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On Some Maps Concerning β-Closed Sets and Related Groups

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On Some Maps Concerning β-Closed Sets and Related Groups

Sanjay Tahiliani

Abstract- The concept of group of functions, say βch(X,τ) preserving β-closed sets containing homeomorphism group h(X,τ) was studied by Arora,Tahiliani and Maki. In continuation to that, we study some new isomorphisms, mappings, subgroups and their properties.

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I. Introduction and Preliminaries

Throughout this paper we consider spaces on which no separation axiom are assumed unless explicitly stated. The topology of a space(By space we always mean a topological space) is denoted by τ and (X,τ) will be replaced by X if there is no chance of confusion. For A⊆X, the closure and interior of A in X are denoted by Cl(A) and Int(A) respectively. Let A be a subset of the space (X,τ).Then A is said to be β-open [1] if A⊆Cl(Int(Cl(A))). Its complement is β-closed. The family of all β-open sets containing A is denoted by βO(A) and all β-closed sets containing A is denoted by βC(A). A is said to be α-open[6] if A⊆ Int(Cl(Int(A))) and its complement is α-closed. The union of all β-open sets contained in A is called β-interior of A, denoted by βInt(A)[2].

A map f:(X,τ) →(Y,σ) is called β-irresolute[4] if the inverse image of every β-open set in Y is β-open in X. It is called βc-homeomorphism[5] if f is β-irresolute bijection and f^{-1} is β-irresolute.

II. Subgroups of βCH(X;τ)

For a topological space (X,τ) we have h(X;τ)={f | f:(X,τ) →(X,τ) is a homeomorphism}[5] and βch(X;τ)={f | f:(X,τ) →(X,τ) is a βc-homeomorphism}[5].

In this section, we investigate some structures of βch(H;τ|H) for a subspace (H,τ|H) of (X,τ) using two subgroups of βch(X,τ), say βch(X, X “ H; τ) and βch_0(X, X “ H; τ) below.

Definition 2.1 For a topological space (X,τ) and subset H of X, we define the following families of maps:

(i) βch(X, X \ H; τ)={a| a∈ βch(X;τ) and a(X \ H)= X \ H}.

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(ii) $\beta ch_0(X, X \setminus H; \tau)= \{a| a\in \beta ch(X, X \setminus H; \tau)$ and $a(x)=x$ for every $x\in X \setminus H\}$.

Theorem 2.2 Let H be a subset of a topological space (X,τ). Then

(i) The family $\beta ch(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X,\tau)$.

(ii) The family $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X, X \setminus H; \tau)$ and hence $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X,\tau)$.

Proof.(i) It is shown obviously that $\beta ch(X, X \setminus H; \tau)$ is a non empty subset of $\beta ch(X,\tau)$, because $1_X\in \beta ch(X, X \setminus H; \tau)$. Moreover, we have that $\omega_X(a,b^{-1})=b^{-1}o\ a\in \beta ch(X, X \setminus H; \tau)$ for any elements a, $b\in \beta ch(X, X$ " $H; \tau)$,where $\omega_X = \omega|(\beta ch(X, X \setminus H; \tau) \times \beta ch(X, X \setminus H; \tau))$ as ω is the binary operation of the group $\beta ch(X,\tau)$. Evidently, the identity map 1_X is the identity element of $\beta ch(X, X \setminus H; \tau)$.

(ii) It is shown that $\beta ch_0(X, X \setminus H; \tau)$ is a non empty subset of $\beta ch(X, X \setminus H; \tau)$ because $1_X\in \beta ch_0(X, X \setminus H; \tau)$.We have that $\omega_{X,0}(a,b^{-1})=b^{-1}o\ a\in \beta ch_0(X, X \setminus H; \tau)$ for any elements a, $b\in \beta ch_0(X, X \setminus H; \tau)$,where $\omega_{X,0}= \omega_X|($ —$(\beta ch_0(X, X \setminus H; \tau) \times \beta ch_0(X, X \setminus H; \tau))($ ω_X is the binary operation of the group $\beta ch(X, X \setminus H; \tau))$. Thus $\beta ch_0(X, X \setminus H; \tau)$ is a subgroup of $\beta ch(X, X \setminus H; \tau)$ and the identity map 1_X is the identity element of $\beta ch_0(X, X \setminus H; \tau)$. By using (i), $\beta ch_0(X, X \setminus H; \tau)$ forms a subgroup of $\beta ch(X,\tau)$.

Let H and K be the subsets of X and Y respectively. For a map $f:X\to Y$ satisfying a property $K=f(H)$, we define the following map $r_{H,K}(f):H\to K$ by $r_{H,K}(f)(x)=f(x)$ for every $x\in H$. Then, we have that $j_K o\ r_{H,K}(f)= f|H:H\to Y$, where $j_K:K\to Y$ be an inclusion defined by $j_K(y)=y$ for every $y\in K$ and $f|H:H\to Y$ is a restriction of f to H defined by $(f|H)(x)=f(x)$ for every $x\in H$. Especially, we consider the following case that $X=Y$, $H=K\subseteq X$ and $a(H)=H$,$b(H)=H$ for any maps a,b: $X\to X$. Thus $r_{H,H}(boa)= r_{H,H}(b)\ o\ r_{H,H}(a)$ holds. Moreover, if a map $a:X\to X$ is a bijection such that $a(H)=H$, then $r_{H,H}:H\to H$ is bijective and $r_{H,H}(a^{-1})= (r_{H,H}(a))^{-1}$.

We recall well known properties on β-open sets of subspace topological spaces.

Theorem 2.3. For a topological space (X,τ) and subsets H and U of X and $A\subseteq H,V\subseteq H$ and $B\subseteq H$, the following properties hold:

(i) Arbitrary union of β-open sets of (X,τ) is β-open in (X,τ). The intersection of an open set of (X,τ) and a β-open set in (X,τ) is β-open in (X,τ).

(ii) (ii-1). If A is β-open in (X,τ) and $A\subseteq H$, then A is β-open in a subspace $(H,\tau|H)$.

(ii-2). If $H\subseteq X$ is open or α-open in (X,τ) and a subset $U\subseteq X$ is β-open in (X,τ), then $H\cap U$ is β-open in a subspace $(H,\tau|H)$.

(iii). Let $V\subseteq H\subseteq X$.

(iii-1). If H is β-open in (X,τ),then $Int_H(V) \subseteq \beta Int(V)$ holds.

(iii-2). If H is β-open in (X,τ) and V is β-open in a subspace $(H,\tau|H)$ then V is β-open in (X,τ).

(iv). Let $B\subseteq H\subseteq X$. If H is β-closed in (X,τ) and B is β-closed in a subspace $(H,\tau|H)$, then B is β-closed in (X,τ).

(v). (v-1).Assume that H is a open subset of (X,τ) .Then,

Notes

$\beta O(X,\tau)|H \subseteq \beta O(H,\tau|H)$ holds, where $\beta O(X,\tau)|H=\{W\cap H|\ W\in \beta O(X,\tau)\}$.
(v-2). Assume that H is a β-open subset of (X,τ) .Then,
$\beta O(H,\tau|H) \subseteq \beta O(X,\tau)|H$ holds.
(v-3). Assume that H is a β-open subset of (X,τ) .Then,
$\beta O(H,\tau|H) = \beta O(X,\tau)|H$ holds.

Proof. (i). Clear from Remark 1.1 of [1] and Theorem 2.7 of [3].
(ii).(ii-1). Clear.(ii-2).Its Lemma 2.5 of [1].
(iii-1). Let $x\in Int_H(V)$. There exists a subset $W(x) \in \tau$ such that $W(x) \cap H\subseteq V$. By (i), $W(x) \cap H\in \beta O(X,\tau)$. This shows that $x\in \beta\, Int(V)$ and so $Int_H(V) \subseteq \beta Int(V)$.
(iii-2) and (iv). Its clear from Lemma 2.7 of [1].
(v). (v-1). Let $V\in \beta O(X,\tau)|H$. For some set $W\in \beta O(X,\tau)$, $V=W\cap H$ and so we have $W\cap H\in \beta O(H,\tau|H)$(from ii-2). Hence $V\in \beta O(H,\tau|H)$ holds.
(v-2). Let $V\in \beta O(H,\tau|H)$. Since $H\in \beta O(X,\tau)$, we have $V\in \beta O(X,\tau)$ by (iii-2). Thus $V= V\cap H\in \beta O(X,\tau)|H$.
(v-3). It follows from (v-1) and (v-2).

Lemma 2.4. (i). If $f:(X,\tau) \rightarrow(Y,\sigma)$ is β-irresolute and a subset H is α-open in (X,τ),then $f|H:(H,\tau|H) \rightarrow(Y,\sigma)$ is β-irresolute.
(ii). Let (1) and (2) be properties of two maps $k:(X,\tau) \rightarrow(K,\sigma|K)$,where $K\subseteq Y$, and $j_K ok$: $(X,\tau) \rightarrow(Y,\sigma)$ as follows:
(1). $k:(X,\tau) \rightarrow(K,\sigma|K)$ is β-irresolute.
(2). $j_K ok$: $(X,\tau) \rightarrow(Y,\sigma)$ is β-irresolute.

Then, the following implication and equivalence hold:
(ii-1). Under the assumption that K is α-open in (Y,σ) ,(1)⇒(2).
(ii-2). Conversely, under the assumption that K is β-open in (Y,σ) ,(2)⇒(1).
(ii-3). Under the assumption that K is β-open in (Y,σ) ,(1)⇔(2).
(iii) If $f:(X,\tau) \rightarrow(Y,\sigma)$ is β-irresolute and a subset H is α-open in (X,τ) and f(H) is β open in (Y,σ),then $r_{H,f(H)}(f)$: $(H,\tau|H) \rightarrow(f(H), \sigma|f(H))$ is β-irresolute.

Proof.(i). Let $V\in\beta O(Y,\sigma)$. Then, we have $(f\mid H)^{-1}(V)=f^{-1}(V) \cap H$ and $(f\mid H)^{-1}(V) \in \beta O(H,\tau|H)$.(Theorem 2.3 (ii-2)).
(ii) (ii-1) (1)⇒(2).Let $V\in \beta O(Y,\sigma)$.Since $(j_K ok)^{-1}(V)= k^{-1}(V\cap K)$ and $V\cap K\in \beta O(K,\sigma|K)$ (Theorem 2.3 (ii-2)),we have that $(j_K ok)^{-1}(V) \in \beta O(X,\tau)$ and hence $j_K ok$ is β-irresolute.
(ii-2) (2) ⇒(1).Let $U\in \beta O(K,\sigma|K)$.Since $U\in \beta O(Y,\sigma)$(Theorem 2.3 (iii-2)),we have $k^{-1}(U)=(j_K ok)^{-1}(U) \in \beta O(X,\tau)$.Thus k is β-irresolute.
(ii-3).Obvious in the view of fact that every α-open set is β-open,it is obtained by (ii-1) and(ii-2).
(iii) By (i), $f|H:(H,\tau|H) \rightarrow(Y,\sigma)$ is β-irresolute.The map $r_{H,f(H)}(f)$ is β-irresolute, because $f|H=j_{f(H)}o\ r_{H,f(H)}(f)$ holds.

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Definition 2.5. For an α-open subset H of (X,τ), the following maps $(r_H)^*$: βch(X, X “ H; τ) → βch(H;τ|H) and $(r_H)^*_{,0}$: βch_0(X, X “ H; τ) → βch(H;τ|H) are well defined as follows(Lemma 2.4 (iii)),respectively:

$(r_H)^*(f)= r_{H,H}(f)$ for every f∈ βch(X, X “ H; τ);

$(r_H)^*_{,0}(g)= r_{H,H}(g)$ for every g∈ βch_0(X, X “ H; τ).Indeed ,in Lemma 2.4 (iii),we assume that X=Y, τ=σ and H=f(H).

Then, under the assumption that H is α-open hence β-open in (X,τ), it is obtained that $r_{H,H}(f)$ ∈ βch(H;τ|H) holds for any f∈βch(X, X “ H; τ)(resp. f∈ βch_0(X, X “ H; τ)).

We need the following lemma and then we prove that $(r_H)^*$ and $(r_H)^*_{,0}$ are onto homomorphisms under the assumptions that H is α-open and α-closed in (X,τ).

Let $X=U_1\cup U_2$ for some subsets U_1 and U_2 and $f_1:(U_1,\tau|U_1) \to(Y,\sigma)$ and $f_2:(U_2,\tau|U_2) \to(Y,\sigma)$ be the two maps satisfying a property $f_1(x)=f_2(x)$ for every x∈ $U_1\cap U_2$.Then, a map $f_1\nabla f_2$ is well defined as follows:

$$(f_1\nabla f_2)(x)= f_1(x) \text{ for every } x\in U_1 \text{ and } (f_1\nabla f_2)(x)= f_2(x) \text{ for every } x\in U_2.$$

We call this map a combination of f_1 and f_2.

Lemma 2.6. For a topological space (X,τ),we assume that X= $U_1\cup U_2$, where U_1 and U_2 are subsets of X and $f_1:(U_1,\tau|U_1) \to(Y,\sigma)$ and $f_2:(U_2,\tau|U_2) \to(Y,\sigma)$ be the two maps satisfying a property $f_1(x)=f_2(x)$ for every x∈ $U_1\cap U_2$.Then if $U_i\in\beta O(X,\tau)$ for each i∈{1,2} and f_1 and f_2 are β-irresolute, then its combination $f_1\nabla f_2$: (X,τ) →(Y,σ) is β-irresolute.

Proof. Its on similar lines in ([1], Theorem 2.8).

Theorem 2.7. Let H be a subset of a topological space (X,τ).

(i) (i-1).If H is α-open in (X,τ),then the maps $(r_H)^*$: βch(X, X“H;τ) → βch(H;τ|H) and $(r_H)^*_{,0}$: βch_0(X, X \ H; τ) → βch(H;τ|H) are homomorphism of groups. Morever $(r_H)^*$ | βch_0(X, X \ H; τ)= $(r_H)^*_{,0}$ holds(Definition 2.5).

(i-2). If H is α-open and α-closed in (X,τ),then the maps $(r_H)^*$: βch(X, X“H; τ) → βch(H;τ|H) and $(r_H)^*_0$: βch_0(X, X “ H; τ) → βch(H;τ|H) are onto homomorphism of groups.

(ii) For an α-open subset H of (X,τ),we have the following isomorphisms of groups:

(ii-1). βch(X, X \ H; τ) |Ker$(r_H)^*$ is isomorphic to Im$(r_H)^*$;

(ii-2). βch_0(X, X \ H; τ) is isomorphic to Im$(r_H)^*_{,0}$ holds.

where Ker$(r_H)^*$={a∈ βch(X, X \ H; τ) |$(r_H)^*(a)=1_X$” is a normal subgroup of βch(X, X \ H; τ); Im$(r_H)^*$=–$(r_H)^*(a)$ | a∈ βch(X, X \ H; τ)”and Im$(r_H)^*$,0=–$(r_H)^*_{,0}$ (b) | b∈ βch_0(X, X \ H; τ)}are subgroups of βch(X,τ).

(iii) For an α-open and α-closed subset H of (X,τ),we have the following isomorphisms of groups:

(iii-1). βch(H;τ|H) is isomorphic to βch(X, X \ H; τ) |Ker$(r_H)^*$.

(iii-2). βch(H;τ|H) is isomorphic to βch_0(X, X \ H; τ).

Notes

Proof. (i).(i-1). Let a,b∈ βch(X, X \ H; τ). Since H is α-open in (X,τ),the maps $(r_H)^*$ and $(r_H)^*_{,0}$ are well defined(Definition 2.5). Then we have that $(r_H)^*(\omega_X(a,b))= (r_H)^*(boa)= r_{H,H}(boa)= r_{H,H}(b) \circ r_{H,H}(a)= \omega_X((r_H)^*(a), (r_H)^*(b))$ hold, where ω_H is a binary operation of βch(H;τ|H) ([5] Theorem 4.4 (iv)).Thus $(r_H)^*$ is a homomorphism of groups. For the map $(r_H)^*_0$: $\beta ch_0(X, X \setminus H; \tau) \to \beta ch(H;\tau|H)$,we have that $(r_H)^*_{,0}(\omega_{X,0}(a,b))= (r_H)^*_{,0}(boa)= r_{H,H}(boa)= r_{H,H}(b) \circ r_{H,H}(a)= \omega_X((r_H)^*(a), (r_H)^*(b))$ hold, where ω_X is a binary operation of βch(H;τ|H)(Theorem 2.3 (ii)). Thus $(r_H)^*_{,0}$ is also a homomorphism of groups. It is obviously shown that $(r_H)^* \mid \beta ch_0(X, X$ " $H; \tau)= (r_H)^*_{,0}$ holds.(Definitions 2.1 and 2.5).

Notes

(i-2). In order to prove that $(r_H)^*$ and $(r_H)^*_0$ are onto, let h∈ βch(H;τ|H). Let j_H: (H;τ|H) →(X,τ) and $J_{X \text{“} H}$: (X \ H, τ| X \ H) →(X,τ) be the inclusions defined $j_H(x)=x$ for every x∈H and $J_{X \text{“} H}(x)=x$ for every x∈ X " H. We consider the combination $h_1=(j_H o h) \nabla(j_{X \text{“} H} o 1_{X \text{“} H})$: (X,τ) →(X,τ). By Lemma 2.4 (ii-1),under the assumption of α-openness on H, it is shown that two maps j_H oh : (H;τ|H) →(X,τ) and $j_H\, oh^{-1}$: (H;τ|H) →(X,τ) are β-irresolute; moreover under the assumption of α-openness on X \ H, $J_{X \text{“} H}$ o $1_{X \text{“} H}$: (X \ H,τ | X \ H) →(X,τ) is β-irresolute. Using lemma 2.6, for a β-open cover {H, X \ H} of X, the combination above h_1: (X,τ) →(X,τ) is β-irresolute. Since h_1 is bijective, its inverse map $h_1^{-1}=(j_H oh^{-1}) \nabla(j_{X \text{“} H} o 1_{X \text{“} H})$ is also β-irresolute. Thus under the assumption that both H and X " H are β-open in (X,τ), we have h_1∈ βch(X,τ). Since $h_1(x)=x$ for every point x∈ X \ H, we conclude that h_1∈ βch_0(X, X \ H; τ) and so h_1∈βch(X, X \ H; τ). Moreover, $(r_H)^*_{,0}(h_1)= (r_H)^*(h_1)= r_{H,H}((h_1)=h$, hence $(r_H)^*$ and $(r_H)^*_{,0}$ are onto, under the assumption that H is α-open and α-closed subset of (X,τ).

(ii). By (i-1) above and the first isomorphism theorem of group theory, it is shown that there are group isomorphism below, under the assumption that H is α-open in (X,τ):

(*). βch(X, X " H; τ) $|Ker(r_H)^*$ is isomorphic to $Im(r_H)^*$; and

(**). βch_0(X, X " H; τ) $|Ker(r_H)^*_0$ is isomorphic to $Im(r_H)^*_{,0}$

where $Ker(r_H)^*_0$ =–a∈βch_0(X, X \ H; τ) $|(r_H)^*_0(a)=1_X$}. Moreover, under the assumption of α-openness on H, it is shown that $Ker(r_H)^*_0 =\{1_H\}$.Therefore, using (**) above, we have the isomorphism (ii-2).

(iii).By (i-2) above, it is shown that $(r_H)^*$ and $(r_H)^*_{,0}$ are onto homomorphism of groups, under the assumption that H is α-open and α-closed in (X,τ). Therefore, by (ii) above, the isomorphisms (iii-1) and (iii-2) are obtained.

Remark 2.8. Under the assumption that H is α-open and α-closed in (X, τ), Theorem 2.7 (iii) is proved. Let (X,τ) be a topological space where X={a,b,c} and τ=– φ, X, {a},{b,c}}, and (H;τ|H) is a subspace of (X,τ),where H={a}.Then βO(X,τ)=P(X)(the power set of X) and H is α -open and α -closed in (X,τ).We apply Theorem 2.7 (iii) to the present case, we have the group isomorphisms. Directly, we obtain the following date on groups: βch(X,τ) is isomorphic to S_3,the symmetric group of degree 3, βch(X, X \ H; τ)={1_X, h_a},$Ker(r_H)^*$={1_X, h_a}, βch(H; τ \ H)=–1_H} and so βch_0(X, X \ H; τ)= –1_X},where h_a:(X,τ) →(X,τ) is a map defined by $h_a(a)=a$, $h_a(b)=c$ and $h_a(c)=b$. Therefore in this example, we have βch(H;τ|H) is isomorphic to βch(X, X \ H; τ)

$|Ker(r_H)^*$ and $\beta ch(H;\tau|H)$ is isomorphic to $\beta ch_0(X, X \setminus H; \tau)$. Moreover we have $h(X,\tau)=\{1_X, h_a\}$.

(iii). Even if a subset H of a topological space (X,τ) is not α-closed and it is α-open, we have the possibilities to investigate isomorphisms of groups corresponding to a subspace $(H, \tau|H)$ and $(r_H)^*$ using Theorem 5.7(ii). For example, Let (X,τ) be a topological space where $X=\{a,b,c\}$ and $\tau=\{\varphi, X, \{a,b\}\}$,and $(H;\tau|H)$ is a subspace of (X,τ),where $H=\{a,b\}$.Then $\beta O(X,\tau)=P(X)$(the power set of X) and H is α-open but not α-closed in (X,τ). By theorem 2.7(i)(i-1), the maps $(r_H)^*$: $\beta ch(X, X$ " $H;\tau) \rightarrow \beta ch(H;\tau|H)$ and $(r_H)^*{}_{,0}$: $\beta ch_0(X, X \setminus H; \tau) \rightarrow \beta ch(H;\tau|H)$ are homomorphism of groups and by theorem 5.7(ii) two isomorphisms of groups are obtained:

(*-1). $\beta ch(X, X$ " $H; \tau)/Ker(r_H)^*$ is isomorphic to $Im(r_H)^*$. (*-2). $\beta ch_0(X, X \setminus H; \tau)/Ker(r_H)^*$ is isomorphic to $Im(r_H)^*{}_{,0}$.

We need notation on maps as follows: let h_c: $(X,\tau) \rightarrow (X,\tau)$ and $t_{a,b}$: $(H,\tau|H) \rightarrow (H,\tau|H)$ are the maps defined by $h_c(a)=b$, $h_c(a)=b$, $h_c(c)=c$ and $t_{a,b}(a)=b$, $t_{a,b}(b)=a$, respectively. Then it is directly shown that $\beta ch(X, X \setminus H; \tau)=-1_X, h_c\}$which is isomorphic to Z_2, $(h_c)^2= 1_X$, and $Ker(r_H)^*=-a\in \beta ch(X, X \setminus H; \tau)$— $(r_H)^*(a)= 1_H\}=\{a\in -1_X, h_c\}$—$(r_H)^*(a)= 1_H \}=\{1_X\}$because $(r_H)^*(1_X)= 1_H$ and $(r_H)^*(h_c)= t_{a,b}$ not equal to 1_H. By using (*-1) above, $Im(r_H)^*$ is isomorphic to $\beta ch(X, X \setminus H; \tau) =\{1_X, h_c\}$and so $Im(r_H)^*=\{ 1_H, r_{H,H}(h_c)\}=-1_H, t_{a,b}\}$. Since $Im(r_H)^*\subseteq \beta ch(H;\tau|H) \subseteq\{1_H, t_{a,b}\}$,we have that $Im(r_H)^* = \beta ch(H,\tau|H)= -1_H, t_{a,b}\}$ and hence $(r_H)^*$ is onto. Namely, we have an isomorphism $(r_H)^*$: $\beta ch(X, X$ " $H; \tau)$ is isomorphic to $\beta ch(H;\tau|H)$ which is isomorphic to Z_2. Morever it is shown that $\beta ch_0(X, X \setminus H; \tau)=-a\in \beta ch(X, X \setminus H; \tau)$—$a(x)=x$ for any $x\in -c\}\}=-1_X, h_c\}= \beta ch(X, X \setminus H; \tau)$ hold and so $(r_H)^*= (r_H)^*{}_{,0}$ holds.

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Notes

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