The Mechanism of Generation of Nonlinear Interactions of Microscopic Particles and its Properties in Quantum Systems

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Abstract- The mechanisms of generation of the nonlinear interactions in the quantum systems are studied using a nonlinear Schrodinger equation, which can describe the states and properties of motion of the microscopic particles. The investigations show that the interactions arises from the interaction between the moved particle and background field, the difficulties, limitations and approximations of the quantum mechanics and its roots and reasons are first revealed and pointed out. The quantum mechanics simplifies and blots out the real motions and interactions of the microscopic particles including the nonlinear interactions, thus the particles have only a wave feature, not corpuscle feature. In view of these problems and difficulties we add the nonlinear interaction, \( b|\phi|^2\phi \), into the dynamic equation and use again it to describe the particles. Thus the wave or dispersive effect of the particles is suppressed by the nonlinear interactions, thus its localization and wave-corpuscle duality are displayed and exhibited naturally.

Keywords: microscopic particle, nonlinear interaction, nonlinear Schrodinger equation, quantum mechanics, mechanism, counteraction.

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The Mechanism of Generation of Nonlinear Interactions of Microscopic Particles and its Properties in Quantum Systems

Pang Xiao Feng

Abstract: The mechanisms of generation of the nonlinear interactions in the quantum systems are studied using a nonlinear Schrödinger equation, which can describe the states and properties of motion of the microscopic particles. The investigations show that the interactions arises from the self-action and interaction between the moved particle and background field. The difficulties, limitations and approximations of the quantum mechanics and its roots and reasons are first revealed and pointed out. The quantum mechanics simplifies and blots out the real motions and interactions of the microscopic particles including the nonlinear interactions, thus the particles have only a wave feature, not corpuscle feature. In view of these problems and difficulties we add the nonlinear interaction, $\hbar |\phi|^2 \phi$, into the dynamic equation and use again it to describe the particles. Thus the wave or dispersive effect of the particles is suppressed by the nonlinear interactions, thus its localization and wave-corpse feature are displayed and exhibited naturally. Subsequently, we investigate the features of the particles and find that the nonlinear interactions exist always, if and only if the real motions of the particles and background fields as well as the interactions between them are considered simultaneously. On the basis of analyses of the real motions and interactions of the microscopic particles, we find that the nonlinear interactions exist always, if and only if the real motions of the particles and background fields as well as the interactions between them are considered simultaneously. The nonlinear interactions are considered simultaneously. On the basis of analyses of the real motions and interactions of the microscopic particles, we find that the nonlinear interactions exist always, if and only if the real motions of the particles and background fields as well as the interactions between them are considered simultaneously. Finally we discuss the properties of the nonlinear interactions in the two mechanisms and obtain the relations between the action and counteraction of the two nonlinear interactions of the particles and background fields. We find that although the nonlinear interaction accepted by the particles and background field can generate simultaneously in the nonlinear quantum systems, the two nonlinear interactions do not completely satisfy the law of action and counteraction of the forces in the classical mechanics. This shows clearly that the nature of the microscopic particles in the nonlinear quantum systems are different from those of classical particles, even though the microscopic particles have also the corpuscle feature. This conclusion is of important and interest to physics and nonlinear science.

Keywords: microscopic particle, nonlinear interaction, nonlinear Schrödinger equation, quantum mechanics, mechanism, counteraction.

1. Introduction, the Limitations of Quantum Mechanics

As are known, the quantum mechanics established by several great scientists, such as Bohr, Born, Schrödinger and Heisenberg, etc., in the early 1900s [1-6] is the foundation of modern science and used extensively to study the properties and rules of motion of microscopic particles. In the theory the states of microscopic particles are often described by a Schrödinger equation:

\[ i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(\vec{r}, t) \psi \]  

(1)

where $\hbar^2 \nabla^2 / 2m$ is the kinetic energy operator, $V(\vec{r}, t)$ is the externally applied potential operator, $m$ is the mass of the particles, $\psi(\vec{r}, t)$ is a wave function describing the states of the particles, \( \vec{r} \) is the coordinate or position of the particle. Equation (1) is a wave equation, if only the externally applied potential is known, we can find the solutions of the equation. However, for all externally applied potentials, its solutions are always a linear or dispersive wave, for example, at $V(\vec{r}, t) = 0$, the solution is a plane wave as follows:

\[ \psi(\vec{r}, t) = A \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \]  

(2)

where $k$ is the wavevector of the wave, $\omega$ is its frequency, and $A$ is its amplitude. This solution denotes the state of a freely moving microscopic particle with an eigenenergy of

\[ E = \frac{p^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2), (-\infty < p_x, p_y, p_z < \infty) \]  

(3)

This energy is continuous, this means that the probability of the particle to appear at any point in the space is a constant, thus the microscopic particle cannot be localized, can only propagate freely in a wave in total space. Then the particle has nothing about corpuscle feature.

If the free particle is artificially confined in a small finite space, such as, a rectangular box of dimension $a$, $b$ and $c$, then the solution of Eq.(1) is a standing wave of

\[ \psi(\vec{r}, t) = \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \]  

(4)

where $k$ is the wavevector of the wave, $\omega$ is its frequency, and $A$ is its amplitude. This solution denotes the state of a freely moving microscopic particle with an eigenenergy of

\[ E = \frac{p^2}{2m} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2), (-\infty < p_x, p_y, p_z < \infty) \]  

(5)

This energy is continuous, this means that the probability of the particle to appear at any point in the space is a constant, thus the microscopic particle cannot be localized, can only propagate freely in a wave in total space. Then the particle has nothing about corpuscle feature.

If the free particle is artificially confined in a small finite space, such as, a rectangular box of dimension $a$, $b$ and $c$, then the solution of Eq.(1) is a standing wave of
(6) means that all interactions among the particles and between the particles and background fields, such as the lattices in solid matter, must belong in the externally applied potential \( V(\mathbf{r}, t) \) no matter what forms of linear, nonlinear and other complicated interactions\(^{13-16}\). In other word, when the forms of the external potential field \( V(\mathbf{r}, t) \) are changed successively, we find that the dispersion and decaying nature of the microscopic particle always persists. This means that the external potential field \( V(\mathbf{r}, t) \) can only change the shape of the microscopic particle, such as, its amplitude and velocity, but not its wave nature because its nature are only determined by the kinetic energy term, \( h^2 \nabla^2 / 2m = \mathbf{p}^2 / 2m \), with dispersive effect in Eq.(1). Because microscopic particles are always in motion, then the dispersion effect always exists, but there is not an interaction, which can obstruct and suppress the dispersive effect of kinetic energy and make the microscopic particles the localization in Eqs.(1) and (6) in quantum mechanics, thus the particles have only the dispersive or wave feature, not the corpuscle feature. These are just the roots resulting in the wave feature, or speaking, the reason why the microscopic particles have only in quantum mechanics. Therefore, a key problem solving the difficulties and limitations of the quantum mechanics is to seek for a new interaction between the particles, which obstruct and suppress the dispersive effect of kinetic energy.

II. The Nonlinear Interactions can Suppress the Dispersive Effect and make the Particles the Localization

As mentioned above, in order to solve the difficulties and limitations of quantum mechanics we must seek for an interaction to obstruct and suppress the dispersive effect of kinetic energy to make the microscopic particles the localization. Thus we must make much account of the nonlinear interaction among the particles, which could play the above role because it can distort and collapse also the wave\(^{18-11}\) eventually can balance and cancel the dispersive effect. Thus the nature of the particle could be changed, its feature of corpuscle could be displayed\(^{13-18}\). In the light of this idea we now evaluate and add further the nonlinear interaction of \( b|\psi|^{2} \phi \) related to the states of the particles into Eq.(1), thus the dynamic equation of microscopic particles should be replaced by the following nonlinear Schrodinger equation.

\[
\begin{align*}
\frac{i \hbar}{\partial t} \phi &= - \frac{\hbar^2}{2m} \nabla^2 \phi + b|\psi|^2 \phi + V(\mathbf{r}, t) \phi \\
&= -(\mathbf{p}^2 / 2m) \phi + V(\mathbf{r}, t) \phi 
\end{align*}
\]
where \( x = x \sqrt{2m/\hbar}, t = t/\hbar \). We now assume the solution of Eq. (8) to be of the form

\[
\phi(x', t') = \phi(x', t') e^{i\theta(x', t')}
\]

Inserting Eq. (9) into Eq. (8) we can obtain

\[
\begin{align*}
\phi_{xx'} - \varphi \phi_{x'} - \varphi \phi_{t'} - b \varphi^2 \phi &= 0, \quad (b > 0) \\
\varphi_{xx'} + 2 \varphi_x \phi_{x'} + \varphi_{t'} &= 0
\end{align*}
\]

If let \( \theta = \theta(x' - v_x t'), \varphi = \phi(x' - v_x t') \), then Equations (10)-(11) become

\[
\begin{align*}
\varphi_{xx'} - v_x \varphi_{x'} - \varphi \varphi_{x'} - b \varphi^2 \varphi &= 0 \\
\varphi_{xx'} + 2 \varphi_x \varphi_{x'} - v_x \varphi_{t'} &= 0
\end{align*}
\]

If fixing the time \( t' \) and further integrating Eq. (13) with respect to \( x' \) we can get

\[
\varphi^2 (2 \varphi_{x'} - v_x) = A(t')
\]

Now let integral constant \( A(t') = 0 \), then we can get \( \varphi_{x'} = v_x/2 \). Again substituting it into Eq. (13), and further integrating this equation we then yield

\[
\int_{\phi_0}^{\phi} \frac{d\phi}{Q(\phi)} = x' - v_x t'
\]

where \( Q(\phi) = -b\varphi^2 + (v_x^2 - 2v_x v_x)\varphi^2 + c' \).

When \( c' = 0, v_x^2 - 2v_x v_x > 0 \), then \( \varphi = \pm \phi_0, \phi_0 = [(v_x^2 - 2v_x v_x) / 2b] \) is the roots of \( Q(\phi) = 0 \) except for \( \varphi = 0 \). Thus from Eq. (15) we obtain the solution of Eq. (10)-(11) is

\[
\phi(x', t') = \phi_0 \sec \left[ \frac{b}{2} \phi_0 (x' - v_x t') \right].
\]

Then the solution of nonlinear Schrodinger equation in Eq. (8) eventually is of the form

\[
\phi(x, t) = A_0 \sec \left( \frac{A_0 \sqrt{bm}}{\hbar} [(x - x_0) - vt] \right) e^{i\omega t} e^{i\theta(x, t)}/\hbar
\]

where \( A_0 = \sqrt{mv_x^2 / 2 - E} / \sqrt{2b} \), \( v \) is the velocity of motion of the particle, \( E = \hbar \omega \). The solution in Eq. (16) can be also found by the inverse scattering method \([13, 14, 18-22]\). This solution is completely different from Eq. (2), and consists of a envelop and carrier waves, the former is \( \phi(x, t) = A_0 \sec \left( \frac{A_0 \sqrt{bm}}{\hbar} [(x - x_0) - vt] \right) \) which is a bell-type non-topological soliton with an amplitude \( A_0 \), the latter is the \( \exp \left( \pm i\omega t \varphi(x - x_0) - Et / \hbar \right) \). This solution is shown in Fig. 1(a). Therefore, the particle described by nonlinear Schrodinger equation (8) is a soliton\([22-23]\). The envelop \( \phi(x, t) \) is a slow varying function and the mass centre of the particle, the position of the mass centre is just at \( x_0 \). \( A_0 \) is its amplitude, and its width is given by \( W = 2\pi \hbar / \sqrt{mbA_0} \). Thus, the size of the particle is \( A_0 W = 2\pi \hbar / \sqrt{mb} \) and a constant. This shows that the particle has exactly a mass centre and determinant size, thus the particle localized at \( x_0 \). According to the soliton theory\([22-23]\), the bell-type soliton in Eq. (16) can move freely over a macroscopic distances in a uniform velocity \( v \) in space-time retaining its form, energy, momentum and other quasi-particle properties. Just so, the vector \( \vec{r} \) or \( x \) has definitively physical significance, and denotes exactly the positions of the particle at time \( t \). Then, the wave-function \( \phi(\vec{r}, t) \) or \( \phi(x, t) \) can represent exactly the states of the particle at the position \( \vec{r} \) or \( x \) and time \( t \). These features are consistent with the concept of particles. Thus the feature of corpuscle of the particle is displayed clearly and outright.
we now re-write the solution Eq. (16) as the following form
\[ \phi(x,t) = \frac{2}{\sqrt{b}} k \sech \left( 2k \left[ (x' - x_0) - v_c t \right] \right) e^{i \left[ k(x - x_0) - v_c t \right] / 2} \tag{17} \]
where \( 2^{1/2} k/b = A_0 \), \( A_0 = \sqrt{\frac{v_c^2 - 2v_e v_c}{2b}} \), \( v_e \) is the group velocity of the electron, \( v_c \) is the phase speed of the carrier wave. For a certain system, \( v_e \) and \( v_c \) are determinant and do not change with time. From the above results we see clearly that the particle is a soliton. According to the soliton theory\[22-23\], the soliton has determinant mass, momentum and energy, which can be represented by\[14,22-23\]
\[ N_x = \int_{-\infty}^{\infty} |\phi|^2 dx' = 2\sqrt{2} A_0 \]
\[ p = -i \int_{-\infty}^{\infty} (\phi^* \phi_x - \phi \phi_x^*) dx' = 2\sqrt{2} A_0 v_c = N_x v_c = \text{const} \]
\[ E = \int_{-\infty}^{\infty} \left[ |\phi|^2 - \frac{1}{2} |\phi|^4 \right] dx' = E_0 + \frac{1}{2} M_{sol} v^2 \tag{18} \]
in such a case, where \( M_{sol} = N_x = 2\sqrt{2} A_0 \) is just effective mass of the particle, which is a constant. Obviously, the energy, mass and momentum of the particle cannot be dispersed in its motion. This manifests again that the particle represented by \( \phi(x',t) \) or \( \phi(x,t) \) in the system has a particle feature. This means that the nonlinear interaction, \( b |\phi|^2 \phi \), related to the wave function of the particle balance and suppress really the dispersion effect of the kinetic term in Eq. (1) to make the particle to be eventually localized\[13-14,22-26\].

However, the envelope of the solution in Eq.(16) or (17) is a solitary wave. It has a certain wavevector and frequency as shown in Fig. 1(b), and can propagate in space-time, which is accompanied with the carrier wave. The feature of propagation depends only on the concrete nature of the particle. Figure 1(b) shows the width of the frequency spectrum of the envelope \( \phi(x,t) \), the frequency spectrum has a localized structure around the carrier frequency \( \omega_0 \). Thus, the particle has exactly a wave-corpuscle duality\[13-14,22-26\], which is first obtained. The Equation (16) or (17) and Figure 1a are just a most beautiful and perfect representation of wave-corpuscle duality of the particle. This consists also of de Broglie relation of wave-corpuscle duality and Davisson and Germer’s experimental result of electron diffraction on double seam in 1927\[6-8\].

From the above investigations we see clearly that the nonlinear interactions result in the localization and wave-corpuscle duality of the microscopic particles, which are completely different from those of the quantum mechanics, thus this could solve the difficulties, problems and disputations existed in the quantum mechanics. In this theoretical framework the states and properties of microscopic particles are described by the nonlinear Schrodinger equation(7), instead of the linear Schrodinger equation (1). Therefore, when the nonlinear interactions are introduced in the quantum mechanics, not only the quantum mechanics but also the natures of microscopic particles are considerably changed.

III. THE MECHANISM OF GENERATION OF NONLINEAR INTERACTION OF MICROSCOPIC PARTICLES IN THE QUANTUM SYSTEMS

At present, a key problem is how does the above interaction generate? or speaking, what is the mechanism of generation of the nonlinear interaction? Why does the quantum mechanics consider not this interaction? For replying these problems we return to look at the method solving the questions of quantum mechanics, especially in the atoms and molecules of many electrons and complicated solids, using linear Schrodinger equation (1). As known, we, in general, use an averagely external applied potential \( V(r, t) \) to replace all real interactions among the particles or between the particles and background fields including the nonlinear interactions through applying average field approximation, free electron approximation, Born-
Oppenheimer approximation, Hartree-Fock approximation, and so on [6-8, 13-16]. The essences of these approximations are to freeze or blot or simplify the real motions of background fields or lattices in solids and of nuclei in atoms and molecules. Thus the quantum mechanics simplify the motions of microscopic particles, the Schrodinger equation include not truly all practical interactions among the particles and between the particles and background fields or nuclei, then the nonlinear interactions of particles are also blotted and neglected. Thus the microscopic particles have only the wave feature. For example, the Coulomb potential of $V(r) = -\frac{e^2}{r}$, which was used in the calculation of the states of the electron in hydrogen atom using Eq. (1), is just an average potential, in which the origin of coordinate is fitted at the nucleon, then the nucleon was thought to be static, thus the inherent motion of the nucleon is completely frozen and rubbed. Obviously, this is not reasonable and appropriate to practical case. In fact, the nucleon is always in harmonic vibration around its equilibrium position. The motion of the nucleon result necessarily in change of distance between the electron and nucleon, thus the Coulomb potential is correspondingly varied, i.e., the Coulomb potential relates closely to the positions of the nucleon, then the nature of the electron are also changed. We now assume that the displacement of the nucleon leaving the equilibrium position is $\vec{u}$, thus the Coulomb potential between the electron and nucleon should now be the form of $V(r, \vec{u}) = -\frac{e^2}{r + \vec{u}}$, instead of $V(r) = -\frac{e^2}{r}$. Since

$$\frac{1}{r+u} = \frac{1}{r(1+u/r)} = \left(\frac{1}{r} - \frac{u}{r^2} + \ldots\right) = \frac{1}{r} - \frac{1}{r^2} \frac{d}{dr} \left(\frac{1}{r} + \ldots\right),$$

then the motion equation of the electron in hydrogen atom should be of the form

$$i\hbar \frac{\partial}{\partial t} \phi = -\frac{\hbar^2}{2m} \nabla^2 \phi - \frac{e^2}{r} \phi - \chi \phi u$$

(20)

We here use $\phi(r, t)$ to represent the state of the electron due to its difference from those denoted by Eq. (1). Where $\chi = -\frac{e^2}{d} \frac{d}{dr} \left(-\frac{1}{r}\right) = \frac{e^2}{r^2}$ denotes a coupling coefficient between the electron and nucleon, in essence, is the Coulomb force between them, $m$ is mass of the electron. Obviously, the change of the Coulomb force result in the variations of state of both the electron and the nucleon. Thus the nucleon is now in the forced vibration, instead of harmonic vibration. On account of the change of the Coulomb force relates closely to both the distance between them and the density of the electrons, then the Coulomb force accepted by the nucleon are different, when the electron is in different positions. Thus the vibration of the nucleon are changed under a action of Coulomb force arising from the gradient fluctuation of distribution or density of the electron. In such a case it is easy to obtain that the vibration of the nucleon should satisfy the equation:

$$M \left(\frac{\partial^2 u}{\partial t^2} - v_0^2 \frac{\partial^2 u}{\partial x_i^2}\right) = \alpha^2 a_0^2 \chi \frac{\partial^2}{\partial x_i^2} \left|\phi\right|^2, (i = 1, 2, 3)$$

(21)

in continuity approximation, where $M$ is mass of the nucleon, $a_0$ is the Bohr radius, $\alpha$ is a fraction. Equation (21) shows that the Coulomb force accepted by the nucleon is represented by $(\alpha a_0^2 \chi \frac{\partial^2}{\partial x_i^2} \left|\phi\right|^2$ in its motion process. From (21) we can obtain

$$u = -\frac{\chi a_0^2}{M (\nu^2 - \nu_0^2)} \left|\phi\right|^2$$

(22)

Inserting Eq. (22) into Eq. (20) we obtain the equation of motion of the electron to be

$$i\hbar \frac{\partial}{\partial t} \phi = -\frac{\hbar^2}{2m} \nabla^2 \phi - \frac{e^2}{r} \phi - b \left|\phi\right|^2 \phi$$

(23)

This is a concrete representation of the nonlinear Schrodinger equation of Eq. (23) in the hydrogen atom, where $b = \frac{\alpha^2 a_0^2 \chi^2}{M (\nu^2 - \nu_0^2)}$ is the nonlinear coupling coefficient or interaction energy between the electron and nucleon in the system and is proportional to the square of Coulomb force and inversely proportional to kinetic energy of the nucleon.

From the above investigation we know that if and only we consider seriously and completely real motions and interactions of the electron and nucleon in hydrogen atom, then the nonlinear interaction of the electron, $b \left|\phi\right|^2 \phi$, will occur certainly, thus the motion of the electron satisfies the nonlinear Schrodinger equation (7) or (23), instead of linear Schrodinger equation (1). In such a case, the electron is no longer a wave, and is localized, has a wave-corpuscle duality. Obviously, the coupling interaction in the hydrogen atom is the Coulomb force between the electron and nucleon, which exists permanently, thus the nonlinear interaction, $b \left|\phi\right|^2 \phi$, in Eq. (23) also exists always. Therefore, the motion of the electron in the hydrogen atom should be depicted by the nonlinear Schrodinger equation (23), instead of linear Schrodinger equation (1).

The generation of nonlinear interaction mentioned above in the hydrogen atom is referred to as the self-interaction mechanism. Naturally, it can generalize into other atoms and molecules of many electrons and some solid matter. The general form of
this mechanism can be represented by the following equations of motion and interactions of particles. The motion of studied particle, which is denoted by $\phi(\vec{r}, t)$, can be depicted by

$$i\hbar \frac{\partial}{\partial t} \phi = -\frac{\hbar^2}{2m} \nabla^2 \phi + V(\vec{r}, t)\phi - \chi \phi F$$  \hspace{1cm} (24)

The motion of the background field or nuclei is represented by

$$M\left(\frac{\partial^2}{\partial t^2} - v_0^2 \frac{\partial^2}{\partial x_i^2}\right) = \chi \frac{\partial^2}{\partial x_i^2} |\phi|^2, \hspace{1cm} (i = 1, 2, 3)$$  \hspace{1cm} (25)

where $F$ is the wave vector of the background field (phonon) or nuclei, $\chi$ is a coupling coefficient between them. The relation between the two motion modes is

$$F = \frac{\chi}{M(v^2 - v_0^2)} |\phi|^2$$  \hspace{1cm} (26)

Thus the nonlinear Schrodinger equation of the studied particle can be obtained as follows

$$i\hbar \frac{\partial}{\partial t} \phi = -\frac{\hbar^2}{2m} \nabla^2 \phi + V(\vec{r}, t)\phi - b|\phi|^2 \phi$$ \hspace{1cm} (27)

where $b = \frac{\chi^2}{M(v^2 - v_0^2)}$.

In fact, the nonlinear interaction of the particles, $b|\phi|^2 \phi$, can be also generated by another mechanism in solid, condensed matter, polymer and macromolecules, in which the interaction between the particle and background field, such as, the lattice field, results in the deformation of the latter, which provides a potential well for the particle and makes it localization. Therefore, we refer to it as self-trapping mechanism. In the mechanism the microscopic particles are possibly the electron or exciton or polaron, its interactions with the background field may be the electron-phonon coupling, or dipole-dipole interaction, and so on. The dynamical equations of the electron (or, exciton or polaron) and vibration of the background field or lattice in this model can be often represented, respectively, by

$$i\hbar \frac{\partial}{\partial t} \phi = -\frac{\hbar^2}{2m} \nabla^2 \phi + V(\vec{r}, t)\phi - \chi \phi \frac{\partial F}{\partial x_i}$$ \hspace{1cm} (28)

and

$$M\left(\frac{\partial^2}{\partial t^2} - v_0^2 \frac{\partial^2}{\partial x_i^2}\right) = \chi \frac{\partial}{\partial x_i} |\phi|^2$$ \hspace{1cm} (29)

where $\phi$ is the wave function of state of the electron, $F$ is the characteristic quantity of vibration of the background field (or phonon). The coupling between the two modes of motion is caused by the deformation of the background field through the electron-phonon coupling, or dipole-dipole interaction. The equation (28) describes the influences of change of the background field arising from its deformation on the states of the electrons. Equation (29) indicates the changes of vibration of the background field with velocity $v_0$ under the influence of the distribution or density of the electrons. Where $\chi$ is the coupling coefficient between them and represents the change of interaction energy between the electron and background field due to an unit variation of the field. From Eq.(29) we can obtain

$$\frac{\partial F}{\partial x_i} = \frac{\chi^2}{M(v^2 - v_0^2)} |\phi|^2$$ \hspace{1cm} (30)

Inserting (30) into Eq.(28) yields the nonlinear Schrodinger equation (7), where $b = \frac{\chi^2}{M(v^2 - v_0^2)}$ is a nonlinear coupling coefficient. The equation (7) is just the nonlinear Schrodinger equation of the electron in the system in the case, where the nonlinear interaction $b|\phi|^2 \phi$ is formed by the electron-phonon or dipole-dipole interactions between the motion of electron (or exciton or polaron) and vibration of background field in virtue of mechanism of self-trapping or self-condensation. Thus the electron(or exciton or polaron) is localized and has a wave-corpuscle duality. In such a case these particles satisfy no longer the quantum mechanics, but are described by the nonlinear Schrodinger equation (7).

From the above investigations from Eq.(19) to Eq.(30) we see that if and if only the real motions of these particles and the interactions between them are considered, then the nonlinear interactions exist certainly in the systems. Thus the motions of the microscopic particles should be depicted by the nonlinear Schrodinger equation (7), cannot be manifested by linear Schrodinger equation (1) or quantum mechanics. On account of any physical systems including the hydrogen atom are composed of many electrons and many bodies, in which the interactions among the particles or between the particles and background fields exist always, thus the nonlinear interactions mentioned above occur permanently. Therefore, the nonlinear Schrodinger equations are a correct and universal dynamic equation for the microscopic particles in all physical systems. This is just a new studied result. However, the microscopic particles in the atoms and molecules of many electrons and bodies and solids should be, in general, described by the following nonlinear Schrodinger equation:

$$i\hbar \frac{\partial}{\partial t} \phi = -\frac{\hbar^2}{2m} \nabla^2 \phi + V(\vec{r}, t)\phi - b|\phi|^2 \phi + A(\phi(\vec{r}, t))$$ \hspace{1cm} (31)

where $A(\phi(\vec{r}, t))$ is a function of $\phi(\vec{r}, t)$ and represents differently complicated interactions related to the wave
functions including higher order nonlinear interactions, such as $D\phi^3 \phi$, which is caused by the interactions of many electrons or many bodies. Thus we can calculate and study correctly and really the states and properties of motion of the microscopic particles in atoms, molecules, solids, condensed matter, polymers and biomolecules, and so on, using Eq.(7) or Eq.(31).

IV. The Relation between the Action and Counteraction of the Nonlinear Interaction in Nonlinear Quantum Systems

As known, there is a law of the action and counteraction for the force in classical mechanics, this law shows that the sizes of the acted force and counteracted force are equal, but their directions are inverse with each other. Is this law for the above nonlinear interactions correct? We have the reasons to doubt the correctness for the nonlinear interactions because the nonlinear interactions occur in microscopic or quantum systems and relate to the wave function of state of the particles or bodies. This is the reason why we here investigate seriously and deeply the relation between the action and counteraction of the nonlinear interaction.

We first establish and determine this relation from Eqs.(7) and (28)-(30) in one-dimensional case. We know that at $V(x,t)=0$ in one-dimensional case, equation (7) becomes as Eq.(8), then the state of the particle described by Eq.(8) is represented in Eq.(16) or (17). Inserting Eq.(16) into Eq.(30) we can obtain

$$\text{Eq.(30), and the particle and the background field also are simultaneously localized under the actions of the nonlinear interactions, when the motions of the particle and the background field and their interactions are simultaneously considered as Eqs.(28)-(29) in the self-trapping mechanism. The nonlinear interactions accepted by the particle and the background field are represented by } -b\phi^2 \phi \text{ and } +D'F^3 \text{, respectively, their symbols are inverse, but the sizes are different because they relate all to themselves wave functions of states and have same powers, respectively, as well as } b\phi^2 \phi \text{ has the dimension of energy, but } D'F^3 \text{ is the dimension of force. This is just the relation between the action and counteraction for the nonlinear interactions in the self-trapping mechanism in nonlinear quantum systems. Therefore, this rule is different from the law of the action and counteraction for the force in classical mechanics.}

We now study the relation between the action and counteraction for the nonlinear interactions in the self-interaction mechanism in nonlinear quantum systems from Eqs.(7) –(16) and Eqs.(24)-(27) in one-dimensional case. Inserting Eq.(16) into Eq.(26) we can obtain

$$F(x,t) = \frac{A^2 \chi}{M(v^2 - v_0^2)} \sec h^2 \left\{ A_0 \sqrt{D'F} \frac{\hbar}{h} [(x - x_0) - vt] \right\}$$

(32)

Substituting Eqs.(16) and (32) into Eq.(29) we can get

$$M \left( \frac{\partial^2 F}{\partial t^2} - v_0^2 \frac{\partial^2 F}{\partial x^2} \right) = -CF + D'F^3$$

(33)

where $C = \frac{2A^2 M(v^2 - v_0^2)b m}{h^2}$, $D' = \frac{2M^3(v^2 - v_0^2)^2(b m)^2}{\chi^2 h^4}$

Equation (33) is just the equation of motion of the background field. Obviously, it is also a nonlinear equation of the field particle, its solution is represented by Eq.(34), which is also a soliton, thus the field particle is also localized. Clearly, its localization is also caused by the nonlinear interaction $D'F^2$. Thus we can draw also a conclusion from the above investigations in Eqs.(7)-(16), Eqs.(24)-(27) and (34)-(35) that the nonlinear interactions accepted by the particle and the background field in the system generate simultaneously through the cross correlation in Eq.(26), and the particle and the background field particle also are simultaneously localized under the actions of the nonlinear interactions, when the motions of the particle and the background field particle and their interactions are simultaneously considered as Eq.(24)-(25) in the self-interaction mechanism. The nonlinear interactions accepted by the particle and the background field are represented by $-b\phi^2 \phi \text{ and } -D'F^2 \text{, respectively, the former is the dimension of energy, the latter is the$
dimension of force. As distinct from the above results in the self-trapping mechanism, the symbol of the nonlinear interactions accepted by the particle is same with those of the background field, but their powers are also different, the former is three powers, the latter is two powers, therefore, their sizes are also different, and the localization of the background field particle, which is denoted by Eq.(34), is enhanced in the self-interaction mechanism.

From this investigation of the relation of the action and counteraction of the nonlinear interaction we know that the interactional particles or bodies or matters (background fields) have and can accept simultaneously the nonlinear interactions in the nonlinear quantum systems, but their nonlinear interactions accepted do not completely satisfy the law of action and counteraction of the forces in the classical mechanics, although the microscopic particles in the nonlinear quantum systems have also corpuscle feature. This shows clearly that the natures of the particles in the nonlinear quantum systems are different from those of classical particles. This conclusion is of important interest to physics and nonlinear science.

V. Conclusion

We here first studied and revealed the difficulties, limitations and approximations of the quantum mechanics and the roots and reasons of these difficulties. The reasons of these difficulties and problems are that the quantum mechanics simplifies and blots out the real motions of the microscopic particles and background field particle and interactions between them including the nonlinear interactions, and use an average fields to replace a much variety of real interactions by using various different approximate ways,. thus the particles have only a wave feature, not corpuscle feature. In view of these problems and difficulties we add the nonlinear interaction, $\hbar \phi \phi$ into the dynamic equation of the microscopic particles, and use a nonlinear Schrodinger equation to depict their states of motion. Thus the wave or dispersive effect of the microscopic particles is suppressed by the nonlinear interactions, its localization and wave- corpuscle duality are displayed and exhibited naturally. Subsequently, we seek and investigate the mechanism of generation of the nonlinear interactions of the particles, and find that the nonlinear interactions of the particles exist always, when the real motions of the particles and background fields and the interactions between them are considered simultaneously. On the basis of analyses of dynamics of the electrons and nucleon or lattice (background) fields in atoms or lattices we give two mechanisms of self-interaction and self-trapping for the generation of nonlinear interactions and corresponding representations. of their dynamic equations. Finally we discuss the properties of the nonlinear interactions in the two mechanisms and obtain the relations between the action and counteraction of the two nonlinear interactions of the particles and background fields. We find that although the nonlinear interaction accepted by the particles and background field particle can generated simultaneously in the nonlinear quantum systems, the two nonlinear interactions do not completely satisfy the law of action and counteraction of the forces in the classical mechanics. This shows clearly that the natures of the microscopic particles in the nonlinear quantum systems are different from those of classical particles, even though the microscopic particles have also the corpuscle feature in nonlinear quantum systems. This shows clearly that the natures of the particles in the nonlinear quantum systems are different from those of classical particles. This conclusion is of important interest to physics and nonlinear science.

In the meanwhile, we obtained a correct and universal nonlinear Schrodinger equation in Eq.(31), which considers various interactions involving the different nonlinear interactions in the atoms and molecules of many electrons, solid, condensed matter, polymers, biomolecules. Thus we can calculate truthfully the natures and properties of the microscopic particles in these complicated systems by the nonlinear Schrodinger equation, but original quantum mechanics is difficult or impossible to describe these systems.

Régénèrences

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