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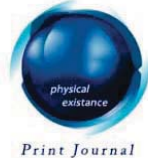
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Abstract- This communication investigates the effect of magnetic field on oscillatory flow with the combined effects of fluctuating heat and mass transfer past vertical parallel porous flat plates. It is assumed that vertical channel is rotating with angular velocity Ω . The periodic suction velocity is assumed at the plate and other plate oscillating with periodic free stream velocity. The governing equations are solved by adopting complex variable notations. The analytical expressions for velocity and temperature fields are obtained using perturbation technique. The effects of various parameters on mean primary, mean secondary velocity, mean temperature, mean concentration, transient velocity, transient temperature, transient concentration and rate of heat and mass transfer in terms of amplitude and phase differences have been discussed and shown graphically.

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I. INTRODUCTION

Magnetohydrodynamics (MHD) (magneto fluid dynamics or hydromagnetics) is the study of the dynamics of electrically conducting fluids. Examples of such fluids are include plasmas, liquid metals, and salt water or electrolytes. The word magnetohydrodynamics (MHD) is derived from magneto- meaning magnetic field, hydro- meaning liquid, and -dynamics meaning movement. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. MHD applies quite well to astrophysics, content of the universe is made up of plasma, including stars, the interplanetary medium (space between the planets), the interstellar medium (space between the stars), the intergalactic medium, nebulae and jets. Sunspots are caused by the Sun's magnetic fields, the solar wind is also governed by MHD. However, magnetohydrodynamic effects transfer the Sun's angular momentum into the outer solar system, slowing its rotation. MHD is related to engineering problems such as plasma confinement, liquid-metal cooling of nuclear reactors, and electromagnetic casting. The working principle involves electrification of the propellant (gas or water) which can then be directed by a magnetic field, pushing the vehicle in

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the opposite direction. An important task in cancer research is developing more precise methods for delivery of medicine to affected areas. One method involves the binding of medicine to biologically compatible magnetic particles (e.g. ferrofluids), which are guided to the target via careful placement of permanent magnets on the external body.

The flow problems of an electrically conducting fluids under the influence of magnetic field have attracted the interest of many authors in view of its applications to geophysics, astrophysics, engineering, and to the boundary layer control in the field of aerodynamics. On the other hand in view of the increasing technical applications using magnetohydrodynamics effect, it is desirable to extend many of the available viscous hydrodynamic solution to include the effects of magnetic field for those cases when the viscous fluid is electrically conducting. Rossow [1], Greenspan and Carrier [2] have studied extensively the hydromagnetic effects on the flow past a plate with or without injection/suction. The hydromagnetic channel flow and temperature field was investigated by Attia and Kotab [3]. Hossain et al. [4] have studied the MHD free convection flow when the surface kept at oscillating surface heat flux.

In view of applications of the flow through porous medium with the effect of magnetic field, attract attention of a number of scholars. Aldoss et al [5], Helmy [6] and Kim [7] studied the magnetohydrodynamic mixed convection from a vertical plate in a porous medium. Unsteady free convection flow with the combined effect of thermal and mass diffusion in the presence of magnetic field and Hall effect is investigated by Takhar et.al [8]. Ahmed et al [9] studied the thermal diffusion effect on a three-dimensional MHD free convection with mass transfer flow from a porous vertical plate and Chamkha [10] also investigates MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The MHD flow between two parallel horizontal porous plate also investigated by Chaudhary et al [11]. Sharma [12] studied the simultaneous thermal and mass diffusion in three dimensional mixed convection flows in the presence of porous medium. Singh et.al [13] and Lai and Kulacki [14] have been studied the free convective flow past vertical wall. Nield [15] studied convection flow through porous medium with inclined temperature gradient. Kelleher et al. [16] studied the heat transfer response of laminar free convection boundary layers along vertical heated plates to surface temperature oscillations. Sharma et al [17] studied the unsteady free convection oscillatory flow through porous medium with periodic temperature variation. Also the oscillatory Couette flows in a rotating system have been studied by Jana and Datta [18] Muzumder [19], and Ganapathy [20]. Raptis and Peridikis [21] also studied the oscillatory flow through porous medium in the presence of convection.

Therefore the object of the present paper is to investigate the oscillatory flow through rotating porous vertical channel in the presence of magnetic field with fluctuating thermal and mass diffusion assuming periodic suction velocity at the plate and other plate which is also fluctuating with periodic free stream velocity about a non zero constant mean. The analytical solutions for mean primary, mean secondary velocity, transient velocity, transient temperature and concentration are obtained using regular perturbation technique. The effect of various parameters on flow characteristic are discussed and shown graphically.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider an oscillatory free convective flow of a conducting viscous incompressible fluid through highly porous medium bounded between two infinite vertical porous plates distance d

apart. The periodic suction velocity is applied at the stationary plate $z^* = 0$ and other plate at $z^* = d$, which is oscillating in its own plane with a velocity U^* about a non zero constant mean velocity U_0 . The origin is assumed to be at the plate $z^* = 0$ and the channel is oriented vertically upward along the x^* -axis. The channel rotates as a rigid body with uniform angular velocity Ω^* about z^* -axis, which is perpendicular to the vertical plane confined with a viscous fluid occupying the porous region. Since the plates are infinite in extent, all the physical quantities except the pressure, depend only on z^* and t^* . Denoting the velocity components u^*, v^*, w^* in the x^*, y^*, z^* directions, respectively, temperature by T^* and concentration by C^* . The flow in porous medium involves small velocities permitting the neglect of heat due to viscous dissipation in governing equation. A magnetic field of constant intensity is applied perpendicular to the channel.

The basic equation of magnetofluidynamics and conventional fluid dynamics are different by only additional force term due to electromagnetic field. The Maxwell's equations have to be satisfied in the entire field. In order to derive the basic equations for the problem under consideration, the following assumptions are made:

1. The flow is steady and laminar and the magnetic field is applied perpendicular to the plate.
2. The fluid under consideration is viscous, incompressible and finitely conducting with constant physical properties.
3. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
4. Hall Effect, electrical and polarization effects are neglected.

The equation expressing the conservation of mass and energy transfer in rotating frame of reference are given by

$$w^* = -w_0(1 + \varepsilon \cos \omega^* t^*), \quad (1)$$

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + g\beta(T^* - T_d^*) \\ & + g\beta_c(C^* - C_d^*) + \nu \frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\nu u^*}{k^*} - \frac{(\vec{J} \times \vec{B})}{\rho}, \end{aligned} \quad (2)$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\nu v^*}{k^*} - \frac{(\vec{J} \times \vec{B})}{\rho}, \quad (3)$$

where the fourth term on the right hand side of equations (2-3) is the Lorentz force due to magnetic field \vec{B} , and is given by

$$\vec{J} \times \vec{B} = \sigma(\vec{v} \times \vec{B}) \times \vec{B} \quad (4)$$

Using (4) in equations (2) and (3), we have

$$\begin{aligned} \frac{\partial u^*}{\partial t^*} + w^* \frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* = & -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + g\beta(T^* - T_d^*) \\ & + g\beta_c(C^* - C_d^*) + \nu \frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\nu u^*}{k^*} - \frac{\sigma B^2}{\rho} u^* \end{aligned} \quad (5)$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\nu v^*}{k^*} - \frac{\sigma B^2}{\rho} v^*, \quad (6)$$

$$\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T^*}{\partial z^{*2}}, \quad (7)$$

$$\frac{\partial C^*}{\partial t^*} + w^* \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}}. \quad (8)$$

where g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β_c is the volumetric coefficient of expansion for concentration, T^* is the temperature, T_d^* is the temperature in free stream, ν is the kinematic viscosity, Ω^* is the angular velocity, k^* is the permeability, C_p is the specific heat at constant pressure, p^* is the pressure, ρ is the density, t^* is the time and κ is the thermal conductivity, ω^* frequency of fluctuations.

The boundary conditions of the problem are

$$\left. \begin{aligned} z = 0: u^* = 0, \quad v^* = 0, \quad T^* = T_0^* + \varepsilon(T_0^* - T_d^*) \cos \omega^* t^*, \\ C^* = C_0^* + \varepsilon(C_0^* - C_d^*) \cos \omega^* t^* \\ z = d: u^* = v^* = U^* = U_0(1 + \varepsilon \cos \omega^* t^*), \quad T^* = T_d^*, C^* = C_d^* \end{aligned} \right\} \quad (9)$$

Considering $u + iv = U$ and eliminating the pressure gradient from (5) and (6), we have

$$\begin{aligned} \frac{\partial U^*}{\partial t^*} + w^* \frac{\partial U^*}{\partial z^*} + 2i\Omega^* U^* = g\beta(T^* - T_d^*) \\ + g\beta_c(C^* - C_d^*) + \nu \frac{\partial^2 U^*}{\partial z^{*2}} - \frac{\nu U^*}{k^*} - \frac{\sigma B^2}{\rho} U^*, \end{aligned} \quad (10)$$

We introduce the following non-dimensional quantities as:

$$z = \frac{z^*}{d}, \quad u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{U_0}, \quad \omega = \frac{d^2 \omega^*}{\nu}, \quad \theta = \frac{(T^* - T_d^*)}{(T_0^* - T_d^*)},$$

$$k = \frac{k^*}{d^2}, \quad t = \omega^* t^*, \quad \lambda (\text{Suction parameter}) = \frac{d w_0}{\nu},$$

$$\alpha (\text{Thermal diffusivity}) = \frac{\kappa}{\rho C_p},$$

$$M (\text{Hartmann Number}) = \sqrt{\frac{\sigma B^2 d^2}{\rho \nu U_0}}, \quad Sc (\text{Schmidt Number}) = \frac{\nu}{D},$$

$$Gr (\text{Grashof number}) = \frac{g\beta(T_0^* - T_d^*)d^2}{\nu U_0}, \quad Pr (\text{Prandtl number}) = \frac{\nu}{\alpha},$$

$$C = \frac{(C^* - C_d^*)}{(C_0^* - C_d^*)},$$

$$Gc \text{ (modified Grashof number)} = \frac{g \beta_c (C_0^* - C_d^*) d^2}{\nu U_0}$$

Substituting these non-dimensional quantities in equations (7), (8) and (10), we get

$$\omega \frac{\partial U}{\partial t} - (1 + \varepsilon \cos t) \lambda \frac{\partial U}{\partial z} + 2iRU = Gr \lambda^2 \theta + Gc \lambda^2 C + \frac{\partial^2 U}{\partial z^2} - \frac{U}{k} - M^2 U, \tag{11}$$

$$\omega \frac{\partial \theta}{\partial t} - (1 + \varepsilon \cos t) \lambda \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2}, \tag{12}$$

$$\omega \frac{\partial C}{\partial t} - (1 + \varepsilon \cos t) \lambda \frac{\partial C}{\partial z} = \frac{1}{Sc} \frac{\partial^2 C}{\partial z^2}, \tag{13}$$

The corresponding boundary conditions (9) become

$$\left. \begin{aligned} z = 0: U = 0, \quad \theta = 1 + \varepsilon \cos t, \quad C = 1 + \varepsilon \cos t \\ z = d: U = 1 + \varepsilon \cos t, \quad \theta = 0, \quad C = 0. \end{aligned} \right\} \tag{14}$$

III. SOLUTION OF THE PROBLEM

In order to solve the problem, we assume the solutions of the following form because amplitude $\varepsilon (<< 1)$ of the variation of temperature is very small

$$\left. \begin{aligned} U(z,t) &= U_0(z) + \varepsilon U_1(z) e^{-it} + \dots \\ \theta(z,t) &= \theta_0(z) + \varepsilon \theta_1(z) e^{-it} + \dots \\ C(z,t) &= C_0(z) + \varepsilon C_1(z) e^{-it} + \dots \end{aligned} \right\} \tag{15}$$

Substituting (15) in equations (11), (12) and (13), and equating the coefficient of identical powers of ε and neglecting those of $\varepsilon^2, \varepsilon^3$ etc., we get

$$U_0'' + \lambda U_0' - 2iRU_0 - \frac{U_0}{k} - M^2 U_0 = -Gr \lambda^2 \theta_0 - Gc \lambda^2 C_0, \tag{16}$$

$$U_1''' + \lambda U_1' - 2iRU_1 + i\omega U_1 - \frac{U_1}{k} - M^2 U_1 = -Gr \lambda^2 \theta_1 - Gc \lambda^2 C_1 - \lambda U_0', \tag{17}$$

$$\theta_0'' + \lambda Pr \theta_0' = 0, \tag{18}$$

$$\theta_1''' + \lambda Pr \theta_1' + i\omega Pr \theta_1 = -\lambda Pr \theta_0' \tag{19}$$

$$C_0'' + \lambda Sc C_0' = 0, \tag{20}$$

$$C_1''' + \lambda Sc C_1' + i\omega Sc C_1 = -\lambda Sc C_0' \tag{21}$$

The corresponding boundary conditions (14) reduce to

$$\left. \begin{aligned} z = 0: U_0 = 0, \quad U_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad C_0 = 1, \quad C_1 = 1 \\ z = d: U_0 = 1, \quad U_1 = 1, \quad \theta_0 = 0, \quad \theta_1 = 0, \quad C_0 = 0, \quad C_1 = 0. \end{aligned} \right\} \tag{22}$$

Solving equations (16) to (21) under corresponding boundary conditions (22), we get

$$U_0(z) = n_{17} e^{n_{11}z} + n_{16} e^{n_{12}z} + n_{13} e^{-\lambda Pr z} + n_{14} e^{-\lambda Sc z} + n_{15} \tag{23}$$

$$U_1(z) = n_{31} e^{n_{18}z} + n_{30} e^{n_{19}z} + n_{20} e^{n_{11}z} + n_{21} e^{n_{12}z} + n_{22} e^{-\lambda Pr z} + n_{23} e^{-\lambda Sc z} + n_{24} e^{n_2 z} + n_{25} e^{n_1 z} + n_{26} e^{n_7 z} + n_{27} e^{n_6 z} \tag{24}$$

$$\theta_0(z) = \frac{1}{(1 - e^{-\lambda Pr})} (e^{-\lambda Pr z} - e^{-\lambda Pr}) \tag{25}$$

$$\theta_1(z) = n_4 e^{n_2 z} + n_5 e^{n_1 z} + n_3 e^{-\lambda Pr z} \tag{26}$$

$$C_0(z) = \frac{1}{(1 - e^{-\lambda Sc})} (e^{-\lambda Sc z} - e^{-\lambda Sc}) \tag{27}$$

$$C_1(z) = n_9 e^{n_7 z} + n_{10} e^{n_6 z} + n_8 e^{-\lambda Sc z} \tag{28}$$

where

$$n_1 = \frac{1}{2} \left[-\lambda Pr + \sqrt{\lambda^2 Pr^2 - 4i\omega Pr} \right]$$

$$n_2 = \frac{1}{2} \left[-\lambda Pr - \sqrt{\lambda^2 Pr^2 - 4i\omega Pr} \right]$$

$$n_3 = \frac{\lambda^2 Pr}{i(1 - e^{-\lambda Pr})\omega}$$

$$n_4 = \frac{e^{n_1} - n_3 (e^{n_1} - e^{-\lambda Pr})}{(e^{n_1} - e^{n_2})}$$

$$n_5 = - \left[\frac{e^{n_2} - n_3 (e^{n_2} - e^{-\lambda Pr})}{(e^{n_1} - e^{n_2})} \right]$$

$$n_6 = \frac{1}{2} \left[-\lambda Sc + \sqrt{\lambda^2 Sc^2 - 4i\omega Sc} \right]$$

$$n_7 = \frac{1}{2} \left[-\lambda Sc - \sqrt{\lambda^2 Sc^2 - 4i\omega Sc} \right]$$

$$n_8 = \frac{\lambda^2 Sc}{i(1 - e^{-\lambda Sc})\omega}$$

$$n_9 = \frac{e^{n_6} - n_8 (e^{n_6} - e^{-\lambda Sc})}{(e^{n_6} - e^{n_7})}$$

$$n_{10} = - \left[\frac{e^{n_7} - n_8 (e^{n_6} - e^{-\lambda Sc})}{(e^{n_6} - e^{n_7})} \right]$$

$$n_{11} = \frac{1}{2} \left[-\lambda + \sqrt{\lambda^2 + 4 \left(2iR + \frac{1}{k} + M^2 \right)} \right]$$

$$n_{12} = \frac{1}{2} \left[-\lambda - \sqrt{\lambda^2 + 4 \left(2iR + \frac{1}{k} + M^2 \right)} \right]$$

$$n_{13} = - \frac{Gr \lambda^2}{(1 - e^{-\lambda Pr}) [\lambda^2 Pr^2 - \lambda^2 Pr - (2iR + \frac{1}{k} + M^2)]}$$

$$n_{14} = - \frac{Gc \lambda^2}{(1 - e^{-\lambda Sc}) [\lambda^2 Sc^2 - \lambda^2 Sc - (2iR + \frac{1}{k} + M^2)]}$$

$$n_{15} = - \frac{Gr \lambda^2 e^{-\lambda Pr}}{(1 - e^{-\lambda Pr}) (2iR + \frac{1}{k} + M^2)} - \frac{Gc \lambda^2 e^{-\lambda Sc}}{(1 - e^{-\lambda Sc}) (2iR + \frac{1}{k} + M^2)}$$

$$n_{16} = \frac{1 + (n_{13} + n_{14} + n_{15}) e^{n_{11}} - n_{13} e^{-\lambda Pr} - n_{14} e^{-\lambda Sc} - n_{15}}{e^{n_{12}} - e^{n_{11}}}$$

$$n_{17} = -n_{16} - n_{13} - n_{14} - n_{15}$$

$$n_{18} = \frac{1}{2} \left[-\lambda + \sqrt{\lambda^2 - 4 \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)} \right]$$

$$n_{19} = \frac{1}{2} \left[-\lambda - \sqrt{\lambda^2 - 4 \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)} \right]$$

$$n_{20} = - \frac{\lambda n_{11} n_{17}}{n_{11}^2 + \lambda n_{11} + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)}$$

$$n_{21} = - \frac{\lambda n_{12} n_{16}}{n_{12}^2 + \lambda n_{12} + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)}$$

$$n_{22} = \frac{\lambda^2 n_{13} Pr}{\lambda^2 Pr^2 - \lambda^2 Pr + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)} - \frac{\lambda^2 n_3 Gr}{\lambda^2 Pr^2 - \lambda^2 Pr + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)}$$

$$n_{23} = \frac{\lambda^2 n_{14} Sc}{\lambda^2 Sc^2 - \lambda^2 Sc + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)} - \frac{\lambda^2 n_8 Gc}{\lambda^2 Sc^2 - \lambda^2 Sc + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)}$$

$$n_{24} = - \frac{Gr \lambda^2 n_4}{n_2^2 + \lambda n_2 + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)}$$

$$n_{25} = - \frac{Gr \lambda^2 n_5}{n_1^2 + \lambda n_1 + \left(i\omega - 2iR - \frac{1}{k} - M^2 \right)}$$

$$n_{26} = - \frac{Gc \lambda^2 n_9}{n_7^2 + \lambda n_7 + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{27} = - \frac{Gc \lambda^2 n_{10}}{n_6^2 + \lambda n_6 + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{28} = n_{20} + n_{21} + n_{22} + n_{23} + n_{24} + n_{25} + n_{26} + n_{27}$$

$$n_{29} = 1 - n_{20} e^{n_{11}} - n_{21} e^{n_{12}} - n_{22} e^{-\lambda Pr} - n_{23} e^{-\lambda Sc} - n_{24} e^{n_2} - n_{25} e^{n_1} - n_{26} e^{n_7} - n_{27} e^{n_6}$$

$$n_{30} = \frac{n_{29} + n_{28} e^{n_{18}}}{e^{n_{19}} - e^{n_{18}}}, \quad n_{31} = -n_{30} - n_{28}.$$

IV. DISCUSSION AND CONCLUSIONS

a) Steady Flow

We take $U_0 = u_0 + i v_0$ in equation (23) and subsequent comparison of the real and imaginary parts gives the mean primary velocity u_0 and mean secondary velocity v_0 . The mean primary velocity is presented in Fig. 1 for fixed values of Gr, Gc and Sc=0.60 (for CO₂) in air (Pr = 0.71). The graph reveals that velocity increases with increasing suction parameter λ and reverse effect is observed for R (rotation parameter) and k (permeability of porous medium). This shows that the porosity and rotation of porous medium exert retarding influence on the primary flow. Fig.2 also shows mean primary velocity for different values of Gr (Grashof Number), Gc(Modified Grashof Number) and Sc(Schmidt Number). It is observed from the figure that the mean primary velocity increases rapidly with increasing either Gr or Gc. The magnitude of velocity is lesser in case of Sc=0.78 (NH₃) than that of Sc=0.60 (CO₂). Furthermore the mean primary velocity increases in the vicinity of the plate. It is interesting to note that if we increase magnetic field parameter M (Hartmann Number) , i.e. medium become conducting then the mean primary velocity become fluctuate sinusoidally.

The mean secondary velocity profiles is shown in Fig. 3 for the fixed values of Gr, Gc Sc and Pr=0.71(air). It is observed that it increases with increasing R while reverse phenomena is observed for λ . It is interesting to note that mean primary velocity increases while mean secondary velocity decreases with R and λ . It is also observed that due to increase in k mean secondary velocity decreases upto middle half of the channel then it increases. Fig.4 also showed the mean secondary velocity for different values of parameters. It is observed that it decreases with increasing either Gr, Gc and Sc. The magnitude is lower in case of NH₃ than that of CO₂. The amount of secondary velocity is much lower for Gc than that of Gr. Also due to increase intensity of magnetic field the mean secondary velocity fluctuating.

The mean temperature and concentration is presented in Fig.9. It is observed that both decreases with increasing λ . The mean temperature and concentration decreases exponentially, the magnitude of concentration is less in case of NH₃ than that of CO₂

b) *Unsteady Flow*

Replacing the unsteady parts

$U_1(z,t) = M_r + iM_i$, $\theta_1(z,t) = T_r + iT_i$, and $C_1(z,t) = C_r + iC_i$ respectively in equations (24), (26) and (28) we get

$$[U(z,t), \theta(z,t), C(z,t)] = [U_0(z), \theta_0(z), C_0(z)] + \varepsilon e^{-it} [(M_r + iM_i), (T_r + iT_i), (C_r + iC_i)] \quad (29)$$

The primary, secondary velocity fields, temperature and concentration in terms of the fluctuating components are

$$u(z,t) = u_0 + \varepsilon (M_r \cos t + M_i \sin t) \quad (30)$$

$$v(z,t) = v_0 + \varepsilon (M_i \cos t - M_r \sin t) \quad (31)$$

$$\theta(z,t) = \theta_0 + \varepsilon (T_r \cos t + T_i \sin t) \quad (32)$$

$$C(z,t) = C_0 + \varepsilon (C_r \cos t + C_i \sin t) \quad (33)$$

Taking $t = \frac{\pi}{2}$ in equations (30) to (33) we get the expression for transient primary velocity, transient secondary velocity transient temperature and concentration as

$$u\left(z, \frac{\pi}{2}\right) = u_0(z) - \varepsilon M_i(z), \quad (34)$$

$$v\left(z, \frac{\pi}{2}\right) = v_0(z) + \varepsilon M_r(z), \quad (35)$$

$$\theta\left(z, \frac{\pi}{2}\right) = \theta_0(z) - \varepsilon T_i(z) \quad (36)$$

$$C\left(z, \frac{\pi}{2}\right) = C_0(z) - \varepsilon C_i(z) \quad (37)$$

The transient primary velocity component is shown in Fig.5 for fixed values of Pr, Gr, Gc, Sc and ω . It is observed that it decreases with increasing either R and k while transient primary velocity increases with increasing suction parameter λ . It is interesting to note that initially there is decrease in transient primary and then it increasing near the other plate which is fluctuating with free stream velocity. The transient primary velocity shift from positive to negative due to increase in intensity of magnetic field. Fig.6 also shows that due to increase in Gr and Gc the transient primary velocity increases. An increase in ω , the frequency of fluctuation transient velocity behave sinusoidally. The transient primary velocity increases with increasing Sc near the plate upto $z < 0.6$ than it decreases. It is interesting to note that due to increase in M, transient velocity is fluctuating sinusoidally.

The transient secondary velocity profiles is given in Fig.7 for different values of R, k and λ . It is observed that transient secondary velocity increases with increasing either R and k, while it decreasing with increase in λ . The amount of decrease in velocity is much lower due to increase in permeability of the porous medium. Physically this is true because the porous material offers resistance to the flow, so velocity decreases in porous medium. Fig.8 also represented transient secondary velocity for different values of Gr, Gc, Sc and ω . The graph reveals that transient secondary velocity decreases with either Gr and Gc. It is interesting to note that value of

transient secondary velocity is greater in case of NH_3 than that of CO_2 . Furthermore velocity decreases rapidly in the vicinity of the plate with ω than it increases near the other the plate which is fluctuating with free stream velocity. The transient secondary velocity Fig.7-8, become fluctuating with increasing M .

Transient temperature and transient concentration are given in Fig.10. It is observed that transient temperature and concentration both are decreasing with suction parameter. The temperature and concentration are decreasing exponentially with distance apart vertical channel.

Heat Transfer: In the dynamics of viscous fluid one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat exchange between the body and the fluid. Since at the boundary the heat exchanged between the fluid and the body is only due to conduction, according to Fourier's law, we have

$$q_w^* = -\kappa \left(\frac{\partial T^*}{\partial z^*} \right)_{z^*=0} \quad (38)$$

where z^* is the direction of the normal to the surface of the body. We can calculate the dimensionless coefficient of heat transfer in terms of Nusselt Number as follows

$$Nu = -\frac{q_w^* d}{\kappa (T_0^* - T_d^*)} = \left(\frac{\partial \theta}{\partial z} \right)_{z=0} = \left(\frac{\partial \theta_0}{\partial z} \right)_{z=0} + \varepsilon e^{-it} \left(\frac{\partial \theta_1}{\partial z} \right)_{z=0} \quad (39)$$

In terms of the amplitude and phase the rate of heat transfer can be written as:

$$Nu = \left(\frac{\partial \theta_0}{\partial z} \right)_{z=0} + \varepsilon |H| \cos(\phi - t) \quad (40)$$

where

$$H = H_r + i H_i = \text{coefficient of } \varepsilon e^{-it} \text{ in equation (40)}$$

$$|H| = \sqrt{H_r^2 + H_i^2}, \quad \tan \phi = H_i / H_r.$$

Mass Transfer: According to Fick's Law the dimensionless coefficient of mass transfer at the plate in terms of Shearwood Number is given as follows

$$Sh = \left(\frac{\partial C}{\partial z} \right)_{z=0} = \left(\frac{\partial C_0}{\partial z} \right)_{z=0} + \varepsilon e^{-it} \left(\frac{\partial C_1}{\partial z} \right)_{z=0} \quad (41)$$

In terms of the amplitude and phase the rate of mass transfer can be written as:

$$Sh = \left(\frac{\partial C_0}{\partial z} \right)_{z=0} + \varepsilon |S| \cos(\varphi - t) \quad (42)$$

where

$$S = S_r + i S_i = \text{coefficient of } \varepsilon e^{-it} \text{ in equation (42)}$$

$$|S| = \sqrt{S_r^2 + S_i^2}, \quad \tan \varphi = S_i / S_r.$$

The amplitudes of rate of heat and mass transfer in presented in Fig.11. The graph reveals that both are increases with increasing ω the frequency of fluctuations upto $\lambda < 0.8$ than they decreases for higher values of suction parameter. It is also observed that amplitude of mass transfer is higher in case of NH_3 than that of CO_2 .

Fig.12 shows the phases of rate of heat and mass transfer. It is observed from the figure that phases of heat and mass transfer increases with increasing ω . The phase of mass transfer increases with increase in Sc . The magnitude is higher for NH_3 than that of CO_2 . It is also observed from the figure that phases of heat and mass transfer increases for $\lambda < 0.5$ than they decreases and become negative for small values of ω .

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