\sin\gamma},$$
$$t_\beta=S_\gamma+S_\alpha=\frac{S\sin\beta}{\sin\gamma\sin\alpha},\quad t_\gamma=S_\alpha+S_\beta=\frac{S\sin\gamma}{\sin\alpha\sin\beta}.$$

The sum of the coordinates of the point *D, E* resp. *F* is t_α, t_β resp. t_γ. The length of the segment *AD* is:

$$AD=\frac{1}{t_\alpha}\sqrt{\left(a^2+t_\alpha\right)^2 S_A+\left(S_\gamma+S_C\right)^2 S_B+\left(S_\beta+S_B\right)^2 S_C}$$
$$=\frac{1}{t_\alpha}\sqrt{\left(a^2+2t_\alpha\right)\left(a^2S_A+S_{BC}\right)+S_A t_\alpha^2+S_B S_\gamma^2+S_C S_\beta^2}$$
$$=\frac{1}{t_\alpha}\sqrt{\left(a^2+2t_\alpha\right)S^2+S_A t_\alpha^2+S_B S_\gamma^2+S_C S_\beta^2}$$
$$=\frac{1}{t_\alpha}\sqrt{\left(a^2+2t_\alpha\right)S^2+S_A S^2\frac{\sin^2\alpha}{\sin^2\beta\sin^2\gamma}+S_B S^2\frac{\cos^2\gamma}{\sin^2\gamma}+S_C S^2\frac{\cos^2\beta}{\sin^2\beta}}$$
$$=\frac{1}{\sin\alpha}\sqrt{a^2\sin^2\beta\sin^2\gamma+2S\sin\alpha\sin\beta\sin\gamma+S_A\sin^2\alpha+S_B\sin^2\beta\cos^2\gamma+S_C\cos^2\beta\sin^2\gamma}$$
$$=\frac{1}{\sin\alpha}\sqrt{S_A\sin^2\alpha+S_B\sin^2\beta+S_C\sin^2\gamma+2S\sin\alpha\sin\beta\sin\gamma}$$

© 2013 Global Journals Inc. (US)

$$= \frac{2R\sqrt{\lambda}}{\sin\alpha} = \frac{2R\sqrt{\lambda_0}}{\sin\alpha} = \frac{\sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma + 2S\right)}}{\sin\alpha}.$$

For this reason:

$$AD = \sqrt{\frac{\sin\beta\sin\gamma}{\sin\alpha}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma + 2S\right)}, \quad (3)$$

$$BE = \sqrt{\frac{\sin\gamma\sin\alpha}{\sin\beta}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma + 2S\right)}, \quad (4)$$

$$CF = \sqrt{\frac{\sin\alpha\sin\beta}{\sin\gamma}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma + 2S\right)}. \quad (5)$$

Based on the formulas (3), (4) and (5):

$$AD\cdot\sin\alpha = BE\cdot\sin\beta = CF\cdot\sin\gamma = \sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma + 2S\right)}. \quad (6)$$

b) The distance of the first generalized Fermat point from the vertices of the reference triangle

In what follows we calculate the lengts of the segments AK, BK and CK. The sum of coordinates of the point K is:

$$\Phi = \left(S_\beta + S_B\right)\left(S_\gamma + S_C\right) + \left(S_\gamma + S_C\right)\left(S_\alpha + S_A\right) + \left(S_\alpha + S_A\right)\left(S_\beta + S_B\right)$$
$$= S_{BC} + S_{CA} + S_{AB} + S_{\beta\gamma} + S_{\gamma\alpha} + S_{\alpha\beta} + \left(S_B + S_C\right)S_\alpha + \left(S_C + S_A\right)S_\beta + \left(S_A + S_B\right)S_\gamma$$
$$= a^2S_\alpha + b^2S_\beta + c^2S_\gamma + 2S^2.$$

Therefore

$$\lambda = \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma + 2S^2\right) = \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\Phi.$$

The length of the segment AK is:

$$AK = \frac{1}{|\Phi|}\sqrt{\left[\Phi - \left(S_\beta + S_B\right)\left(S_\gamma + S_C\right)\right]^2 S_A + \left(S_\gamma + S_C\right)^2\left(S_\alpha + S_A\right)^2 S_B + \left(S_\alpha + S_A\right)^2\left(S_\beta + S_B\right)^2 S_C}$$
$$= \frac{|S_\alpha + S_A|}{|\Phi|}\sqrt{\left(S_B + S_C + S_\beta + S_\gamma\right)^2 S_A + \left(S_\gamma + S_C\right)^2 S_B + \left(S_\beta + S_B\right)^2 S_C}$$
$$= \frac{|S_\alpha + S_A|}{|\Phi|}\sqrt{\left(a^2 + t_\alpha\right)^2 S_A + \left(S_\gamma + S_C\right)^2 S_B + \left(S_\beta + S_B\right)^2 S_C}$$

$$= \frac{|S_\alpha + S_A|}{|\Phi|}AD\cdot t_\alpha = \frac{|S_\alpha + S_A|}{|\Phi|}\frac{2R\cdot t_\alpha\sqrt{\lambda}}{\sin\alpha} = \frac{\sin\alpha}{2R\sqrt{\lambda}}|S_\alpha + S_A| = \frac{S}{2R\sqrt{\lambda}\sin A}\left|\sin\left(A+\alpha\right)\right|$$

$$= \frac{S}{a\sqrt{\lambda}}\left|\sin\left(A+\alpha\right)\right| = \frac{bc}{2R\sqrt{\lambda}}\left|\sin\left(A+\alpha\right)\right|.$$

Therefore:

$$AK = \frac{bc}{2R\sqrt{\lambda}}\left|\sin\left(A+\alpha\right)\right|, \quad (7) \qquad BK = \frac{ca}{2R\sqrt{\lambda}}\left|\sin\left(B+\beta\right)\right|, \quad (8)$$

Notes

© 2013 Global Journals Inc (US)

$$CK = \frac{ab}{2R\sqrt{\lambda}}\left|\sin(C+\gamma)\right|. \qquad (9)$$

c) Relations referring to the segments AK, BK and CK

Now we will demonstrate the following conditional identity:

$$bc\sin\alpha\sin(A+\alpha)+ca\sin\beta\sin(B+\beta)+ab\sin\gamma\sin(C+\gamma)=4R^2\lambda\,. \qquad (10)$$

Notes

Indeed

$$bc\sin\alpha\sin(A+\alpha)+ca\sin\beta\sin(B+\beta)+ab\sin\gamma\sin(C+\gamma)=$$
$$=4R^2\sin B\sin C\sin\alpha(\sin A\cos\alpha+\sin\alpha\cos A)+4R^2\sin C\sin A\sin\beta(\sin B\cos\beta+\sin\beta\cos B)+$$
$$+4R^2\sin A\sin B\sin\gamma(\sin C\cos\gamma+\sin\gamma\cos C)=$$
$$=4R^2\sin A\sin B\sin C\left[ctgA\sin^2\alpha+ctgB\sin^2\beta+ctgC\sin^2\gamma+(\sin 2\alpha+\sin 2\beta+\sin 2\gamma)/2\right]=$$
$$=4R^2\sin A\sin B\sin C\left(ctgA\sin^2\alpha+ctgB\sin^2\beta+ctgC\sin^2\gamma+2\sin\alpha\sin\beta\sin\gamma\right)=4R^2\lambda.$$

Since $A+\alpha+B+\beta+C+\gamma=2\pi$, from inequalities $A+\alpha>\pi$, $B+\beta>\pi$, $C+\gamma>\pi$ only one can be true. If $A+\alpha<\pi$, $B+\beta<\pi$, $C+\gamma<\pi$, then the K is an inside point of the triangle ABC and based on the (10) we have

$$AD\sin\alpha=BE\sin\beta=CF\sin\gamma=2R\sqrt{\lambda}=AK\sin\alpha+BK\sin\beta+CK\sin\gamma\,. \qquad (11)$$

If for instance $A+\alpha>\pi$, then

$$AD\sin\alpha=BE\sin\beta=CF\sin\gamma=2R\sqrt{\lambda}=-AK\sin\alpha+BK\sin\beta+CK\sin\gamma\,. \qquad (12)$$

III. The Second Generalized Fermat Point of the Triangle

We build on the sides of the triangle ABC inwards the similar triangles BCL, CAM and ABN in that way, that

$BAN\angle=\alpha=CAM\angle$, $CBL\angle=\beta=ABN\angle$, $ACM\angle=\gamma=BCL\angle$ (Figure 4). Since

$A+B+C=\pi=\alpha+\beta+\gamma$, therefore $BLC\angle=\alpha$, $CMA\angle=\beta$, $ANB\angle=\gamma$.

According to the Conway's formulas, the barycentric coordinates of the points L, M, N are:

$$L=\left(a^2:S_\gamma-S_C:S_\beta-S_B\right),\ M=\left(S_\gamma-S_C:b^2:S_\alpha-S_A\right),\ N=\left(S_\beta-S_B:S_\alpha-S_A:c^2\right).$$

The equations of the lines AL, BM, CN are: $-\left(S_\beta-S_B\right)y+\left(S_\gamma-S_C\right)z=0$,

$$\left(S_\alpha-S_A\right)x-\left(S_\gamma-S_C\right)z=0,$$
$$-\left(S_\alpha-S_A\right)x+\left(S_\beta-S_B\right)y=0.$$

Since adding whatever two equation, we get the third equation, the lines AL, BM and CN meets each other in a point $T=AL\cap BM\cap CN$. This point T will be called **the second generalized Fermat point of the triangle**.

The barycentric coordinates of the point T are:

$$T=\left(\left(S_\beta-S_B\right)\left(S_\gamma-S_C\right):\left(S_\gamma-S_C\right)\left(S_\alpha-S_A\right):\left(S_\alpha-S_A\right)\left(S_\beta-S_B\right)\right)$$
$$=\left(\frac{1}{S_\alpha-S_A}:\frac{1}{S_\beta-S_B}:\frac{1}{S_\gamma-S_C}\right).$$

© 2013 Global Journals Inc. (US)

We introduce the following notations:

$$\mu = \mu(A,B,C,\alpha,\beta,\gamma) =$$
$$= \sin A \sin B \sin C\left(ctgA\sin^2\alpha + ctgB\sin^2\beta + ctgC\sin^2\gamma - 2\sin\alpha\sin\beta\sin\gamma\right)$$
$$= \frac{1}{4R^2}\left(S_A\sin^2\alpha + S_B\sin^2\beta + S_C\sin^2\gamma - 2S\sin\alpha\sin\beta\sin\gamma\right).$$
$$\mu_0 = \mu(\alpha,\beta,\gamma,A,B,C) =$$
$$= \sin\alpha\sin\beta\sin\gamma\left(ctg\alpha\sin^2 A + ctg\beta\sin^2 B + ctg\gamma\sin^2 C - 2\sin A\sin B\sin C\right)$$
$$= \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)$$
$$= \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma - 2S^2\right).$$

In this case the equality $\mu_0 = \mu$ is valid too.

a) The length of the segments AL, BM and CN

The sum of the coordinates of the point L, M resp. N are t_α, t_β resp. t_γ. The length of the segment AL is:

$$AL = \frac{1}{t_\alpha}\sqrt{\left(a^2 - t_\alpha\right)^2 S_A + \left(S_\gamma + S_C\right)^2 S_B + \left(S_\beta + S_B\right)^2 S_C}$$
$$= \frac{1}{t_\alpha}\sqrt{\left(a^2 - 2t_\alpha\right)\left(a^2S_A + S_{BC}\right) + S_At_\alpha^2 + S_BS_\gamma^2 + S_CS_\beta^2}$$
$$= \frac{1}{t_\alpha}\sqrt{\left(a^2 - 2t_\alpha\right)S^2 + S_At_\alpha^2 + S_BS_\gamma^2 + S_CS_\beta^2}$$
$$= \frac{1}{t_\alpha}\sqrt{\left(a^2 - 2t_\alpha\right)S^2 + S_AS^2\frac{\sin^2\alpha}{\sin^2\beta\sin^2\gamma} + S_BS^2\frac{\cos^2\gamma}{\sin^2\gamma} + S_CS^2\frac{\cos^2\beta}{\sin^2\beta}}$$
$$= \frac{1}{\sin\alpha}\sqrt{a^2\sin^2\beta\sin^2\gamma - 2S\sin\alpha\sin\beta\sin\gamma + S_A\sin^2\alpha + S_B\sin^2\beta\cos^2\gamma + S_C\cos^2\beta\sin^2\gamma}$$
$$= \frac{1}{\sin\alpha}\sqrt{S_A\sin^2\alpha + S_B\sin^2\beta + S_C\sin^2\gamma - 2S\sin\alpha\sin\beta\sin\gamma}$$
$$= \frac{2R\sqrt{\mu}}{\sin\alpha} = \frac{2R\sqrt{\mu_0}}{\sin\alpha} = \frac{\sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}}{\sin\alpha}.$$

For this reason:

$$AL = \sqrt{\frac{\sin\beta\sin\gamma}{\sin\alpha}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}, \qquad (13)$$

$$BM = \sqrt{\frac{\sin\gamma\sin\alpha}{\sin\beta}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}, \qquad (14)$$

$$CN = \sqrt{\frac{\sin\alpha\sin\beta}{\sin\gamma}\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}. \qquad (15)$$

© 2013 Global Journals Inc (US)

Based on the (13), (14) and (15) we obtain:

$$AL\cdot\sin\alpha = BM\cdot\sin\beta = CN\cdot\sin\gamma = \sqrt{\sin\alpha\sin\beta\sin\gamma\left(a^2ctg\alpha + b^2ctg\beta + c^2ctg\gamma - 2S\right)}. \quad (16)$$

b) The distance of the second generalized Fermat point from the vertices of the reference triangle

The sum of coordinates of the point T is:

$$\Gamma = \left(S_\beta - S_B\right)\left(S_\gamma - S_C\right) + \left(S_\gamma - S_C\right)\left(S_\alpha - S_A\right) + \left(S_\alpha - S_A\right)\left(S_\beta - S_B\right)$$
$$= S_{BC} + S_{CA} + S_{AB} + S_{\beta\gamma} + S_{\gamma\alpha} + S_{\alpha\beta} - \left(S_B + S_C\right)S_\alpha - \left(S_C + S_A\right)S_\beta - \left(S_A + S_B\right)S_\gamma$$
$$= -a^2S_\alpha - b^2S_\beta - c^2S_\gamma + 2S^2 = -\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma - 2S^2\right).$$

Therefore:

$$\mu = \frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\left(a^2S_\alpha + b^2S_\beta + c^2S_\gamma - 2S^2\right) = -\frac{\sin\alpha\sin\beta\sin\gamma}{4R^2S}\Gamma.$$

The length of the segment AT is:

$$AT = \frac{1}{|\Gamma|}\sqrt{\left[\Gamma - \left(S_\beta - S_B\right)\left(S_\gamma - S_C\right)\right]^2 S_A + \left(S_\gamma - S_C\right)^2\left(S_\alpha - S_A\right)^2 S_B + \left(S_\alpha - S_A\right)^2\left(S_\beta - S_B\right)^2 S_C}$$
$$= \frac{|S_\alpha - S_A|}{|\Gamma|}\sqrt{\left(S_\beta + S_\gamma - S_B - S_C\right)^2 S_A + \left(S_\gamma - S_C\right)^2 S_B + \left(S_\beta - S_B\right)^2 S_C}$$
$$= \frac{|S_\alpha - S_A|}{|\Gamma|}\sqrt{\left(-a^2 + t_\alpha\right)^2 S_A + \left(S_\gamma - S_C\right)^2 S_B + \left(S_\beta - S_B\right)^2 S_C}$$
$$= \frac{|S_\alpha - S_A|}{|\Gamma|}AL\cdot t_\alpha = \frac{|S_\alpha - S_A|}{|\Gamma|}\frac{2R\cdot t_\alpha\sqrt{\mu}}{\sin\alpha} = \frac{\sin\alpha}{2R\sqrt{\mu}}|S_\alpha - S_A| = \frac{S}{2R\sqrt{\mu}\sin A}\left|\sin\left(A-\alpha\right)\right|$$
$$= \frac{S}{a\sqrt{\mu}}\left|\sin\left(A-\alpha\right)\right| = \frac{bc}{2R\sqrt{\mu}}\left|\sin\left(A-\alpha\right)\right| = \frac{bc}{2R\sqrt{\mu}}\sin|A-\alpha|.$$

Therefore:

$$AT = \frac{bc}{2R\sqrt{\mu}}\sin|A-\alpha|, \quad (17) \qquad BT = \frac{ca}{2R\sqrt{\mu}}\sin|B-\beta|, \quad (18)$$

$$CT = \frac{ab}{2R\sqrt{\mu}}\sin|C-\gamma|. \quad (19)$$

c) Relations referring to the segments AT, BT and CT

Here is valid too the following conditional identity:

$$bc\sin\alpha\sin(A-\alpha) + ca\sin\beta\sin(B-\beta) + ab\sin\gamma\sin(C-\gamma) = -4R^2\mu. \quad (20)$$

Since $A-\alpha+B-\beta+C-\gamma=0$, only two of the $A>\alpha$, $B>\beta$, $C>\gamma$ inequalities

© 2013 Global Journals Inc. (US)

can be true at the same time. For instance if A $<\alpha$, B $>\beta$, C $>\gamma$, then based on the formula (20)

$$AL\sin\alpha = BM\sin\beta = CN\sin\gamma = 2R\sqrt{\mu} = AT\sin\alpha - BT\sin\beta - CT\sin\gamma. \qquad (21)$$

Or if A $<\alpha$, B $<\beta$, C $>\gamma$, then

$$AL\sin\alpha = BM\sin\beta = CN\sin\gamma = 2R\sqrt{\mu} = AT\sin\alpha + BT\sin\beta - CT\sin\gamma. \qquad (22)$$

IV. Speciale Case

In this section we use the following relations:

$$abc(ctgA+ctgB+ctgC) = R\left(a^2+b^2+c^2\right), \qquad (23)$$

$$\sin A\sin B\sin C\left(ctgA+ctgB+ctgC+\sqrt{3}\right) = \\ = 1+\sqrt{3}\sin A\sin B\sin C+\cos A\cos B\cos C, \qquad (24)$$

$$\sin A\sin B\sin C\left(ctgA+ctgB+ctgC-\sqrt{3}\right) = \\ = 1-\sqrt{3}\sin A\sin B\sin C+\cos A\cos B\cos C. \qquad (25)$$

The proof of (23):

$$abc(ctgA+ctgB+ctgC) = abc\left(\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}+\frac{\cos C}{\sin C}\right) = 2Rabc\left(\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}\right) = \\ = R\left(2bc\cos A+2ca\cos B+2ab\cos C\right) = R\left(b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2\right) = \\ = R\left(a^2+b^2+c^2\right).$$

The proof of (24) and (25):

$$\sin A\sin B\sin C\left(ctgA+ctgB+ctgC\pm\sqrt{3}\right) = \sin A\sin B\sin C\left(\frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}+\frac{\cos C}{\sin C}\pm\sqrt{3}\right) = \\ = \cos A\sin B\sin C+\cos B\sin C\sin A+\cos C\sin A\sin B\pm\sqrt{3}\sin A\sin B\sin C = \\ = 1\pm\sqrt{3}\sin A\sin B\sin C+\cos A\cos B\cos C.$$

If $\alpha=\beta=\gamma=\pi/3$, then $K\equiv F_1$ and $T\equiv F_2$, where F_1 resp. F_2 is the first resp. the second Fermat's point of the triangle ABC (X_{13} resp. X_{14} in [1]). In this case

$$\lambda^* = \lambda\left(A,B,C,\frac{\pi}{3},\frac{\pi}{3},\frac{\pi}{3}\right) = \frac{3}{4}\sin A\sin B\sin C\left(ctgA+ctgB+ctgC+\sqrt{3}\right) = \\ = \frac{3}{32R^2}\left(a^2+b^2+c^2+2\sqrt{3}S\right) = \frac{3}{4}\left(1+\sqrt{3}\sin A\sin B\sin C+\cos A\cos B\cos C\right),$$

$$\mu^* = \mu\left(A,B,C,\frac{\pi}{3},\frac{\pi}{3},\frac{\pi}{3}\right) = \frac{3}{4}\sin A\sin B\sin C\left(ctgA+ctgB+ctgC-\sqrt{3}\right) = \\ = \frac{3}{32R^2}\left(a^2+b^2+c^2-2\sqrt{3}S\right) = \frac{3}{4}\left(1-\sqrt{3}\sin A\sin B\sin C+\cos A\cos B\cos C\right).$$

© 2013 Global Journals Inc (US)

Based on (6) we have

$$AX_1 = BY_1 = CZ_1 = \frac{4R}{\sqrt{3}}\sqrt{\lambda^*} = \frac{\sqrt{a^2+b^2+c^2+2\sqrt{3}S}}{\sqrt{2}}. \quad (26)$$

Notes

According to (7), (8) and (9)

$$AF_1 = \frac{bc}{2R\sqrt{\lambda^*}}\left|\sin\left(A+\frac{\pi}{3}\right)\right|, \quad (27) \qquad BF_1 = \frac{ca}{2R\sqrt{\lambda^*}}\left|\sin\left(B+\frac{\pi}{3}\right)\right|, \quad (28)$$

$$CF_1 = \frac{ab}{2R\sqrt{\lambda^*}}\left|\sin\left(C+\frac{\pi}{3}\right)\right|. \quad (29)$$

If $A<2\pi/3$, $B<2\pi/3$, $C<2\pi/3$, then

$$AX_1 = BY_1 = CZ_1 = \frac{4R}{\sqrt{3}}\sqrt{\lambda^*} = AF_1 + BF_1 + CF_1. \quad (30)$$

If for instance $A>2\pi/3$, then

$$AX_1 = BY_1 = CZ_1 = \frac{4R}{\sqrt{3}}\sqrt{\lambda^*} = -AF_1 + BF_1 + CF_1. \quad (31)$$

Based on (16) we have

$$AX_2 = BY_2 = CZ_2 = \frac{4R}{\sqrt{3}}\sqrt{\mu^*} = \frac{\sqrt{a^2+b^2+c^2-2\sqrt{3}S}}{\sqrt{2}}. \quad (32)$$

According to (17), (18) and (19)

$$AF_2 = \frac{bc}{2R\sqrt{\mu^*}}\sin\left|A-\frac{\pi}{3}\right|, \quad (33) \qquad BF_2 = \frac{ca}{2R\sqrt{\mu^*}}\sin\left|B-\frac{\pi}{3}\right|, \quad (34)$$

$$CF_2 = \frac{ab}{2R\sqrt{\mu^*}}\sin\left|C-\frac{\pi}{3}\right|. \quad (35)$$

If for instance $A<\pi/3$, $B>\pi/3$, $C>\pi/3$, then

$$AX_2 = BY_2 = CZ_2 = \frac{4R}{\sqrt{3}}\sqrt{\mu^*} = AF_2 - BF_2 - CF_2. \quad (36)$$

Or if A $<\pi/3$, B $<\pi/3$, C $>\pi/3$, then

$$AX_2 = BY_2 = CZ_2 = \frac{4R}{\sqrt{3}}\sqrt{\mu^*} = AF_2 + BF_2 - CF_2. \quad (37)$$

© 2013 Global Journals Inc. (US)

Other arrangements of the triangles *BCD, CAE* and *ABF* are possible. To investigate the valabilities of the above metric relations in this cases is a possible subject for further researches.

References Références Referencias

[1] C. Kimberling, Triangle Centers and Central Triangles, *Congr. Numer.* **129**, 1-295, 1998.

[2] H. Mowaffaq, An Advanced Calculus Approach to Finding the Fermat Point, *Math. Mag.* **67**, 29-34, 1994.

[3] Shay Gueron and Ran Tessler, The Fermat-Steiner Problem, *Monthly* **109**, 443-451, 2002

[4] P. G. Spain, The Fermat Point of a Triangle, *Math. Mag.* **69**, 131-133, 1996.

[5] J. Tong and Y. S. Chua, The Generalized Fermat's Point, *Math. Mag.* **68**, 214-215, 1995.

The Metric Characterization of the Generalized Fermat Points

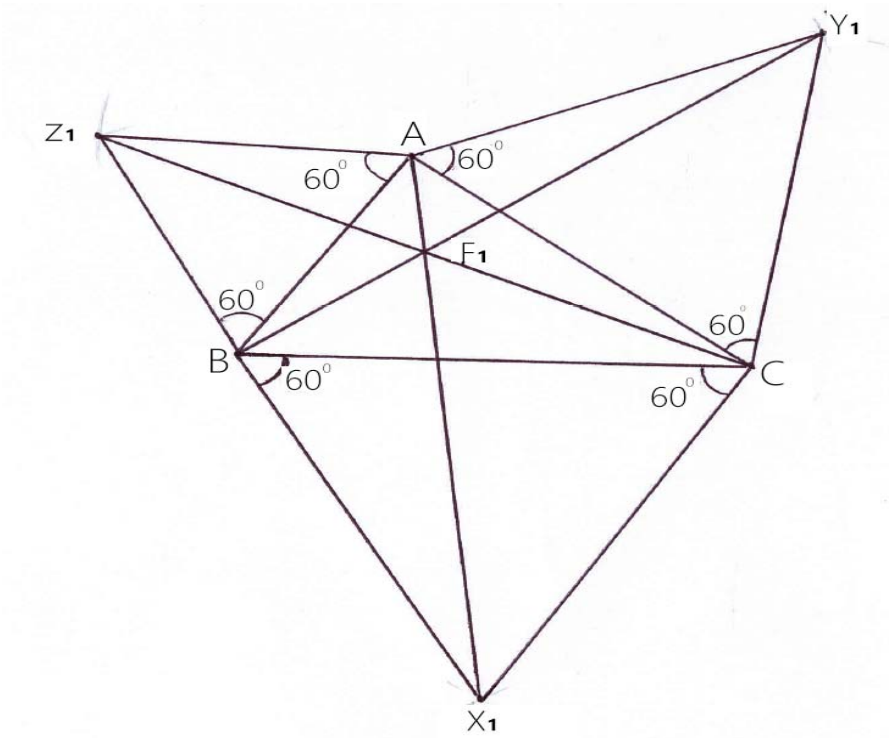


Figure 1

Notes

© 2013 Global Journals Inc (US)

Notes

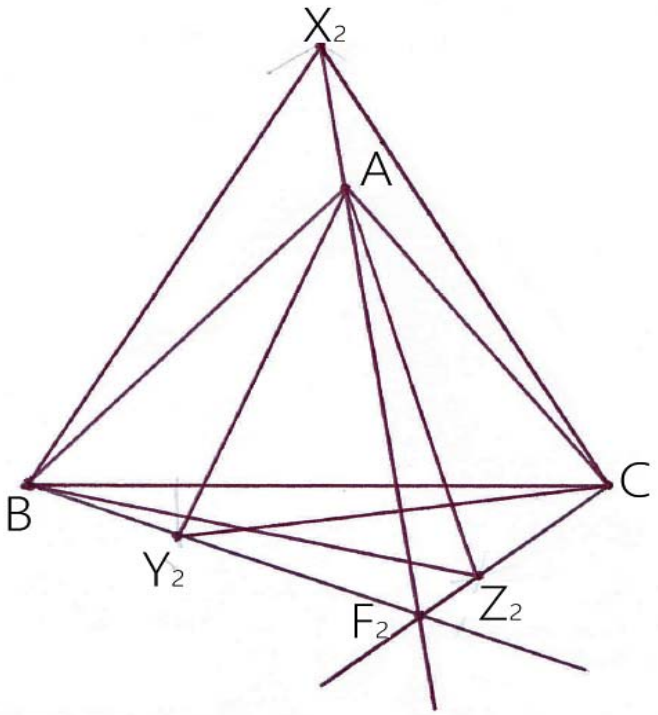


Figure 2

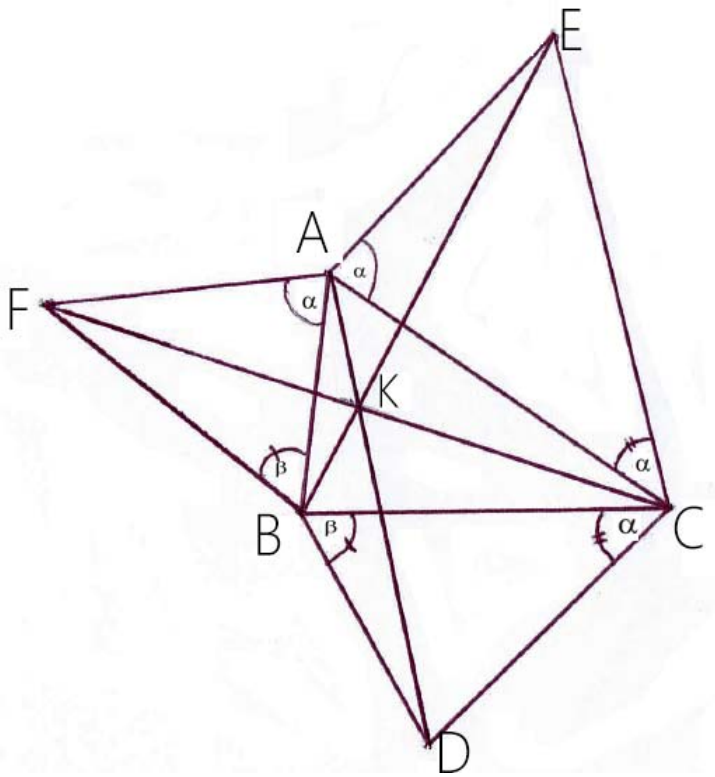


Figure 3

© 2013 Global Journals Inc. (US)

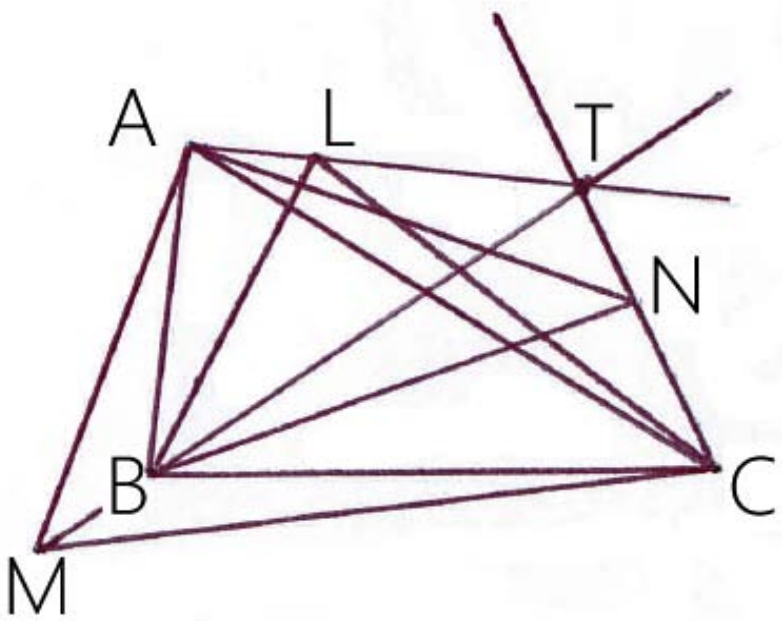


Figure 4

Notes

© 2013 Global Journals Inc (US)

Global Journals Inc. (US) Guidelines Handbook 2013

www.GlobalJournals.org

FELLOWS

FELLOW OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (FARSS)

Global Journals Incorporate (USA) is accredited by Open Association of Research Society (OARS), U.S.A and in turn, awards "FARSS" title to individuals. The 'FARSS' title is accorded to a selected professional after the approval of the Editor-in-Chief/Editorial Board Members/Dean.



- The "FARSS" is a dignified title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FARSS or William Walldroff, M.S., FARSS.

FARSS accrediting is an honor. It authenticates your research activities. After recognition as FARSB, you can add 'FARSS' title with your name as you use this recognition as additional suffix to your status. This will definitely enhance and add more value and repute to your name. You may use it on your professional Counseling Materials such as CV, Resume, and Visiting Card etc.

The following benefits can be availed by you only for next three years from the date of certification:

FARSS designated members are entitled to avail a 40% discount while publishing their research papers (of a single author) with Global Journals Incorporation (USA), if the same is accepted by Editorial Board/Peer Reviewers. If you are a main author or co-author in case of multiple authors, you will be entitled to avail discount of 10%.

Once FARSB title is accorded, the Fellow is authorized to organize a symposium/seminar/conference on behalf of Global Journal Incorporation (USA). The Fellow can also participate in conference/seminar/symposium organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent.





You may join as member of the Editorial Board of Global Journals Incorporation (USA) after successful completion of three years as Fellow and as Peer Reviewer. In addition, it is also desirable that you should organize seminar/symposium/conference at least once.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.



© Copyright by Global Journals Inc.(US) | Guidelines Handbook

The FARSS can go through standards of OARS. You can also play vital role if you have any suggestions so that proper amendment can take place to improve the same for the benefit of entire research community.



As FARSS, you will be given a renowned, secure and free professional email address with 100 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.



The FARSS will be eligible for a free application of standardization of their researches. Standardization of research will be subject to acceptability within stipulated norms as the next step after publishing in a journal. We shall depute a team of specialized research professionals who will render their services for elevating your researches to next higher level, which is worldwide open standardization.



The FARSS member can apply for grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.
Once you are designated as FARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria. After certification of all your credentials by OARS, they will be published on your Fellow Profile link on website https://associationofresearch.org which will be helpful to upgrade the dignity.

The FARSS members can avail the benefits of free research podcasting in Global Research Radio with their research documents. After publishing the work, (including published elsewhere worldwide with proper authorization) you can upload your research paper with your recorded voice or you can utilize chargeable services of our professional RJs to record your paper in their voice on request.



The FARSS member also entitled to get the benefits of free research podcasting of their research documents through video clips. We can also streamline your conference videos and display your slides/ online slides and online research video clips at reasonable charges, on request.

© Copyright by Global Journals Inc.(US)| Guidelines Handbook



The FARSS is eligible to earn from sales proceeds of his/her researches/reference/review Books or literature, while publishing with Global Journals. The FARSS can decide whether he/she would like to publish his/her research in a closed manner. In this case, whenever readers purchase that individual research paper for reading, maximum 60% of its profit earned as royalty by Global Journals, will be credited to his/her bank account. The entire entitled amount will be credited to his/her bank account exceeding limit of minimum fixed balance. There is no minimum time limit for collection. The FARSS member can decide its price and we can help in making the right decision.

The FARSS member is eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get remuneration of 15% of author fees, taken from the author of a respective paper. After reviewing 5 or more papers you can request to transfer the amount to your bank account.



MEMBER OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (MARSS)

The ' MARSS ' title is accorded to a selected professional after the approval of the Editor-in-Chief / Editorial Board Members/Dean.



The "MARSS" is a dignified ornament which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., MARSS or William Walldroff, M.S., MARSS.

MARSS accrediting is an honor. It authenticates your research activities. After becoming MARSS, you can add 'MARSS' title with your name as you use this recognition as additional suffix to your status. This will definitely enhance and add more value and repute to your name. You may use it on your professional Counseling Materials such as CV, Resume, Visiting Card and Name Plate etc.

The following benefitscan be availed by you only for next three years from the date of certification.



MARSS designated members are entitled to avail a 25% discount while publishing their research papers (of a single author) in Global Journals Inc., if the same is accepted by our Editorial Board and Peer Reviewers. If you are a main author or co-author of a group of authors, you will get discount of 10%.

As MARSS, you will be given a renowned, secure and free professional email address with 30 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.



© Copyright by Global Journals Inc.(US) | Guidelines Handbook



We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The MARSS member can apply for approval, grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.





Once you are designated as MARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria.

It is mandatory to read all terms and conditions carefully.

© Copyright by Global Journals Inc.(US)| Guidelines Handbook

AUXILIARY MEMBERSHIPS

Institutional Fellow of Global Journals Incorporation (USA)-OARS (USA)

Global Journals Incorporation (USA) is accredited by Open Association of Research Society, U.S.A (OARS) and in turn, affiliates research institutions as "Institutional Fellow of Open Association of Research Society" (IFOARS).



The "FARSC" is a dignified title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as "Institutional Board of Open Association of Research Society"-(IBOARS).

The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.

The author fees of such paper may be waived off up to 40%.

The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.





The IBOARS can organize symposium/seminar/conference in their country on behalf of Global Journals Incorporation (USA)-OARS (USA). The terms and conditions can be discussed separately.

The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of "Open Association of Research Society, U.S.A (OARS)" so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.



The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

© Copyright by Global Journals Inc.(US) | Guidelines Handbook

We shall provide you intimation regarding launching of e-version of journal of your stream time to time. This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.



After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:



Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

Open Association of Research Society (US)/ Global Journals Incorporation (USA), as described in Corporate Statements, are educational, research publishing and professional membership organizations. Achieving our individual Fellow or Associate status is based mainly on meeting stated educational research requirements.



Disbursement of 40% Royalty earned through Global Journals : Researcher = 50%, Peer Reviewer = 37.50%, Institution = 12.50% E.g. Out of 40%, the 20% benefit should be passed on to researcher, 15 % benefit towards remuneration should be given to a reviewer and remaining 5% is to be retained by the institution.

We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

Other:

The individual Fellow and Associate designations accredited by Open Association of Research Society (US) credentials signify guarantees following achievements:

- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

© Copyright by Global Journals Inc.(US)| Guidelines Handbook

- In addition to above, if one is single author, then entitled to 40% discount on publishing research paper and can get 10%discount if one is co-author or main author among group of authors.
- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
- The Fellow can become member of Editorial Board Member after completing 3yrs.
- The Fellow can earn 60% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.
- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) First, register yourself using top right corner of Home page then Login. If you are already registered, then login using your username and password.

(II) Choose corresponding Journal.

(III) Click 'Submit Manuscript'. Fill required information and Upload the paper.

(B) If you are using Internet Explorer, then Direct Submission through Homepage is also available.

(C) If these two are not conveninet , and then email the paper directly to dean@globaljournals.org.

Offline Submission: Author can send the typed form of paper by Post. However, online submission should be preferred.



© Copyright by Global Journals Inc.(US)| Guidelines Handbook

Preferred Author Guidelines

MANUSCRIPT STYLE INSTRUCTION (Must be strictly followed)

Page Size: 8.27" X 11'"

- Left Margin: 0.65
- Right Margin: 0.65
- Top Margin: 0.75
- Bottom Margin: 0.75
- Font type of all text should be Swis 721 Lt BT.
- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, "Abstract" word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
- Two Column with Equal Column with of 3.38 and Gaping of .2
- First Character must be three lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

Journals Inc. (US) are being abstracted and indexed (in process) by most of the reputed organizations. Topics of only narrow interest will not be accepted unless they have wider potential or consequences.

2. ETHICAL GUIDELINES

Authors should follow the ethical guidelines as mentioned below for publication of research paper and research activities.

Papers are accepted on strict understanding that the material in whole or in part has not been, nor is being, considered for publication elsewhere. If the paper once accepted by Global Journals Inc. (US) and Editorial Board, will become the copyright of the Global Journals Inc. (US).

Authorship: The authors and coauthors should have active contribution to conception design, analysis and interpretation of findings. They should critically review the contents and drafting of the paper. All should approve the final version of the paper before submission

The Global Journals Inc. (US) follows the definition of authorship set up by the Global Academy of Research and Development. According to the Global Academy of R&D authorship, criteria must be based on:

1) Substantial contributions to conception and acquisition of data, analysis and interpretation of the findings.

2) Drafting the paper and revising it critically regarding important academic content.

3) Final approval of the version of the paper to be published.

All authors should have been credited according to their appropriate contribution in research activity and preparing paper. Contributors who do not match the criteria as authors may be mentioned under Acknowledgement.

Acknowledgements: Contributors to the research other than authors credited should be mentioned under acknowledgement. The specifications of the source of funding for the research if appropriate can be included. Suppliers of resources may be mentioned along with address.

Appeal of Decision: The Editorial Board's decision on publication of the paper is final and cannot be appealed elsewhere.

Permissions: It is the author's responsibility to have prior permission if all or parts of earlier published illustrations are used in this paper.

Please mention proper reference and appropriate acknowledgements wherever expected.

If all or parts of previously published illustrations are used, permission must be taken from the copyright holder concerned. It is the author's responsibility to take these in writing.

Approval for reproduction/modification of any information (including figures and tables) published elsewhere must be obtained by the authors/copyright holders before submission of the manuscript. Contributors (Authors) are responsible for any copyright fee involved.

3. SUBMISSION OF MANUSCRIPTS

Manuscripts should be uploaded via this online submission page. The online submission is most efficient method for submission of papers, as it enables rapid distribution of manuscripts and consequently speeds up the review procedure. It also enables authors to know the status of their own manuscripts by emailing us. Complete instructions for submitting a paper is available below.

Manuscript submission is a systematic procedure and little preparation is required beyond having all parts of your manuscript in a given format and a computer with an Internet connection and a Web browser. Full help and instructions are provided on-screen. As an author, you will be prompted for login and manuscript details as Field of Paper and then to upload your manuscript file(s) according to the instructions.



To avoid postal delays, all transaction is preferred by e-mail. A finished manuscript submission is confirmed by e-mail immediately and your paper enters the editorial process with no postal delays. When a conclusion is made about the publication of your paper by our Editorial Board, revisions can be submitted online with the same procedure, with an occasion to view and respond to all comments.

Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

5.STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and to make suggestions to improve briefness.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

Structure

All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

Color Charges: It is the rule of the Global Journals Inc. (US) for authors to pay the full cost for the reproduction of their color artwork. Hence, please note that, if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a color work agreement form before your paper can be published.

Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

6.1 Proof Corrections

The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

6.2 Early View of Global Journals Inc. (US) (Publication Prior to Print)

The Global Journals Inc. (US) are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

6.3 Author Services

Online production tracking is available for your article through Author Services. Author Services enables authors to track their article - once it has been accepted - through the production process to publication online and in print. Authors can check the status of their articles online and choose to receive automated e-mails at key stages of production. The authors will receive an e-mail with a unique link that enables them to register and have their article automatically added to the system. Please ensure that a complete e-mail address is provided when submitting the manuscript.

6.4 Author Material Archive Policy

Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

6.5 Offprint and Extra Copies

A PDF offprint of the online-published article will be provided free of charge to the related author, and may be distributed according to the Publisher's terms and conditions. Additional paper offprint may be ordered by emailing us at: editor@globaljournals.org .



Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.



© Copyright by Global Journals Inc.(US)| Guidelines Handbook

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.



© Copyright by Global Journals Inc.(US)| Guidelines Handbook

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.



© Copyright by Global Journals Inc.(US)| Guidelines Handbook

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.



© Copyright by Global Journals Inc.(US)| Guidelines Handbook

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals Inc. (US).

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

Index

A

Acoustical · 28
Atanassov · 32, 39

B

Barycentric · 55, 57, 60

C

Cardinality · 34
Chebyshev · 42, 44, 46, 47, 52
Cliff · 36, 37, 38
Combinatorial · 19
Conjectures · 48

G

Gegenbauer · 42, 44, 47, 52
Gegenbaur · 40

H

Hurwitz · 41
Hypergeometric · 21, 22, 40

M

Membranophone · 28

N

Nontrivial · 4
Notwithstanding · 17

P

Pochhammer · 23
Prentice · 30
Putative · 17

R

Recursion · 2, 3, 4, 5, 6, 8, 9, 11, 13, 15, 17, 19, 20

S

Steiner · 54, 68

W

Whittaker · 23, 24, 30, 43



Global Journal of Science Frontier Research

Visit us on the Web at www.GlobalJournals.org | www.JournalofScience.org
or email us at helpdesk@globaljournals.org

ISSN 9755896



© Global Journals