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Certain Results on Bicomplex Topologies and their Comparison

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CERTAINRESULTSONBICOMPLEXTOPOLOGIESANDTHEIRCOMPARISON

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Ref

5. Ringleb, F.: Beitrage Zur Funktionen theorie in Hyperkomplexon Systemen, I, Rend. Circ. Mat. Palermo, 57 (1933), 311-340.

Certain Results on Bicomplex Topologies and their Comparison

Akhil Prakash [α] & Prabhat Kumar [σ]

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I. Introduction

In 1892, Corrado Segre (1860-1924) published a paper [6] in which he treated an infinite set of Algebras whose elements he called bicomplex numbers, tricomplex numbers,…., n-complex numbers. A bicomplex number is an element of the form $(x_1+i_1x_2) + i_2 (x_3+i_1x_4)$, where $x_1,\ldots, x_4$ are real numbers, $i_1^2 = i_2^2 = -1$ and $i_1i_2 = i_2i_1$.

Segre showed that every bicomplex number $z_1+i_2z_2$ can be represented as the complex combination

$$(z_1-i_1z_2)\left[\frac{1+i_1i_2}{2}\right]+(z_1+i_1z_2)\left[\frac{1-i_1i_2}{2}\right]$$

Srivastava [8] introduced the notations ${}^1\xi$ and ${}^2\xi$ for the idempotent components of the bicomplex number $\xi = z_1+i_2z_2$, so that

$$\xi = {}^1\xi.\frac{1+i_1i_2}{2}+{}^2\xi.\frac{1-i_1i_2}{2}$$

Michiji Futagawa seems to have been the first to consider the theory of functions of a bicomplex variable [1, 2] in 1928 and 1932.

The hyper complex system of **Ringleb** [5] is more general than the Algebras; he showed in 1933 that Futagawa system is a special case of his own.

In 1953 James D. Riley published a paper [4] entitled "Contributions to theory of functions of a bicomplex variable".

Throughout, the symbols $\mathbb{C}_2$, $\mathbb{C}_1$, $\mathbb{C}_0$ denote the set of all bicomplex, complex and real numbers respectively.

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In $\mathbb{C}_2$-besides 0 and 1- there are exactly two non-trivial idempotent elements denoted as e_1 and e_2 and defined as

$$e_1 = \frac{1+i_1i_2}{2} \text{ and } e_2 = \frac{1-i_1i_2}{2}$$

Obviously $(e_1)^n = e_1$, $(e_2)^n = e_2$

$$e_1 + e_2 = 1,\ e_1.e_2 = 0$$

Every bicomplex number ξ has unique idempotent representation as complex combination of e_1 and e_2 as follows

$$\xi = z_1+i_2z_2 = (z_1\text{-}i_1z_2)e_1 + (z_1+i_1z_2)e_2$$

The complex numbers $(z_1\text{-}i_1z_2)$ and $(z_1+i_1z_2)$ are called idempotent component of ξ, and are denoted by ${}^1\xi$ and ${}^2\xi$ respectively *(cf. Srivastava [8]).*

Thus $\xi = {}^1\xi\ e_1 + {}^2\xi\ e_2$

a) h_1, h_2 image and Cartesian idempotent set

The h_1 and h_2 image of a set X are denoted as ${}^1X$ and ${}^2X$ respectively and defined as

$$h_1(X) = {}^1X = \{z:\ ze_1+we_2 \in X\} = \{{}^1\xi : \xi\in X\}$$

$$h_2(X) = {}^2X = \{w:\ ze_1+we_2 \in X\} = \{{}^2\xi : \xi\in X\}$$

The Cartesian idempotent product of ${}^1X$ and ${}^2X$ is the set which is the subset of $\mathbb{C}_2$ and denoted as ${}^1X \times_e {}^2X$ and defined as

$${}^1X \times_e {}^2X = \{ze_1+we_2:\ z \in {}^1X,\ w \in {}^2X\}$$

If $X = {}^1X \times_e {}^2X$ then X is said to be Cartesian idempotent set *(cf. Srivastava [8]).*

II. Certain Results from Topologies on Bicomplex Space

a) Norm, Complex and Idempotent topologies on $\mathbb{C}_2$[9].

2.1.1 Norm topology

The norm of a bicomplex number $\xi = z_1+i_2z_2 = x_1+i_1x_2+i_2x_3+i_1i_2x_4 = {}^1\xi\ e_1+{}^2\xi\ e_2$ is defined as

$$\|\xi\| = (x_1^2 + x_2^2+x_3^2 + x_4^2)^{1/2}$$

$$= (|z_1|^2+|z_2|^2)^{1/2}$$

$$= \sqrt{\frac{|{}^1\xi|^2 + |{}^2\xi|^2}{2}}$$

Since $\mathbb{C}_2$ is modified normed algebra w.r.t this norm therefore for $\delta > 0$, the δ-ball centered at x is the set

$B(x,\delta) = \{y\in\mathbb{C}_2 : \|x - y\|<\delta\}$ of all points y whose distance from x is less than δ, it is called the δ-ball centered at x.

The collection '$\boldsymbol{B_N}$' of all δ-balls $B(x,\delta)$, for $x\in\mathbb{C}_2$ and $\delta>0$ is a basis for a topology on $\mathbb{C}_2$. The topology generated by $\boldsymbol{B_N}$ is called norm topology on $\mathbb{C}_2$ and denoted by $\Im_N$.

Ref

9. Srivastava, Rajiv K.: Certain topological aspects of the Bicomplex space, Bull. Pure & Appl. Maths., 2(2)(2008),222-234.

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2.1.2 Theorem

If X is a Cartesian idempotent set in $\mathbb{C}_2$ then X is open (w.r.t. norm topology) if and only if ${}^1X$ and ${}^2X$ are open in complexplane **(cf. Price [3])**.

2.1.3 Complex topology

The norm of a complex number 'z' is defined as $\|z\| = |z|$

Since $\mathbb{C}_1$ is a normed algebra w.r.t. this norm therefore the collection '$\boldsymbol{B}$' of all circular disk $S(z,\delta)$, $z\in\mathbb{C}_1$ and $\delta > 0$ will be a basis for a topology on $\mathbb{C}_1$, where $S(z,\delta) = \{w\in\mathbb{C}_1: |z-w|<\delta\}$Therefore

$\boldsymbol{B}\times\boldsymbol{B} = \{S_1\times S_2: S_1, S_2\in\boldsymbol{B}\}$ will be a basis for some topology on $\mathbb{C}_1\times\mathbb{C}_1$.Since $C_2\cong C_1\times C_1$ therefore $\boldsymbol{B}_C = \{S_1\times_C S_2: S_1, S_2\in\boldsymbol{B}\}$ will be a basis for some topology on $\mathbb{C}_2$, where

$$S_1\times_C S_2 = \{\eta = w_1+i_2w_2: w_1\in S_1,\ w_2\in S_2\}$$

If $S_1 = S_1(z_1,r_1)$ and $S_2 = S_2(z_2,r_2)$

Then $S_1\times_C S_2 = S_1(z_1,r_1)\times_C S_2(z_2,r_2)$

$$= \{\eta = w_1+i_2w_2: |z_1-w_1|< r_1,\ |z_2-w_2|< r_2\}$$

The set $S_1(z_1,r_1)\times_C S_2(z_2,r_2)$ is denoted by $C(\xi = z_1+i_2z_2;r_1,r_2)$ and this set $C(\xi = z_1+i_2z_2;\ r_1,r_2)$ is called *open complex discus* centered at ξ and associated radii r_1 and r_2.

Therefore

$$C(\xi = z_1+i_2z_2;\ r_1,\ r_2) = \{\eta = w_1+i_2w_2: |z_1-w_1|< r_1,\ |z_2-w_2|< r_2\}$$

Thus $\boldsymbol{B}_C =$ Set of all open complex discus

$$= \{C(\xi;\ r_1,\ r_2): \xi\in\mathbb{C}_2 \text{ and } r_1,\ r_2 > 0\}$$

The topology generated by '$\boldsymbol{B}_C$' is called complex topology and denoted by T_C.

2.1.4 Idempotent topology

Since $\boldsymbol{B}\times\boldsymbol{B} = \{S_1\times S_2: S_1, S_2\in\boldsymbol{B}\}$ is a basis for some topology on $\mathbb{C}_1\times\mathbb{C}_1$ and $C_2\cong C_1\times C_1$ therefore $\boldsymbol{B}_I = \{S_1\times_e S_2: S_1, S_2\in\boldsymbol{B}\}$ will be a basis for some topology on $\mathbb{C}_2$, Where

$$S_1\times_e S_2 = \{\eta = {}^1\eta\, e_1 + {}^2\eta\, e_2: {}^1\eta\in S_1,\ {}^2\eta\in S_2\}$$

If $S_1 = S_1(z_1,r_1)$ and $S_2 = S_2(z_2,r_2)$

Then $S_1\times_e S_2 = S_1(z_1,r_1)\times_e S_2(z_2,r_2)$

$= \{\eta = {}^1\eta\, e_1 + {}^2\eta\, e_2: \left|{}^1\eta - z_1\right| < r_1,\ \left|{}^2\eta - z_2\right| < r_2\}$. The set $S_1(z_1,r_1)\times_e S_2(z_2,r_2)$ is denoted by $D(\xi = z_1e_1+z_2e_2;\ r_1,r_2)$ and this set $D(\xi = z_1e_1+z_2e_2;\ r_1,r_2)$ is called *open idempotent discus* centered at ξ and associated radii r_1 and r_2.

Therefore $D(\xi;\ r_1,\ r_2) = \{\eta: \left|{}^1\eta - {}^1\xi\right| < r_1, \left|{}^2\eta - {}^2\xi\right| < r_2\}$

Thus $\boldsymbol{B}_I =$ Set of all open idempotent discus

$$= \{D(\xi;\ r_1,\ r_2): \xi\in\mathbb{C}_2 \text{ and } r_1,\ r_2 > 0\}$$

The topology generated by '$\boldsymbol{B}_I$' is called idempotent topology and denoted by T_I.

Ref

3. Price, G.B.: An introduction to multicomplex spaces and Functions, Marcel Dekker Inc., New York, 1991.

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b) Order topology on $\mathbb{C}_2$

Singh [7] has developed certain orders on $\mathbb{C}_2$. He defined three types of ordering in $\mathbb{C}_2$, viz., Real dictionary order $<_{R,}$ Complex dictionary order $<_{c,}$ Idempotent dictionary order$<_{ID}$. With the help of these three relations he has defined three order topologies on $\mathbb{C}_2$. The order topology induced by real dictionary order is called as real order topology $\mathfrak{T}_1$, the order topology generated by Complex dictionary order is called complex order topology $\mathfrak{T}_1^*$ and the topology induced by Idempotent dictionary order is called idempotent order topology $\mathfrak{T}_1^{\#}$ on $\mathbb{C}_2$.

In the present paper, $\boldsymbol{B_1}$, $\boldsymbol{B_1^*}$ ***and*** $\boldsymbol{B_1^{\#}}$ denotes the basis of $\mathfrak{T}_1$, $\mathfrak{T}_1^*$and$\mathfrak{T}_1^{\#}$ respectively. The sets $(\xi, \eta)_R$, $(\xi, \eta)_C$ and $(\xi, \eta)_{ID}$ are the open interval with respect to Real dictionary order, Complex dictionary order and Idempotent dictionary order relation.

c) Product and metric topology on $\mathbb{C}_2$

Singh [7]defined three product topologies on $\mathbb{C}_2$,viz., Real product topology $\mathfrak{T}_2$,Complex product topology $\mathfrak{T}_2^*$ and Idempotent product topology $\mathfrak{T}_2^{\#}$.
$\boldsymbol{B_2}$, $\boldsymbol{B_2^*}$ and $\boldsymbol{B_2^{\#}}$ denotes the basis of $\mathfrak{T}_2$, $\mathfrak{T}_2^*$ and $\mathfrak{T}_2^{\#}$ respectively. Where

$$\boldsymbol{B_2} = \{S_1 \times_R S_2 \times_R S_3 \times_R S_4 : S_1, S_2, S_3, S_4 \text{ are in } \widetilde{\boldsymbol{B}}\}$$

$$S_1 \times_R S_2 \times_R S_3 \times_R S_4 = \{\xi = x_1 + i_1 x_2 + i_2 x_3 + i_1 i_2 x_4 : x_1 \in S_1,\ x_2 \in S_2,\ x_3 \in S_3,\ x_4 \in S_4\}$$

$\widetilde{\boldsymbol{B}}$ is the collection all open intervals in $\mathbb{C}_0$

$$\boldsymbol{B_2^*} = \{S_5 \times_c S_6 : S_5, S_6 \in \boldsymbol{B^*}\}$$

$\boldsymbol{B^*}$ is the collection of all open intervals in $\mathbb{C}_1$

$$S_5 \times_c S_6 = \{\xi = z_1 + i_2 z_2 : z_1 \in S_5,\ z_2 \in S_6\}$$

$$\boldsymbol{B_2^{\#}} = \{S_7 \times_e S_8 : S_7, S_8 \in \boldsymbol{B^*}\}$$

$$S_7 \times_e S_8 = \{z_1 e_1 + z_2 e_2 : z_1 \in S_7,\ z_2 \in S_8\}$$

There are three metrics on $\mathbb{C}_2$, viz., real metric, complex metric and idempotent metric. With the help of these three metric *Singh [7]* defined three metric topologies on $\mathbb{C}_2$.The topology generated by the real metric is known as the real metric topology $\mathfrak{T}_3$, the topology generated by the complex metric is called complex metric topology $\mathfrak{T}_3^*$ and the topology generated by the idempotent metric is called idempotent metric topology $\mathfrak{T}_3^{\#}$.

In this present paper, $\boldsymbol{B_3}$, $\boldsymbol{B_3^*}$and $\boldsymbol{B_3^{\#}}$ denotes the basis of $\mathfrak{T}_3$, $\mathfrak{T}_3^*$ and $\mathfrak{T}_3^{\#}$ respectively.

2.3.1 Theorem

The norm topology $\mathfrak{T}_N$ and idempotent topology $\mathfrak{T}_I$ on $\mathbb{C}_2$ are equivalent to each other *Srivastava [9].*

2.3.2 Theorem

The norm topology $\mathfrak{T}_N$ and complex topology $\mathfrak{T}_C$ on $\mathbb{C}_2$ are equivalent to each other *Srivastava [9].*

Ref

7. Singh, Sukhdev: Certain order topologies on the bicomplex Space,M.Phil Dissertation, Dr. B. R. Ambedkar University, Agra, 2009.

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Theorems 2.3.1 and 2.3.2 imply that

2.3.3 Corollary

The complex topology τ_C and idempotent topology τ_I on $\mathbb{C}_2$ are equivalent.

2.3.4 Theorem

The real order topology τ_1 and real product topology τ_2 on $\mathbb{C}_2$ are equivalent to each other [7].

2.3.5 Theorem

The real product topology τ_2 and real metric topology τ_3 on $\mathbb{C}_2$ are equivalent to each other [7].

Theorems 2.3.4 and 2.3.5 imply that

2.3.6 Corollary

The real order topology τ_1 and real metric topology τ_3 on $\mathbb{C}_2$ are equivalent to each other.

2.3.7 Theorem

The complex order topology τ_1^* and complex product topology τ_2^* on $\mathbb{C}_2$ are equivalent to each other [7].

2.3.8 Theorem

The complex product topology τ_2^* and complex metric topology τ_3^* on $\mathbb{C}_2$ are equivalent to each other [7].

In view of Theorems 2.3.7 and 2.3.8, we have

2.3.9 Corollary

The complex order topology τ_1^* and complex metric topology τ_3^* on $\mathbb{C}_2$ are equivalent to each other.

2.3.10 Theorem

The idempotent order topology $\tau_1^{\#}$ and idempotent product topology $\tau_2^{\#}$ on $\mathbb{C}_2$ are equivalent to each other [7].

2.3.11 Theorem

The idempotent product topology $\tau_2^{\#}$ and idempotent metric topology $\tau_3^{\#}$ on $\mathbb{C}_2$ are equivalent to each other [7].

As the idempotent order topology on $\mathbb{C}_2$ and the idempotent product topology on $\mathbb{C}_2$ are equivalent to each other. Also, the idempotent product topology and idempotent metric topology are equivalent to each other, therefore we have.

2.3.12 Corollary

The idempotent order topology $\tau_1^{\#}$ and idempotent metric topology $\tau_3^{\#}$ on $\mathbb{C}_2$ are equivalent to each other.

2.3.13 Corollary

As the real dictionary ordering of the bicomplex number is same as the complex dictionary ordering of the bicomplex numbers therefore the real order topology τ_1 is equivalent to the complex order topology τ_1^* on C_2.

Theorems 2.3.4, 2.3.5, 2.3.7, 2.3.8 and Corollary 2.3.6, 2.3.9, 2.3.13 imply that

2.3.14 Corollary

The real order, real product, real metric, complex order, complex product and complex metric topology on $\mathbb{C}_2$ are equivalent to each other.

Ref

7. Singh, Sukhdev: Certain order topologies on the bicomplex Space,M.Phil Dissertation, Dr. B. R. Ambedkar University, Agra, 2009.

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2.3.15 Corollary

The real dictionary ordering and complex ordering of the bicomplex numbers is different from the idempotent ordering of the bicomplex numbers therefore the idempotent order topology can be neither equivalent to the real order topology nor to the complex order topology on C_2.

III. Comparison of Various Topologies on Bicomplex Space

Notes

This section is our contribution to the theory of bicomplex topology and contains some important results from topological structures on the bicomplex space. In this section, we have tried to develop some relation between various topological structures on the bicomplex space.

a) Comparison of the idempotent order topology and norm topology on the bicomplex space

3.1.1 Lemma

The set $({}^1\xi e_1 + ({}^2\xi - i_1\delta)e_2, {}^1\xi e_1 + ({}^2\xi + i_1\delta)e_2)_{ID}$ is the proper subset of $B(\xi, r)$ where $0 < \delta < \sqrt{2}$and $r > 0$

Proof- Let suppose

$$\eta \in ({}^1\xi e_1 + ({}^2\xi - i_1\delta)e_2, {}^1\xi e_1 + ({}^2\xi + i_1\delta)e_2)_{ID} \qquad \ldots(1)$$

$$\Rightarrow {}^1\eta = {}^1\xi \text{ and } {}^2\xi - i_1\delta < {}^2\eta < {}^2\xi + i_1\delta$$

Since ${}^2\xi \in \mathbb{C}_1$ therefore consider ${}^2\xi = a + i_1 b$

where a, b $\in \mathbb{C}_0$

$\Rightarrow {}^2\eta = a + i_1 q$ where b- $\delta < q < b + \delta$

Now $\left|{}^2\xi - {}^2\eta\right| = |(a + i_1 b) - (a + i_1 q)|$

$= \mp(b - q)$

Since b- $\delta < q < b + \delta$ therefore $\mp(b - q) < \delta$

$\Rightarrow \left|{}^2\xi - {}^2\eta\right| < \delta$

Since $0 < \delta < \sqrt{2}\, r$

$$\Rightarrow \left|{}^2\xi - {}^2\eta\right| < \sqrt{2}\, r$$

$$\Rightarrow \frac{\left|{}^2\xi - {}^2\eta\right|}{\sqrt{2}} < r \qquad \ldots(2)$$

Now $\|\xi - \eta\| = \sqrt{\dfrac{\left|{}^1\xi - {}^1\eta\right|^2 + \left|{}^2\xi - {}^2\eta\right|^2}{2}}$

$$= \sqrt{\frac{0 + \left|{}^2\xi - {}^2\eta\right|^2}{2}} \text{ ; since } {}^1\eta = {}^1\xi$$

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Notes

From (2), $\|\xi - \eta\| < r$

$$\Rightarrow \eta \in B(\xi, r) \qquad \ldots(3)$$

Now consider an element ζ in $\mathbb{C}_2$ such that ${}^1\zeta = {}^1\xi + r$ and ${}^2\zeta = {}^2\xi$

Then $\sqrt{\dfrac{\left|{}^1\xi - {}^1\zeta\right|^2 + \left|{}^2\xi - {}^2\zeta\right|^2}{2}} = \dfrac{r}{\sqrt{2}} < r$

Therefore $\zeta \in B(\xi, r)$ ...(4)

Since ${}^1\zeta \neq {}^1\xi$

$$\therefore \zeta \notin ({}^1\xi e_1 + ({}^2\xi - i_1\delta)e_2, {}^1\xi e_1 + ({}^2\xi + i_1\delta)e_2)_{ID} \qquad \ldots(5)$$

Hence the set $({}^1\xi e_1 + ({}^2\xi - i_1\delta)e_2, {}^1\xi e_1 + ({}^2\xi + i_1\delta)e_2)_{ID}$ is a proper subset of $B(\xi, r)$ where $0 < \delta < \sqrt{2}\, r$

3.1.2 Lemma

If $(\zeta, \Psi)_{ID}$ is an open interval in $\mathbb{C}_2$ such that ${}^1\zeta \neq {}^1\Psi$ then there exist no open ball $B(\xi, r)$; $r < \infty$ which contain the set $(\zeta, \Psi)_{ID}$.

Proof- Let $B(\xi, r)$ be an arbitrary open ball in $\mathbb{C}_2$ such that $r < \infty$

Let η be the arbitrary element of $B(\xi, r)$

$$\Rightarrow \eta \in B(\xi, r) \qquad \ldots(6)$$

$$\Rightarrow \|\xi - \eta\| < r$$

$$\Rightarrow \sqrt{\frac{\left|{}^1\xi - {}^1\eta\right|^2 + \left|{}^2\xi - {}^2\eta\right|^2}{2}} < r$$

$$\Rightarrow \left|{}^1\xi - {}^1\eta\right| < \sqrt{2}\, r\ ,\ \left|{}^2\xi - {}^2\eta\right| < \sqrt{2}\, r$$

$$\Rightarrow \eta \in D(\xi; \sqrt{2}r, \sqrt{2}r) \qquad \ldots(7)$$

From (6), (7)

Therefore $B(\xi, r) \subseteq D(\xi; \sqrt{2}r, \sqrt{2}r)$ ...(8)

Let us consider an arbitrary open interval $(\zeta, \Psi)_{ID}$ in $\mathbb{C}_2$ which contain the element η

$$\Rightarrow \eta \in (\zeta, \Psi)_{ID}$$

$$\Rightarrow \zeta <_{ID} \eta <_{ID} \Psi$$

Since $\zeta <_{ID} \eta$

Therefore either ${}^1\zeta < {}^1\eta$ or ${}^1\zeta = {}^1\eta$, ${}^2\zeta < {}^2\eta$

Since $\eta <_{ID} \Psi$

Therefore either ${}^1\eta < {}^1\Psi$ or ${}^1\eta = {}^1\Psi$, ${}^2\eta < {}^2\Psi$

Since ${}^1\zeta \neq {}^1\Psi$ therefore there will be only three possibilities.

Case 1st - If ${}^1\zeta < {}^1\eta$ and ${}^1\eta < {}^1\Psi$

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Consider an element $y \in \mathbb{C}_2$ such that $^1y = {}^1\eta$ and $\left|{}^2\xi - {}^2y\right| > \sqrt{2}r$

Since $^1y = {}^1\eta$

$$\Rightarrow {}^1\zeta < {}^1y,\ {}^1y < {}^1\Psi$$

$$\Rightarrow \zeta <_{ID} y \text{ and } y <_{ID} \Psi$$

Therefore $y \in (\zeta\ ,\ \Psi)_{ID}$

Since $\left|{}^2\xi - {}^2y\right| > \sqrt{2}r$

$$\Rightarrow y \notin D(\xi;\sqrt{2}r,\sqrt{2}r)$$

From (8), $y \notin B(\xi, r)$

Therefore we have an element $y \in (\zeta\ ,\Psi)_{ID}$ such that $y \notin B(\xi, r)$

$$\Rightarrow (\zeta\ ,\Psi)_{ID} \not\subseteq B(\xi, r)$$

Case 2nd - If $^1\zeta < {}^1\eta$, $^1\eta = {}^1\Psi$ and $^2\eta < {}^2\Psi$

Consider an element $y \in \mathbb{C}_2$ such that $^1\zeta < {}^1y < {}^1\eta$ and $\left|{}^2\xi - {}^2y\right| > \sqrt{2}r$

Since $^1\zeta < {}^1y \Rightarrow \zeta <_{ID} y$

Since $^1y < {}^1\eta$ and $^1\eta = {}^1\Psi \Rightarrow {}^1y < {}^1\Psi$

$$\Rightarrow y <_{ID} \Psi$$

Therefore $y \in (\zeta\ ,\ \Psi)_{ID}$

$$\text{Since } \left|{}^2\xi - {}^2y\right| > \sqrt{2}r$$

$$\Rightarrow y \notin D(\xi;\sqrt{2}r,\sqrt{2}r)$$

From (8), $y \notin B(\xi, r)$

Therefore we have an element $y \in (\zeta\ ,\ \Psi)_{ID}$ such that $y \notin B(\xi, r)$

$$\Rightarrow (\zeta\ ,\Psi)_{ID} \not\subseteq B(\xi, r)$$

Case 3rd - If $^1\zeta = {}^1\eta$, $^2\zeta < {}^2\eta$ and $^1\eta < {}^1\Psi$

Consider an element $y \in \mathbb{C}_2$ such that $^1\eta < {}^1y < {}^1\Psi$ and $\left|{}^2\xi - {}^2y\right| > \sqrt{2}r$

Since $^1\eta < {}^1y$ and $^1\zeta = {}^1\eta \Rightarrow {}^1\zeta < {}^1y$

$\Rightarrow \zeta <_{ID} y$

Since $^1y < {}^1\Psi \Rightarrow y <_{ID} \Psi$

Therefore $y \in (\zeta\ ,\ \Psi)_{ID}$

Since $\left|{}^2\xi - {}^2y\right| > \sqrt{2}r$

$$\Rightarrow y \notin D(\xi;\sqrt{2}r,\sqrt{2}r)$$

From (8), $y \notin B(\xi, r)$

Therefore we have an element $y \in (\zeta\ ,\ \Psi)_{ID}$ such that $y \notin B(\xi, r)$

Notes

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$$\Rightarrow (\zeta, \Psi)_{ID} \not\subseteq B(\xi, r)$$

Finally the ball $B(\xi, r)$; $r < \infty$ cannot contain any open interval $(\zeta, \Psi)_{ID}$ where ${}^1\zeta \neq {}^1\Psi$

Hence $(\zeta, \Psi)_{ID}$ cannot be contained in any ball $B(\xi, r)$; $r < \infty$

3.1.3 Theorem

The Idempotent order topology is strictly finer than Norm topology.

Notes

Proof- Let $X = (\xi, \eta)_{ID}$ be an open interval such that ${}^1\xi = {}^1\eta$

Then $h_1(X) = {}^1X = \{{}^1\xi\}$ and $h_2(X) = {}^2X = ({}^2\xi, {}^2\eta)$

In fact $X = {}^1X \times_e {}^2X$

Since ${}^1X$ is not open in complex plane

Therefore X will not be open w.r.t. Norm topology.

(By Theorem-2.1.2)

Since X is open w.r.t. Idempotent order topology therefore Idempotent order topology and norm topology are not equivalent and Norm topology cannot be finer than Idempotent order topology

Now we want to show for all open ball $B(\xi, r)$ and for all $\eta \in B(\xi, r)$ then there exist $(\zeta, \Psi)_{ID}$ such that $\eta \in (\zeta, \Psi)_{ID} \subseteq B(\xi, r)$

Let us consider an arbitrary ball $B(\xi, r)$ and consider an arbitrary element η of $B(\xi, r)$

$\Rightarrow \eta \in B(\xi, r)$

1st Method- Since $\eta \in B(\xi, r)$ then there exist a ball

$B(\eta, s)$;$s > 0$ such that $B(\eta, s) \subseteq B(\xi, r)$

From Lemma-3.1.1,

$({}^1\eta e_1 + ({}^2\eta - i_1\delta)e_2, {}^1\eta e_1 + ({}^2\eta + i_1\delta)e_2)_{ID}$ will be the proper subset of $B(\eta, s)$

Therefore for $\eta \in B(\xi, r)$ we have a set $({}^1\eta e_1 + ({}^2\eta - i_1\delta)e_2, {}^1\eta e_1 + ({}^2\eta + i_1\delta)e_2)_{ID}$ such that $\eta \in ({}^1\eta e_1 + ({}^2\eta - i_1\delta)e_2, {}^1\eta e_1 + ({}^2\eta + i_1\delta)e_2)_{ID}$ and

$({}^1\eta e_1 + ({}^2\eta - i_1\delta)e_2, {}^1\eta e_1 + ({}^2\eta + i_1\delta)e_2)_{ID}$ is the subset of $B(\xi, r)$

2nd Method- Since $\eta \in B(\xi, r)$

$$\Rightarrow \sqrt{\frac{\left|{}^1\xi - {}^1\eta\right|^2 + \left|{}^2\xi - {}^2\eta\right|^2}{2}} < r$$

$$\Rightarrow \left|{}^1\xi - {}^1\eta\right|^2 + \left|{}^2\xi - {}^2\eta\right|^2 < 2r^2$$

Let $\left|{}^1\xi - {}^1\eta\right|^2 = d_1^2, \left|{}^2\xi - {}^2\eta\right|^2 = d_2^2$

$$\Rightarrow d_1^2 + d_2^2 < 2r^2$$

There exist $d_3 > 0$ such that $d_2 < d_3$ and $d_1^2 + d_3^2 < 2r^2$

Consider a set $Q = ({}^1\eta e_1 + ({}^2\eta - i_1\delta)e_2, {}^1\eta e_1 + ({}^2\eta + i_1\delta)e_2)_{ID}$ where $\delta = d_3 - d_2 > 0$

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Obviously $\eta \in Q$

Let $y \in Q$ ...(9)

$\Rightarrow {}^1y = {}^1\eta$ and $({}^2\eta - i_1\delta) < {}^2y < ({}^2\eta + i_1\delta)$
Let ${}^2\eta = a+i_1b$
Then ${}^2y = a+i_1q$ where b- $\delta < q < b + \delta$
Now $\left|{}^2\xi - {}^2y\right| \leq \left|{}^2\xi - {}^2\eta\right| + \left|{}^2\eta - {}^2y\right|$

$$\Rightarrow \left|{}^2\xi - {}^2y\right| \leq d_2 + |i_1(b-q)|$$

$$\Rightarrow \left|{}^2\xi - {}^2y\right| \leq d_2 \pm (b-q) \qquad \text{...(10)}$$

Since b- $\delta < q < b + \delta$ and $\delta = d_3 - d_2$

Therefore $\pm(b-q) < d_3 - d_2$

From (10), $\left|{}^2\xi - {}^2y\right| < d_3$

Since $\left|{}^1\xi - {}^1y\right| = d_1$

Therefore $\left|{}^1\xi - {}^1y\right|^2 + \left|{}^2\xi - {}^2y\right|^2 < d_1^2 + d_3^2 < 2r^2$

$$\Rightarrow y \in B(\xi, r) \qquad \text{...(11)}$$

From (9), (11) $Q \subseteq B(\xi, r)$

Therefore for $\eta \in B(\xi, r)$ we have a open interval Q w.r.t. idempotent ordering such that $\eta \in Q \subseteq B(\xi, r)$
Hence it proves that Idempotent order topology is strictly finer than Norm topology.

Theorem 3.1.3 together with Theorems 2.3.1, 2.3.2, 2.3.10, 2.3.11 and Corollary 2.3.3, 2.3.12 generate a new corollary which states that

3.1.4 Corollary

The topology $\mathrm{T}_1^{\#}$ (and therefore $\mathrm{T}_2^{\#}$ and $\mathrm{T}_3^{\#}$) on $\mathbb{C}_2$ is strictly finer than the topology T_N (and therefore T_I and T_C) on $\mathbb{C}_2$.

b) Comparison of the idempotent order topology and real order topology on the bicomplex space

3.2.1 Theorem

The Real order topology and Idempotent order topology are not comparable.

Proof- Consider an open interval 'A' w.r.t. real ordering such that A= $(\xi, \eta)_R$ where $\xi = 1+2i_1+4i_2+7i_1i_2$ and $\eta = 2+2i_1+3i_2+4i_1i_2$

Consider an element x in $\mathbb{C}_2$ such that $x = 1+2i_1+5i_2+7i_1i_2$ therefore $\xi <_R x$ and $x <_R \eta$
$\Rightarrow x \in (\xi, \eta)_R$

Let us consider arbitrary open interval $(\zeta, \Psi)_{ID}$ in $\mathbb{C}_2$ (w.r.t. Idempotent ordering) which contain the element x.

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i.e. $x \in (\zeta, \Psi)_{ID} \Rightarrow \zeta <_{ID} x$, $x <_{ID} \Psi$
Since $\zeta <_{ID} x$
Therefore either $^1\zeta < {}^1x$ or $^1\zeta = {}^1x$, $^2\zeta < {}^2x$
Since $x <_{ID} \Psi$
Therefore either $^1x < {}^1\Psi$ or $^1x = {}^1\Psi$, $^2x < {}^2\Psi$
Hence there will be four possibilities.

Notes

Case A- If $^1\zeta < {}^1x$ and $^1x < {}^1\Psi$
Since $x = 1+2i_1+5i_2+7i_1i_2$
Therefore $^1x = 8-3i_1$, $^2x = -6+7i_1$

Consider an element y in $\mathbb{C}_2$ such that $y = 6+2i_1+5i_2+2i_1i_2$
Therefore $^1y = 8-3i_1$, $^2y = 4+7i_1$
Since $^1y = {}^1x$ and $^1\zeta < {}^1x$, $^1x < {}^1\Psi$
Therefore $\zeta <_{ID} y <_{ID} \Psi$

$$\Rightarrow y \in (\zeta, \Psi)_{ID} \qquad \ldots(12)$$

Since $\xi <_R y$ and $\eta <_R y$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(13)$$

From (12), (13) $(\zeta, \Psi)_{ID} \not\subseteq (\xi, \eta)_R$
Case B-If $^1\zeta < {}^1x$, $^1x = {}^1\Psi$ and $^2x < {}^2\Psi$
Consider an element y in $\mathbb{C}_2$ such that $^1\zeta < {}^1y < {}^1x$
Since $^1\zeta < {}^1y \Rightarrow \zeta <_{ID} y$
Since $^1x = {}^1\Psi$, $^1y < {}^1x \Rightarrow y <_{ID} \Psi$

$$\Rightarrow y \in (\zeta, \Psi)_{ID} \qquad \ldots(14)$$

Since $^1x = 8-3i_1$ and $^1y < {}^1x$
$\Rightarrow$ Therefore there are only two possibilities either $^1y = 8 + \delta i_1$ where $\delta < -3$ or $^1y = a + i_1 b$ where $a < 8$, $b \in \mathbb{C}_0$
If $^1y = 8 + \delta i_1$ where $\delta < -3$
Then y will be in the form $y = a_1+i_1a_2+i_2a_3+i_1i_2a_4$
Where $a_1+a_4 = 8$, $a_2-a_3 < -3$
Choose $a_1=7$, $a_4=1$ and choose a_2 and a_3 in such a way that $a_2-a_3 < -3$
Therefore $y = 7+i_1a_2+i_2a_3+i_1i_2$

$$\Rightarrow \xi <_R y,\ \eta <_R y$$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(15)$$

From (14), (15) $(\zeta, \Psi)_{ID} \not\subseteq (\xi, \eta)_R$
If $^1y = a + i_1 b$ where $a < 8$, $b \in \mathbb{C}_0$
Then y will be in the form $y = b_1+i_1b_2+i_2b_3+i_1i_2b_4$, Where $b_1+b_4 < 8$
Choose $b_1 = 8$ and $b_4 = -C$ where $C > 0$
Therefore $y = 8+i_1b_2+i_2b_3-i_1i_2C$

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$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(16)$$

From (12), (14) $(\zeta, \Psi)_{ID} \nsubseteq (\xi, \eta)_R$

Case C-If ${}^1\zeta = {}^1x$, ${}^2\zeta < {}^2x$ and ${}^1x < {}^1\Psi$

Consider an element y in $\mathbb{C}_2$ such that ${}^1x < {}^1y < {}^1\Psi$

Since ${}^1y < {}^1\Psi \Rightarrow y <_{ID} \Psi$

Since ${}^1\zeta = {}^1x$ and ${}^1x < {}^1y \Rightarrow \zeta <_{ID} y$

$$\Rightarrow y \in (\zeta, \Psi)_{ID} \qquad \ldots(17)$$

Since ${}^1x = 8\text{-}3i_1$ and ${}^1x < {}^1y$

Therefore there are only two possibilities either ${}^1y = 8 + \delta i_1$ where $\delta > -3$ or ${}^1y = a + i_1 b$ where $a > 8$, $b \in \mathbb{C}_0$

If ${}^1y = 8 + \delta i_1$ where $\delta > -3$

Then y will be in the form $y = a_1 + i_1a_2 + i_2a_3 + i_1i_2a_4$

Where $a_1 + a_4 = 8$, $a_2\text{-}a_3 > -3$

Choose $a_1 = 7$, $a_4 = 1$ and choose a_2 and a_3 in such a way that $a_2\text{-}a_3 > -3$

Therefore $y = 7 + i_1a_2 + i_2a_3 + i_1i_2$

$$\Rightarrow \xi <_R y, \eta <_R y$$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(18)$$

From (17), (18) $(\zeta, \Psi)_{ID} \nsubseteq (\xi, \eta)_R$

If ${}^1y = a + i_1 b$ where $a > 8$, $b \in \mathbb{C}_0$

Then y will be in the form $y = b_1 + i_1b_2 + i_2b_3 + i_1i_2b_4$, Where $b_1 + b_4 > 8$

Choose $b_1 = 8$ and $b_4 = C$ where $C > 0$

Therefore $y = 8 + i_1b_2 + i_2b_3 \text{-} i_1i_2C$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(19)$$

From (17), (19) $(\zeta, \Psi)_{ID} \nsubseteq (\xi, \eta)_R$

Case D- If ${}^1\zeta = {}^1x$, ${}^2\zeta < {}^2x$ and ${}^1x = {}^1\Psi$, ${}^2x < {}^2\Psi$

Consider an element y in $\mathbb{C}_2$ such that

$${}^1y = {}^1x \text{ and } {}^2\zeta < {}^2y < {}^2\Psi$$

$$\Rightarrow \zeta <_{ID} y,\ y <_{ID} \Psi$$

$$\Rightarrow y \in (\zeta, \Psi)_{ID} \qquad \ldots(20)$$

If $Z = a + i_1 b$ is a complex number then the region of all complex number which is greater than Z or less than Z is define as follows.

Figure-1 shows the region of all complex numbers which is greater than Z and figure-2 shows the region of all complex numbers which is less than Z.

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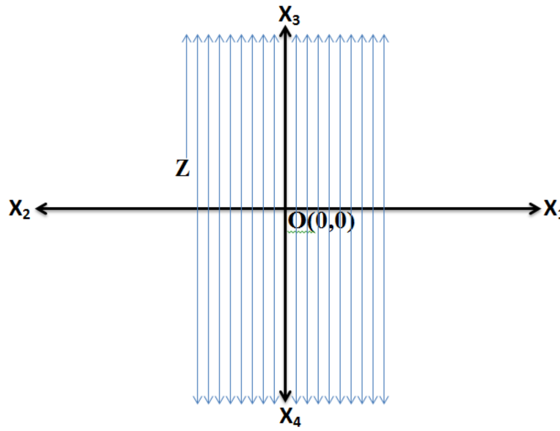

Figure-1

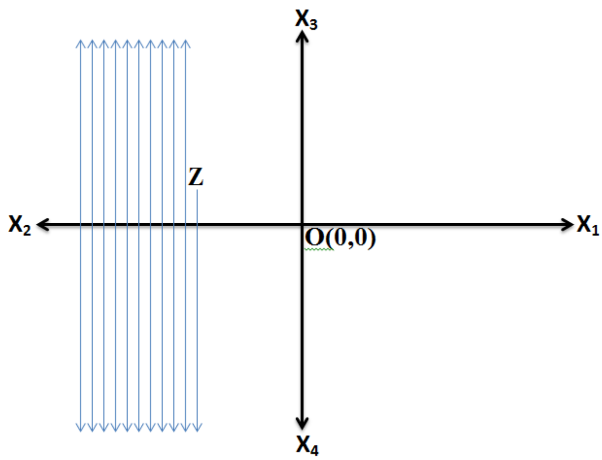

Figure-2

Since ${}^2x = -6+7i_1$ and ${}^2\zeta < {}^2x < {}^2\Psi$
Here there are four possibilities.

Possibility 1[st]-If ${}^2\zeta$ is on the axis **QT** except **Q** point and ${}^2\Psi$ is on the axis **QS** except **Q** point.

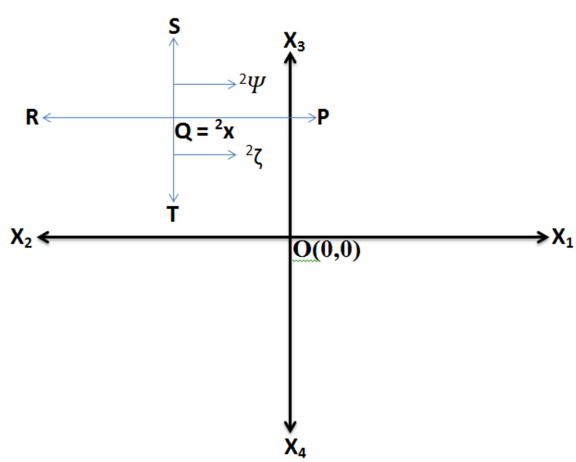

Figure-3

Since ${}^1y = {}^1x$ and ${}^2\zeta < {}^2y < {}^2\Psi$

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Therefore ${}^1y = 8-3i_1$ and ${}^2y = -6+i_1b$ where $b > 7$ or $b < 7$

We will consider only $b < 7$

Then $y = 1+(2-\delta)i_1+(5-\delta)i_2+7i_1i_2$ where $\delta > 0$

$$\Rightarrow y <_R \xi$$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(21)$$

From (20), (21) $(\zeta, \Psi)_{ID} \not\subseteq (\xi, \eta)_R$

Possibility 2nd-If ${}^2\zeta$ is situated in the region **QSRT** except **ST** axis and ${}^2\Psi$ is on the axis **QS** except **Q** point.

Since ${}^1y = {}^1x$ and ${}^2\zeta < {}^2y < {}^2\Psi$

Therefore ${}^1y = 8-3i_1$

Consider ${}^2y = a+i_1b$ where $a < -6$, $b \in \mathbb{C}_0$

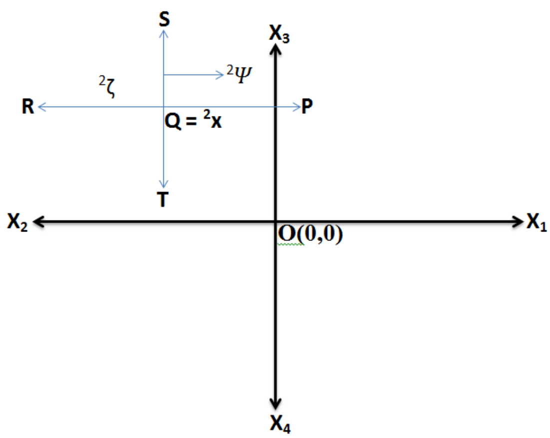

Figure-4

Then $y = (1-\delta_1)+(2-\delta_2)i_1+(5-\delta_2)i_2+(7+\delta_1)i_1i_2$

where $\delta_1 > 0$ and $\delta_2 \in \mathbb{C}_0$

$$\Rightarrow y <_R \xi$$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(22)$$

From (20), (22) $(\zeta, \Psi)_{ID} \not\subseteq (\xi, \eta)_R$

Possibility 3rd- If ${}^2\Psi$ is situated in the region **QSPT** except **ST** axis and ${}^2\zeta$ is on the axis **QT** except **Q** point.

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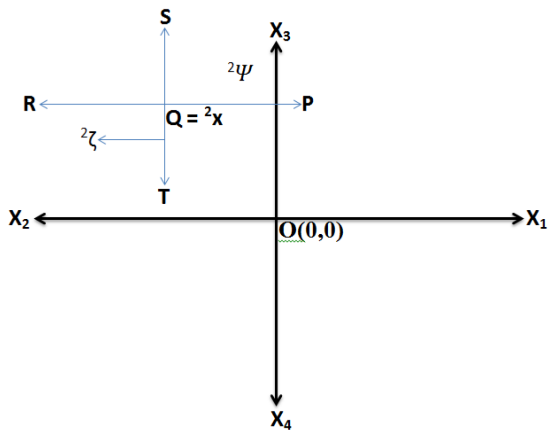

Figure-5

Since $^1y = {}^1x$ and $^2\zeta < {}^2y < {}^2\Psi$
Therefore $^1y = 8-3i_1$
Consider ^2y in such a way that $^2\zeta < {}^2y < {}^2x$
Therefore $^2y = -6+i_1b$ where $b < 7$

$$\Rightarrow y = 1+(2-\delta)i_1+(5-\delta)i_2+7i_1i_2 \text{ where } \delta > 0$$

$$\Rightarrow y <_R \xi$$

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(23)$$

From (20), (23) $(\zeta, \Psi)_{ID} \nsubseteq (\xi, \eta)_R$

Possibility 4th-If $^2\Psi$ is situated in the region **QSPT** except **ST** axis and $^2\zeta$ is situated in the region **QSRT** except **ST** axis.
$^2y = -6+5i_1$ satisfies the condition $^2\zeta < {}^2y < {}^2\Psi$
Therefore on choosing $^2y = -6+5i_1$
Since $^1y = 8-3i_1$ then $y = 1+i_1+4i_2+7i_1i_2$
$\Rightarrow y <_R \xi$

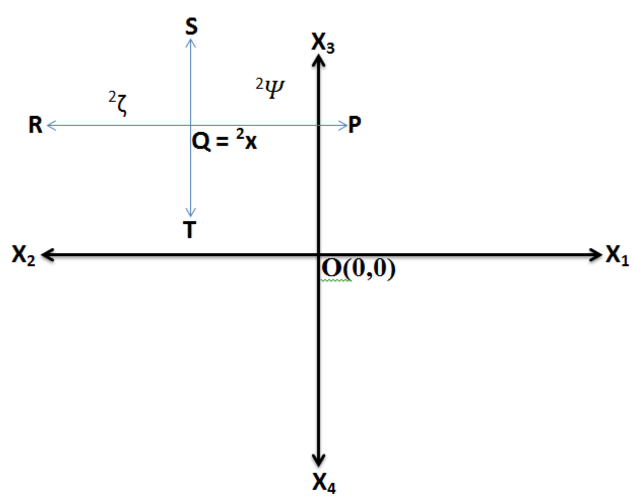

Figure-6

$$\Rightarrow y \notin (\xi, \eta)_R \qquad \ldots(24)$$

From (20), (24) $(\zeta, \Psi)_{ID} \nsubseteq (\xi, \eta)_R$

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Finally all open interval (w.r.t. idempotent ordering) which contain the element x they cannot be subset of the set A.

Since A is open w.r.t. Real order topology therefore it shows that Idempotent order topology is not finer than Real order topology.

Now consider an open interval 'B' w.r.t. idempotent ordering such that $B = (\alpha, \beta)_{ID}$

Where $\alpha = (6\text{-}i_1)e_1+(-2+5i_1)e_2$ and $\beta = (8\text{-}2i_1)e_1+(-6+6i_1)e_2$

Consider an element p in $\mathbb{C}_2$ such that $p=2+2.5i_1+3.5i_2+4i_1i_2$

$$\Rightarrow {}^1p = (6 - i_1),\ {}^2p = (-2 + 6i_1)$$

Since ${}^1\alpha = {}^1p$ and ${}^2\alpha < {}^2p \Rightarrow \alpha <_{ID} p$

Since ${}^1p < {}^1\beta \Rightarrow p <_{ID} \beta$ therefore $p \in (\alpha, \beta)_{ID}$

Let us consider arbitrary open interval $(\phi,\Psi)_R$ in $\mathbb{C}_2$ (w.r.t. Real ordering) which contain the element p.

$$\text{i.e. } p \in (\phi, \Psi)_R \Rightarrow \phi <_R p\ ,\ p <_R \Psi$$

Let $\phi = x_1+i_1x_2+i_2x_3+i_1i_2x_4$ and $\Psi = y_1+i_1y_2+i_2y_3+i_1i_2y_4$

Since $p=2+2.5i_1+3.5i_2+4i_1i_2$ and $\phi <_R p\ ,\ p <_R \Psi$

Therefore there are 16 cases.

Case-(i) If $x_1 < 2$ and $y_1 > 2$

$\Rightarrow \phi = (2\text{-}\ \epsilon)+i_1x_2+i_2x_3+i_1i_2x_4$ and

$\Psi = (2+\delta)+i_1y_2+i_2y_3+i_1i_2y_4$ where $\epsilon, \delta > 0$

Consider an element A in $\mathbb{C}_2$ such that

$$A = 2+2.5i_1+3.5i_2+9i_1i_2$$

$$\Rightarrow A \in (\phi, \Psi)_R \qquad \ldots(25)$$

Since ${}^1A = (11\text{-}i_1)$

$$\Rightarrow {}^1\alpha < {}^1A \text{ and } {}^1\beta < {}^1A$$

$$\Rightarrow \alpha <_{ID} A,\ \beta <_{ID} A$$

$$\Rightarrow A \notin (\alpha, \beta)_{ID} \qquad \ldots(26)$$

From (25), (26) $(\phi,\Psi)_R \not\subseteq (\alpha, \beta)_{ID}$

Case-(ii) If $x_1=2$, $x_2<2.5$ and $y_1>2$

$\Rightarrow \phi = 2+(2.5\text{-}\ \epsilon)i_1+i_2x_3+i_1i_2x_4$ and

$$\Psi = (2+\delta)+i_1y_2+i_2y_3+i_1i_2y_4 \text{ where } \epsilon, \delta > 0$$

Also in this situation $A \in (\phi, \Psi)_R$ ...(27)

Therefore from (26), (27) $(\phi,\Psi)_R \not\subseteq (\alpha, \beta)_{ID}$

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Case-(iii) If $x_1=2$, $x_2=2.5$, $x_3<3.5$ and $y_1>2$

$\Rightarrow \phi = 2+2.5i_1+(3.5-\epsilon)i_2+i_1i_2x_4$ and

$$\Psi = (2+\delta)+i_1y_2+i_2y_3+i_1i_2y_4 \text{ where } \epsilon, \delta > 0$$

$$\Rightarrow A \in (\phi, \Psi)_R \qquad \ldots(28)$$

Therefore from (26), (28) $(\phi,\Psi)_R \not\subseteq (\alpha,\beta)_{ID}$

Case-(iv) If $x_1=2$, $x_2=2.5$, $x_3=3.5$, $x_4<4$ and $y_1>2$

$\Rightarrow \phi = 2+2.5i_1+3.5i_2+(4-\epsilon)i_1i_2$ and

$$\Psi = (2+\delta)+i_1y_2+i_2y_3+i_1i_2y_4 \text{ where } \epsilon, \delta > 0$$

$$\Rightarrow A \in (\phi, \Psi)_R \qquad \ldots(29)$$

Therefore from (26), (29) $(\phi,\Psi)_R \not\subseteq (\alpha,\beta)_{ID}$

Case-(v) If $x_1< 2$, $y_1=2$ and $y_2>2.5$

$\Rightarrow \phi = (2-\epsilon)+i_1x_2+i_2x_3+i_1i_2x_4$ and

$\Psi = 2+(2.5+\delta)i_1+i_2y_3+i_1i_2y_4$ where $\epsilon, \delta > 0$

Since $A = 2+2.5i_1+3.5i_2+9i_1i_2$

$$\Rightarrow A \in (\phi, \Psi)_R \qquad \ldots(30)$$

Therefore from (26), (30) $(\phi,\Psi)_R \not\subseteq (\alpha,\beta)_{ID}$

Case-(vi) If $x_1 = 2$, $x_2 < 2.5$, $y_1 = 2$ and $y_2 > 2.5$

$\Rightarrow \phi = 2+(2.5-\epsilon)i_1+i_2x_3+i_1i_2x_4$ and

$\Psi = 2+(2.5+\delta)i_1+i_2y_3+i_1i_2y_4$ where $\epsilon, \delta > 0$

Also in this situation $A \in (\phi, \Psi)_R$ ...(31)

Therefore from (26), (31) $(\phi,\Psi)_R \not\subseteq (\alpha,\beta)_{ID}$

Case-(vii) If $x_1 = 2$, $x_2 = 2.5$, $x_3 < 3.5$, $y_1 = 2$ and $y_2 > 2.5$

$\Rightarrow \phi = 2+2.5i_1+(3.5-\epsilon)i_2+i_1i_2x_4$ and

$\Psi = 2+(2.5+\delta)i_1+i_2y_3+i_1i_2y_4$ where $\epsilon, \delta > 0$

$$\Rightarrow A \in (\phi, \Psi)_R \qquad \ldots(32)$$

Therefore from (26), (32) $(\phi,\Psi)_R \not\subseteq (\alpha,\beta)_{ID}$

Case-(viii) If $x_1 = 2$, $x_2 = 2.5$, $x_3 = 3.5$, $x_4 < 4$, $y_1 = 2$ and $y_2 > 2.5$

$\Rightarrow \phi = 2+2.5i_1+3.5i_2+(4-\epsilon)i_1i_2$ and

$\Psi = 2+(2.5+\delta)i_1+i_2y_3+i_1i_2y_4$ where $\epsilon, \delta > 0$

$$\Rightarrow A \in (\phi, \Psi)_R \qquad \ldots(33)$$

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Therefore from (26), (33) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(ix) If $x_1 < 2$, $y_1 = 2$, $y_2 = 2.5$ and $y_3 > 3.5$

$\Rightarrow \phi = (2-\epsilon)+i_1x_2+i_2x_3+i_1i_2x_4$ and

$\Psi = 2+2.5i_1+(3.5+\delta)i_2+i_1i_2y_4$ where $\epsilon, \delta > 0$

Since $A = 2+2.5i_1+3.5i_2+9i_1i_2$

$$\Rightarrow A \in (\phi,\Psi)_R \qquad \ldots(34)$$

Therefore from (26), (34) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(x) If $x_1 = 2$, $x_2 < 2.5$, $y_1 = 2$, $y_2 = 2.5$ and $y_3 > 3.5$

$\Rightarrow \phi = 2+(2.5-\epsilon)i_1+i_2x_3+i_1i_2x_4$ and

$\Psi = 2+2.5i_1+(3.5+\delta)i_2+i_1i_2y_4$ where $\epsilon, \delta > 0$

Also in this situation $A \in (\phi,\Psi)_R$ ...(35)

Therefore from (26), (35) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(xi) If $x_1 = 2$, $x_2 = 2.5$, $x_3 < 3.5$, $y_1 = 2$, $y_2 = 2.5$ and $y_3 > 3.5$

$\Rightarrow \phi = 2+2.5i_1+(3.5-\epsilon)i_2+i_1i_2x_4$ and

$\Psi = 2+2.5i_1+(3.5+\delta)i_2+i_1i_2y_4$ where $\epsilon, \delta > 0$

$$\Rightarrow A \in (\phi,\Psi)_R \qquad \ldots(36)$$

Therefore from (26), (36) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(xii) If $x_1 = 2$, $x_2 = 2.5$, $x_3 = 3.5$, $x_4 < 4$, $y_1 = 2$, $y_2 = 2.5$ and $y_3 > 3.5$

$\Rightarrow \phi = 2+2.5i_1+3.5i_2+(4-\epsilon)i_1i_2$ and

$\Psi = 2+2.5i_1+(3.5+\delta)i_2+i_1i_2y_4$ where $\epsilon, \delta > 0$

$$\Rightarrow A \in (\phi,\Psi)_R \qquad \ldots(37)$$

Therefore from(26), (37) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(xiii) If $x_1 < 2$, $y_1 = 2$, $y_2 = 2.5$, $y_3 = 3.5$ and $y_4 > 4$

$\Rightarrow \phi = (2-\epsilon)+i_1x_2+i_2x_3+i_1i_2x_4$ and

$\Psi = 2+2.5i_1+3.5i_2+(4+\delta)i_1i_2$ where $\epsilon, \delta > 0$

Consider an element B in $\mathbb{C}_2$ such that $B = 2+2.5i_1+3.5i_2+3i_1i_2$

$$\Rightarrow B \in (\phi,\Psi)_R \qquad \ldots(38)$$

Since ${}^1B = (5-i_1)$

$$\Rightarrow {}^1B < {}^1\alpha,\ {}^1B < {}^1\beta$$

$$\Rightarrow B <_{ID} \alpha,\ B <_{ID} \beta$$

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$$\Rightarrow B \notin (\alpha, \beta)_{ID} \qquad \ldots(39)$$

From (38), (39) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(xiv) If $x_1 = 2$, $x_2 < 2.5$, $y_1 = 2$, $y_2 = 2.5$, $y_3 = 3.5$ and $y_4 > 4$
$\Rightarrow \phi = 2+(2.5-\epsilon)i_1+i_2x_3+i_1i_2x_4$ and
$\Psi = 2+2.5i_1+3.5i_2+(4+\delta)i_1i_2$ where $\epsilon, \delta > 0$

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Also in this situation $B \in (\phi, \Psi)_R \qquad \ldots(40)$

Therefore from (39), (40) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(xv) If $x_1 = 2$, $x_2 = 2.5$, $x_3 < 3.5$, $y_1 = 2$, $y_2 = 2.5$, $y_3 = 3.5$ and $y_4 > 4$
$\Rightarrow \phi = 2+2.5i_1+(3.5-\epsilon)i_2+i_1i_2x_4$ and
$\Psi = 2+2.5i_1+3.5i_2+(4+\delta)i_1i_2$ where $\epsilon, \delta > 0$

$$\Rightarrow B \in (\phi, \Psi)_R \qquad \ldots(41)$$

Therefore from (39), (41) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Case-(xvi) If $x_1 = y_1 = 2$, $x_2 = y_2 = 2.5$, $x_3 = y_3 = 3.5$ and $x_4 < 4 < y_4$
$\Rightarrow \phi = 2+2.5i_1+3.5i_2+(4-\epsilon)i_1i_2$ and
$\Psi = 2+2.5i_1+3.5i_2+(4+\delta)i_1i_2$ where $\epsilon, \delta > 0$
Consider an element D in $\mathbb{C}_2$ such that $D = 2+2.5i_1+3.5i_2+ai_1i_2$ where $(4-\epsilon) < a < 4$

$$\Rightarrow D \in (\phi, \Psi)_R \qquad \ldots(42)$$

Since ${}^1D = \{(2 + a)-i_1\}$
Since $a < 4 \Rightarrow 2 + a < 6$

$$\Rightarrow {}^1D < {}^1\alpha \Rightarrow D <_{ID} \alpha$$

$$\Rightarrow D \notin (\alpha, \beta)_{ID} \qquad \ldots(43)$$

Therefore from (42), (43) $(\phi,\Psi)_R \nsubseteq (\alpha,\beta)_{ID}$

Hence all open interval (w.r.t. real ordering) which contain the element 'p' they cannot be subset of the set 'B'.

Since B is open w.r.t. Idempotent order topology therefore it shows real order topology is not finer than Idempotent order topology.
Hence it proves that both topologies are not comparable.

Theorem 3.2.1, 2.3.10, 2.3.11and corollary 2.3.12, 2.3.14 submerge together to give a new corollary which is started below.

3.2.2 Corollary

The topology $\mathrm{T}_1^{\#}$ (and therefore $\mathrm{T}_2^{\#}$ and $\mathrm{T}_3^{\#}$) and the topology T_1 (and therefore T_2, T_3, T_1^{*}, T_2^{*} and T_3^{*}) on $\mathbb{C}_2$ are not comparable.

c) Comparison of the real order topology and norm topology on the bicomplex space

3.3.1 Lemma

The set $(\zeta, \Psi)_R$ is the proper subset of $B(\xi = x_1+i_1x_2+i_2x_3+i_1i_2x_4, r)$ where $\zeta = x_1+i_1x_2+i_2x_3+i_1i_2(x_4-\epsilon)$, $\Psi = x_1+i_1x_2+i_2x_3+i_1i_2(x_4+\epsilon)$ and either $\epsilon = \mathrm{Min}(d_1,d_2)$, $d_1^2+d_2^2 < 2r^2$ and $\epsilon > 0$ or $0 < \epsilon < r$

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Proof- Let suppose

$$\eta \in (\zeta, \Psi)_R \qquad \ldots(44)$$

$$\Rightarrow \zeta <_R \eta <_R \Psi$$

$$\Rightarrow \eta = x_1+i_1x_2+i_2x_3+i_1i_2q \text{ where } (x_4-\epsilon) < q < (x_4+\epsilon)$$

Since $\eta = x_1+i_1x_2+i_2x_3+i_1i_2q$

$$\Rightarrow {}^1\eta = (x_1+q)+i_1(x_2-x_3) \text{ and } {}^2\eta = (x_1-q)+i_1(x_2+x_3)$$

Since $\xi = x_1+i_1x_2+i_2x_3+i_1i_2x_4$

$$\Rightarrow {}^1\xi = (x_1+x_4)+i_1(x_2-x_3) \text{ and } {}^2\xi = (x_1-x_4)+i_1(x_2+x_3)$$

Now $\left|{}^1\xi - {}^1\eta\right| = \left|x_4 - q\right| = \pm(x_4 - q)$

Since $(x_4 - \epsilon) < q < (x_4 + \epsilon)$ therefore $\pm(x_4 - q) < \epsilon$

$\Rightarrow \left|{}^1\xi - {}^1\eta\right| < \epsilon$

Similarly $\left|{}^2\xi - {}^2\eta\right| = \pm(x_4 - q) < \epsilon$

Since $\epsilon = Min(d_1, d_2)$ and $d_1^2+d_2^2 < 2r^2$

Therefore $\left|{}^1\xi - {}^1\eta\right|^2 + \left|{}^2\xi - {}^2\eta\right|^2 < 2r^2$

$$\Rightarrow \sqrt{\frac{\left|{}^1\xi - {}^1\eta\right|^2 + \left|{}^2\xi - {}^2\eta\right|^2}{2}} < r$$

Therefore $\|\xi - \eta\| < r$

$$\Rightarrow \eta \in B(\xi = x_1+i_1x_2+i_2x_3+i_1i_2x_4, r) \qquad \ldots(45)$$

Now consider an element P in $\mathbb{C}_2$ such that ${}^1P = {}^1\xi + r$ and ${}^2P = {}^2\xi$

Constitute $\sqrt{\frac{\left|{}^1\xi - {}^1P\right|^2 + \left|{}^2\xi - {}^2P\right|^2}{2}} = \frac{r}{\sqrt{2}} < r$

Therefore $\qquad P \in B(\xi, r) \qquad \ldots(46)$

Since ${}^1\xi = (x_1+x_4)+i_1(x_2-x_3)$ and ${}^2\xi = (x_1-x_4)+i_1(x_2+x_3)$

$\therefore {}^1P = (x_1+x_4+r)+i_1(x_2-x_3)$ and ${}^2P = (x_1-x_4)+i_1(x_2+x_3)$

$\therefore P = (x_1+r/2)+i_1x_2+i_2x_3+i_1i_2(x_4+r/2)$

$$\Rightarrow P \notin (\zeta, \Psi)_R \qquad \ldots(47)$$

From (44), (45), (46) and (47)

Hence the set $(\zeta, \Psi)_R$ is the proper subset of $B(\xi = x_1+i_1x_2+i_2x_3+i_1i_2x_4, r)$

3.3.2 Lemma

If $(\zeta, \Psi)_C$ is an open interval in $\mathbb{C}_2$ such that $u_1 \neq v_1$ or $u_1 = v_1$, $a_3 \neq b_3$ then there exist no open ball $B(\xi, r)$; $r < \infty$ which contain the set $(\zeta, \Psi)_C$.

Or

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If $(\zeta, \Psi)_R$ is an open interval in C_2 such that $a_1 \neq b_1$ or $a_1 = b_1$, $a_2 \neq b_2$ or $a_1 = b_1$, $a_2 = b_2$, $a_3 \neq b_3$ then there exist no open ball $B(\xi, r)$; $r < \infty$ which contain the set $(\zeta, \Psi)_R$.
Where $\zeta = u_1+i_2u_2 = (a_1+i_1a_2) + i_2(a_3+i_1a_4)$ and
$\Psi = v_1+i_2v_2 = (b_1+i_1b_2) + i_2(b_3+i_1b_4)$

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Proof- Let $B(\xi, r)$ be an arbitrary open ball in C_2 such that $r < \infty$
Let suppose $\xi = z_1+i_2z_2 = (x_1+i_1x_2) + i_2(x_3+i_1x_4)$ and
let $\eta = w_1+i_2w_2 = (y_1+i_1y_2) + i_2(y_3+i_1y_4)$ be the arbitrary element of $B(\xi, r)$.

$$\Rightarrow \eta \in B(\xi, r)$$

$$\Rightarrow \|\xi - \eta\| < r$$

$$\Rightarrow \sqrt{|z_1 - w_1|^2 + |z_2 - w_2|^2} < r$$

$$\text{Or } [(x_1\text{-}y_1)^2+(x_2\text{-}y_2)^2+(x_3\text{-}y_3)^2+(x_4\text{-}y_4)^2]^{1/2} < r$$

$$\Rightarrow |z_1 - w_1| < r \text{ and } |z_2 - w_2| < r$$

$$\text{Or } (x_i\text{-}y_i) < r;\ i = 1, 2, 3, 4$$

Let us consider an arbitrary open interval $(\zeta, \Psi)_C$ in C_2 which contain the element η
$\Rightarrow \eta \in (\zeta, \Psi)_C$
$\Rightarrow \zeta <_C \eta <_C \Psi$
Since $\zeta <_C \eta$
Therefore either $u_1 < w_1$ or $u_1 = w_1$, $u_2 < w_2$
Since $\eta <_{ID} \Psi$
Therefore either $w_1 < v_1$ or $w_1 = v_1$, $w_2 < v_2$

Case-A If $u_1 \neq v_1$ then there will be three possibilities.

Possibility-1st: If $u_1 < w_1$ and $w_1 < v_1$
Consider an element $y = q_1+i_2q_2 \in C_2$ such that $q_1 = w_1$ and $|z_2 - q_2| > r$
Since $q_1 = w_1$
$\Rightarrow u_1 < q_1$ and $q_1 < v_1$
$\Rightarrow \zeta <_C y$ and $y <_C \Psi$
Therefore $y \in (\zeta, \Psi)_C$
Since $|z_2 - q_2| > r$
$\Rightarrow y \notin B(\xi, r)$
Therefore we have an element $y \in (\zeta, \Psi)_C$ such that $y \notin B(\xi, r)$

$$\Rightarrow (\zeta, \Psi)_C \not\subseteq B(\xi, r)$$

Possibility-2nd: If $u_1 < w_1$, $w_1 = v_1$ and $w_2 < v_2$
Consider an element $y = q_1+i_2q_2 \in C_2$ such that $u_1 < q_1 < w_1$ and $|z_2 - q_2| > r$
Since $u_1 < q_1 \Rightarrow \zeta <_C y$
Since $q_1 < w_1$ and $w_1 = v_1$

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$\Rightarrow y <_C \Psi$

Therefore $y \in (\zeta, \Psi)_C$

Since $|z_2 - q_2| > r$

$\Rightarrow y \notin B(\xi, r)$

Therefore we have an element $y \in (\zeta, \Psi)_C$ such that $y \notin B(\xi, r)$

$$\Rightarrow (\zeta, \Psi)_C \not\subseteq B(\xi, r)$$

Possibility-3rd: If $u_1 = w_1$, $u_2 < w_2$ and $w_1 < v_1$

Consider an element $y = q_1 + i_2 q_2 \in \mathbb{C}_2$ such that $w_1 < q_1 < v_1$ and $|z_2 - q_2| > r$

Since $u_1 = w_1$ and $w_1 < q_1 \Rightarrow \zeta <_C y$

Since $q_1 < v_1 \Rightarrow y <_C \Psi$

Therefore $y \in (\zeta, \Psi)_C$

Since $|z_2 - q_2| > r$

$\Rightarrow y \notin B(\xi, r)$

Therefore we have an element $y \in (\zeta, \Psi)_C$ such that $y \notin B(\xi, r)$

$\Rightarrow (\zeta, \Psi)_C \not\subseteq B(\xi, r)$

Case-B If $u_1 = v_1$ & $a_3 \neq b_3$

Since $a_3 \neq b_3 \Rightarrow a_3 < b_3$

Consider an element $s = a_1 + i_1 a_2 + i_2 c_3 + i_1 i_2 c_4 \in \mathbb{C}_2$ such that $a_3 < c_3 < b_3$ and $(x_4 - c_4) > r$

Therefore $s \in (\zeta, \Psi)_C$ and $s \notin B(\xi, r)$

Also in this situation $(\zeta, \Psi)_C \not\subseteq B(\xi, r)$

Finally the ball $B(\xi, r)$; $r < \infty$ cannot contain any open interval $(\zeta, \Psi)_C$ where $u_1 \neq v_1$ or $u_1 = v_1$, $a_3 \neq b_3$

Hence $(\zeta, \Psi)_C$ cannot be contained in any ball

$B(\xi, r)$; $r < \infty$

3.3.3 Theorem

The Real order topology is strictly finer than Norm topology.

Proof- Since the Real order topology and Idempotent order topology are not comparable.[By Theorem-3.2.1]

Therefore there exist a set $Q \subseteq \mathbb{C}_2$ which will be open w.r.t. Real order topology and will not be open w.r.t. Idempotent order topology.

Since Q is not open w.r.t. Idempotent order topology.

Therefore, from Theorem-3.1.3

Q will not be open w.r.t. Norm topology

Therefore we have a set Q which is open w.r.t. Real order topology and not open w.r.t. Norm topology.

Therefore Idempotent order topology and norm topology are not equivalent and Norm topology cannot be finer than Idempotent order topology

Now we want to show for all open ball $B(\xi, r)$ and for all $\eta \in B(\xi, r)$ then there exist $(\zeta, \Psi)_R$ such that $\eta \in (\zeta, \Psi)_R \subseteq B(\xi, r)$

Let us consider an arbitrary ball $B(\xi, r)$ and consider an arbitrary element η of $B(\xi, r) \Rightarrow \eta \in B(\xi, r)$

Let $\eta = y_1 + i_1 y_2 + i_2 y_3 + i_1 i_2 y_4$

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1st Method- Since $\eta \in B(\xi, r)$ then there exist a ball
$B(\eta, s); s > 0$ such that $B(\eta, s) \subseteq B(\xi, r)$
Since $\eta = y_1+i_1y_2+i_2y_3+i_1i_2y_4$ then
$(\zeta, \Psi)_R$ will be the proper subset of $B(\eta=y_1+i_1y_2+i_2y_3+i_1i_2y_4, r)$
Where $\zeta = y_1+i_1y_2+i_2y_3+i_1i_2(y_4-\epsilon)$,
$\Psi = y_1+i_1y_2+i_2y_3+i_1i_2(y_4+\epsilon)$ and $\epsilon = Min(d_1,d_2)$ and $d_1^2+d_2^2 < 2r^2$
[By Lemma-3.3.1]
Therefore for $\eta \in B(\xi, r)$ we have a set $(\zeta, \Psi)_R$ such that $\eta \in (\zeta, \Psi)_R$ and $(\zeta, \Psi)_R \subseteq B(\xi, r)$.
2nd Method-Since $\eta \in B(\xi, r)$

$$\Rightarrow \sqrt{\frac{\left|{}^1\xi-{}^1\eta\right|^2+\left|{}^2\xi-{}^2\eta\right|^2}{2}} < r$$

$$\Rightarrow \left|{}^1\xi-{}^1\eta\right|^2+\left|{}^2\xi-{}^2\eta\right|^2 < 2r^2$$

Let $\left|{}^1\xi-{}^1\eta\right|^2 = d_1^{\,2}, \left|{}^2\xi-{}^2\eta\right| = d_2^{\,2}$
$\Rightarrow d_1^2+d_2^2 < 2r^2$
There exist $d_3, d_4 > 0$ such that $d_1 < d_3$, $d_2 < d_4$ and $d_3^2 + d_4^2 < 2r^2$
Consider a set $(\zeta, \Psi)_R$ such that
$\zeta = y_1+i_1y_2+i_2y_3+i_1i_2(y_4-\epsilon)$, $\Psi = y_1+i_1y_2+i_2y_3+i_1i_2(y_4+\epsilon)$ and $\epsilon = Min(d_3-d_1, d_4-d_2)$
Obviously $\eta \in (\zeta, \Psi)_R$

Let $\quad Y \in (\zeta, \Psi)_R \quad \ldots(48)$

$\Rightarrow \zeta <_R Y <_R \Psi$
$\Rightarrow Y = y_1+i_1y_2+i_2y_3+i_1i_2a$ where $(y_4-\epsilon) < a < (y_4+\epsilon)$
Therefore
${}^1Y = (y_1+a)+i_1(y_2-y_3)$ and ${}^2Y = (y_1-a)+i_1(y_2+y_3)$
Since $\eta = y_1+i_1y_2+i_2y_3+i_1i_2y_4$
Therefore ${}^1\eta = (y_1+y_4)+i_1(y_2-y_3)$ and ${}^2\eta = (y_1-y_4) + i_1(y_2+y_3)$
Now $\left|{}^1\xi-{}^1Y\right| \le \left|{}^1\xi-{}^1\eta\right| + \left|{}^1\eta-{}^1Y\right|$

$$\Rightarrow \left|{}^1\xi-{}^1Y\right| \le d_1 + |y_4 - a|$$

$$\Rightarrow \left|{}^1\xi-{}^1Y\right| \le d_1 \pm (y_4 - a)$$

Since $(y_4-\epsilon) < a < (y_4+\epsilon)$ therefore $\pm(y_4 - a) < \epsilon$

$$\Rightarrow \left|{}^1\xi-{}^1Y\right| < d_1+\epsilon$$

Since $\epsilon = Min(d_3-d_1, d_4-d_2)$
Therefore $\left|{}^1\xi-{}^1Y\right| < d_3$
Similarly $\left|{}^2\xi-{}^2Y\right| < d_4$

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Since $d_3^2+d_4^2< 2r^2$

Therefore $\left|{}^1\xi - {}^1Y\right|^2 + \left|{}^2\xi - {}^2Y\right|^2 < 2r^2$

$$\Rightarrow \sqrt{\frac{\left|{}^1\xi - {}^1Y\right|^2 + \left|{}^2\xi - {}^2Y\right|^2}{2}} < r$$

Therefore $\|\xi - Y\| < r$

$$\Rightarrow Y \in B(\xi, r) \qquad \ldots(49)$$

From (48), (49)

$$(\zeta, \Psi)_R \subseteq B(\xi, r)$$

Therefore for $\eta \in B(\xi, r)$ we have a set $(\zeta, \Psi)_R$ such that $\eta \in (\zeta, \Psi)_R$ and $(\zeta, \Psi)_R \subseteq B(\xi, r)$. Hence it proves that real order topology is strictly finer than Norm topology.

Compiling Theorems 3.3.3, 2.3.1, 2.3.2 and corollary 2.3.3, 2.3.14 together, result to new corollary which states that

3.3.4 Corollary

The topology $\mathfrak{T}_1$ (and therefore $\mathfrak{T}_2$, $\mathfrak{T}_3$, $\mathfrak{T}_1^*$, $\mathfrak{T}_2^*$ and $\mathfrak{T}_3^*$) on $\mathbb{C}_2$ is strictly finer than the topology $\mathfrak{T}_N$ (and therefore $\mathfrak{T}_I$ and $\mathfrak{T}_C$) on $\mathbb{C}_2$.

References Références Referencias

1. Futagawa, M.: On the theory of functions of a quaternary Variable, Tohoku Math. J., 29 (1928), 175-222.
2. Futagawa, M.: On the theory of functions of a quaternary Variable, II, Tohoku Math. J., 35 (1932), 69-120.
3. Price, G.B.: An introduction to multicomplex spaces and Functions, Marcel Dekker Inc., New York, 1991.
4. Riley, J.D.: Contributions to the theory of function of a bicomplex variable, Tohoku Math. J., 2nd series 5 (1953), 132-165.
5. Ringleb, F.: Beitrage Zur Funktionen theorie in Hyperkomplexon Systemen, I, Rend. Circ. Mat. Palermo, 57 (1933), 311-340.
6. Segre, C.: Le Rappresentazioni Reali Delle Forme Complesse e Gli enti Iperalgebrici, Math. Ann. 40 (1892), 413-467.
7. Singh, Sukhdev: Certain order topologies on the bicomplex Space,M.Phil Dissertation, Dr. B. R. Ambedkar University, Agra, 2009.
8. Srivastava, Rajiv K.: Bicomplex Numbers: Analysis and applications, Math. Student, 72 (1-4) 2003, 69-87.
9. Srivastava, Rajiv K.: Certain topological aspects of the Bicomplex space, Bull. Pure & Appl. Maths., 2(2)(2008),222-234.

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