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A Note on Identifying Critical Activities in Project Scheduling Via Linear Programming on Spreadsheets, with Incidental Pedagogical Remarks

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6. Heizer, J., Render, B., and Munson, C., *Operations Management*, Boston, Pearson (2020).

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I. Introduction

Critical Path Method (CPM) has wide-ranging applications from business operations [1], to medical procedures [2],[3], to engineering constructions [4], to electric circuitry, computer soft and hard wares [5]. There are multitudes of computer programs to conduct these analyses. As such, the topic has been considered a standard teaching material in many a college curriculum. While industries benefit from efficient computing packages, students of education need to have a fundamental understanding of this theme. Two prevalent pedagogical treatments have been that of drawing a network flow chart to consider "forward/backward passes" [6] and that of conducting linear programming [7], [8]. In either approach, the "slack time" for a non-critical activity is subject to ambiguities. Consider a linear predecessor-successor relation from activity E to F to G, with no "Y-shaped" lateral bifurcation; then if E is not critical, its released time can be passed on to F and/or G. Thus, we limit our scope here to the identification of acritical path of activities without going into any detailed analyses of slack times. We also will not pursue the possibility of the existence of more than one critical path.

From an extensive literature research, we did not find any similar techniques to ours to identify critical activities, with the closest being [9]. We propose a perturbation of the required times of all the activities and subsequent observation of how the

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minimized finishing time has changed. Clearly, one can engage in this tack one activity at a time, but we will demonstrate ways to make the perturbation all at once. We contend that our method here will not only help students quickly identify the critical activities of a project but also contribute to their appreciation of the distinction between "critical" and "non-critical" activities.

In the following, we will illustrate our procedure by an example, which can nevertheless be generalized. As this paper has a pedagogical intent, we will also add a note in the matter of constructing examples of linear programming in class.

II. Analysis

The objective of scheduling an ensemble of activities as contained in a project is to minimize its final completion time but subject to two sets of constraints: [1] the time point to start any activity *j* equals the time point for all its immediate predecessors to deliver "their torches" to *j*(by an analogy to a marathon), and [2] the delivery time point of the "torch" by any activity *k*(to all its immediate successor(s)) minus *k's* starting time point is greater than or equal to *k's* required time (interval) of completion. Accordingly, one can have the following spreadsheet presentation (as an example):

Activities/contacting times	0	?	?	?	?	?	?
A	-1	1					
B	-1		1				
C		-1	1				
D			-1	1			
E			-1		1		
F				-1		1	
G					-1	1	
H						-1	1

for a project with

Activities	Immediate Predecessor(s)
A	-
B	-
C	A
D	B, C
E	B, C
F	D
G	E
H	F, G

where "-1" refers to the starting time point of an activity, and "1," the contacting time to its immediate successor(s), thereof addressing constraint set [1]. For constraint set [2], along with the consequent optimal solution, we may have:

	0	73	110	148	152	188	297	=H23		required time
A	-1	1						=SUMPRODUCT($B$23:$H$23,B24:H24)	>=	73
B	-1		1					=SUMPRODUCT($B$23:$H$23,B25:H25)	>=	41
C		-1	1					=SUMPRODUCT($B$23:$H$23,B26:H26)	>=	37
D			-1	1				=SUMPRODUCT($B$23:$H$23,B27:H27)	>=	38
E			-1		1			=SUMPRODUCT($B$23:$H$23,B28:H28)	>=	37
F				-1		1		=SUMPRODUCT($B$23:$H$23,B29:H29)	>=	40
G					-1	1		=SUMPRODUCT($B$23:$H$23,B30:H30)	>=	36
H						-1	1	=SUMPRODUCT($B$23:$H$23,B31:H31)	>=	109

Ref

1. Ahmad, H. H., Analysis of business networks in project management by using critical path method (CPM), *J. Interdisc. Math.*, 23(4), 2020, 851-855.

or

	0	73	110	148	152	188	297	297		required time
A	-1	1						73	>=	73
B	-1		1					110	>=	41
C		-1	1					37	>=	37
D			-1	1				38	>=	38
E			-1		1			42	>=	37
F				-1		1		40	>=	40
G					-1	1		36	>=	36
H						-1	1	109	>=	109

That is, the least amount of time is 297 units of time (in cell "H3"). A special note that is worth mentioning here is that the six decision variables, with their optimal time points: 73, 110, 148, 152, 188, and 297, do not need to be in increasing order in general; consider an interchange between the two columns headed by 73 and 110; one would nevertheless obtain the identical solution:

	0	110	73	148	152	188	297	297		required time
A	-1		1					73	>=	73
B	-1	1						110	>=	41
C		1	-1					37	>=	37
D		-1		1				38	>=	38
E		-1			1			42	>=	37
F				-1		1		40	>=	40
G					-1	1		36	>=	36
H						-1	1	109	>=	109

A more demanding task now is to identify the critical path associated with the optimal objective function's value of 297. This can be accomplished by a perturbation of the required times as follows:

	0	110	73.1	148	152	188	297	297.10110101		required time
A	-1		1					73.1	>=	73.1
B	-1	1						110.101	>=	41.01
C		1	-1					37.001	>=	37.001
D		-1		1				38.0001	>=	38.0001
E		-1			1			42.0001009	>=	37.00001
F				-1		1		40.000001	>=	40.000001
G					-1	1		36.0000001	>=	36.0000001
H						-1	1	109.00000001	>=	109.00000001

so that the critical activities are identified to be: A, C, D, F, and H from the decimal extension of 297 by 0.1, 0.001, 0.0001, 0.000001, and 0.00000001. We observe from the original solution that the slack times have been known to be integers; hence adding fractional values to the required times does not alter the identification of the critical activities. In principle, this technique can be applied to much greater number of activities by multiplying the required times by a common multiple of a power of 10 and perturbing successively by lower and lower power of 10 across the required times - - provided that one ensures the sum of the time increments is less than any slack time as solved from the original optimization. Otherwise, one may consider adding 0.01ln(2), 0.01ln(3), ..., 0.01ln(19, the eighth prime number) to the required times of A, B, ..., H respectively, so that the perturbed objective optimal value minus the pre-perturbed value = 0.01ln(2*5*7*13*19) = "d" and exp(100*d) = 2*5*7*13*19 recovers the critical activities, A, C, D, F, and H by the unique factorization theorem. In this regard, one can readily obtain 200 prime numbers from the Internet; dividing such a number as the above exp(100*d) by each of the prime numbers as having been assigned to all the activities, one then identifies a critical activity when the quotient is an integer.

As a separate matter of teaching linear programming by examples of 3 decision variables with exactly three constraints (in addition to non-negativity), one often finds not all the decision variables ending in positive values (which may be considered undesirable from a pedagogical perspective), e.g.,

Max 2x + 3y + 4z

(x,y,z) >= 0

s.t. [1] 5x + 6y + 7z <= 400,

[2] 30x + 20y +10z <= 500, and

[3] x + y + z <= 600.

Then one has the optimal solution: x = y = 0 and z = 50, with the objective function's value = 200. The crux of the problem here is that the z-direction yields the greatest ascent to the objective value so that x and y are necessarily zero. Of course, one would quickly think of altering the signs of the coefficients; yet rather than by a haphazard trial and error, we propose a minimization over an unbounded region as constrained by three planes that intersect at a point of (x*, y*, z*) > 0, as the optimal solution, such as P* = (10, 10, 10). Any pair of the three planes intersect into a line, which is to intersect the (x, y)-plane, the (y, z)-plane, or the (z, x)-plane at a point, such as Q = (5, 5, 0), R = (5, 0, 5), or S = (0, 5, 5); then P*Q, P*R, and P*S yield three lines in R^3 and any two of these three lines form a plane. Elementary algebra then leads to the following three equations:

[1] 3x − y − z = 10,

[2] − x + 3y − z = 10, and

[3] − x − y + 3z = 10.

Then the solution to, say,

Min x + 2y + 3z

subject to

[1] 3x − y − z >= 10,

[2] − x + 3y − z >= 10, and

[3] − x − y + 3z >= 10

is x* = y* = z* = 10 with the objective function of value 60, as expected. By the duality theorem of linear programming, we have the following dual:

Max 10u + 10v + 10w

s.t.

[1] 3u − v − w <= 1,

[2] − u + 3v − w <= 2, and

Notes

Global Journal of Science Frontier Research (F) Volume XXI Issue I Version I Year 2021

[3] − u − v + 3w <= 3,

which has the solution: u* = 1.75, v* = 2, and w* = 2.25, necessarily yielding the identical objective function's value of 60.

In this way, an instructor can construct examples of linear programming with the optimal solutions for the decision variables all positive, naturally with the leeway of perturbation of the data without affecting the said qualitative outcome.

Notes

III. Summary

In this note, we have presented (1) a speedy way of identifying critical activities in CPM and (2) a procedure to construct examples of linear programming that may be more illuminating to students. Although the undertone in our exposition here leans toward teaching, we contend that even for professional research/analysis, our aforementioned "perturbation via prime numbers" can bring about a quick resolution of the critical path by simple spreadsheet operations.

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