Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 2 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling

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Abstract- In the present study, we have proposed a class of ratio-cum-product estimators for estimating finite population mean of the study variable in two phase sampling. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Comparison of the proposed class of estimators with other estimators is also worked out theoretically to demonstrate the superiority of the proposed estimator over the other estimators.

Keywords: ratio-cum-product, mean, two phase sampling, asymptotically optimum estimator, bias, mean square error.

GJSFR-F Classification : 62D05

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Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling

Waikhom Warseen Chanu [α] & B. K. Singh [σ]

Abstract- In the present study, we have proposed a class of ratio-cum-product estimators for estimating finite population mean $\bar{Y}$ of the study variable y in two phase sampling. The bias and mean square error of the proposed estimator have been obtained. The asymtotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Comparison of the proposed class of estimators with other estimators is also worked out theoretically to demonstrate the superiority of the proposed estimator over the other estimators.
Keywords: ratio-cum-product, mean, two phase sampling, asymptotically optimum estimator, bias, mean square error.

I. Introduction

The literature on survey sampling describes a great variety of techniques for using auxiliary information in order to obtain improved estimators for estimating some most common population parameter such as population total, population mean, population proportion, population ratio etc. More often we are interested in the estimation of the mean of a certain characteristic of a finite population on the basis of a sample taken from the population following a specified sampling procedure.

Use of auxiliary information has shown its significance in improving the efficiency of estimators of unknown population parameters. Cochran (1940) used auxiliary information in the form of population mean of auxiliary variate at estimation stage for the estimation of population parameters when study and auxiliary variate are positively correlated. In case of negative correlation between study variate and auxiliary variate, Robson (1957) defined product estimator for the estimation of population mean which was revisited by Murthy (1967). Ratio estimator performs better than simple mean estimator in case of positive correlation between study variate and auxiliary variate.

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For further discussion on ratio cum product estimator, the reader is referred to Singh (1967), Shah and Shah (1978), Singh and Tailor (2005), Tailor and Sharma (2009), Tailor and Sharma (2009), Sharma and Tailor (2010), Choudhury and Singh (2011) etc.

Notes

When the population mean $\bar{X}$ of the auxiliary variable x is unknown before start of the survey, it is estimated from a preliminary large sample on which only the auxiliary characteristic x is observed. The value of X in the estimator is then replaced by its estimate. After then a smaller second-phase sample of the variate of interest (study variate) y is then taken. This technique is known as double sampling or two-phase sampling. Neyman (1938) was the first to give the concept of double sampling in connection with collecting information on the strata sizes in a stratified sampling

Consider a finite population $U = (u_1, u_2, u_3, ..., u_N)$ of size N units, y and x are the study and auxiliary variate respectively. When the population mean $\bar{X}$ of x is not known, a first phase sample of size n_1 is drawn from the population on which only the x characteristic is measured in order to furnish a good estimate of $\bar{X}$. After then a second-phase sample of size n $(n < n_1)$ is drawn on which both the variates y and x are measured.

The usual ratio and product estimators in double sampling are:

$$\bar{Y_R}^d = \bar{y}\frac{\bar{x_1}}{\bar{x}}$$

and

$$\bar{Y_P}^d = \bar{y}\frac{\bar{z}}{\bar{z_1}}$$

where $\bar{x}, \bar{y}$ and $\bar{z}$ are the sample mean of x, y and z respectively based on the sample of size n out of the population N units and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ and $\bar{z_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} z_i$ denote the sample mean of x and z based on the first- phase sample of the size n_1.

Singh (1967) improved the ratio and the product methods of estimation by studying the ratio cum product estimator for estimating $\bar{Y}$ as

$$\bar{Y}_{RP} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\frac{\bar{z}}{\bar{Z}}\right)$$

Motivated by Singh (1967), Choudhury and Singh (2011) proposed a modified class of ratio cum product type of estimator for estimating population mean $\bar{Y}$ as

Notes

$$\bar{Y}_{RP}^{(\alpha)} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\frac{\bar{z}}{\bar{Z}}\right)^{\alpha}$$

Motivated by **?**) and as an extension to the work of Choudhury and Singh (2011), we have developed an improved class of ratio-cum-product estimators in double sampling to estimate the population mean $\bar{Y}$ theoretically and studied the properties of the proposed estimator.

II. The Proposed Estimator

The proposed improved class of ratio-cum-product estimators of population mean $\bar{Y}$ in two-phase sampling is given as

$$\bar{Y}_{RP}^{w(d)} = \bar{y}\left(\frac{\bar{x_1}}{\bar{x}}.\frac{\bar{z}}{\bar{z_1}}\right)^{\alpha}. \tag{1}$$

where α is a suitably chosen constant.

To obtain the bias and MSE of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation, we write

$$e_0 = \left(\bar{y} - \bar{Y}\right)/\bar{Y}, \quad e_1 = \left(\bar{x} - \bar{Y}\right)/\bar{X}, \quad e_2 = \left(\bar{x}_1 - \bar{X}\right)/\bar{X}, \quad e_3 = \left(\bar{z} - \bar{Z}\right)/\bar{Z},$$
$$e_4 = \left(\bar{z_1} - \bar{Z}\right)/\bar{Z}.$$

Expressing $Y_{RP}^{\bar{w}(d)}$ in terms of e's and neglecting higher power of e's, we have

$$Y_{RP}^{\bar{w}(d)} = \bar{Y}\left(1+e_0\right)\left\{\left(1+e_2\right)\left(1+e_1\right)^{-1}\left(1+e_3\right)\left(1+e_4\right)^{-1}\right\}^{\alpha}.$$

Assuming the sample size to be large enough so that $|e_1| < 1$, $|e_4| < 1$ and expanding $(1+e_1)^{-1}$, $(1+e_4)^{-1}$ in powers of e_1, e_4, multiplying out and neglecting higher powers of $e's$, we have

$$
\begin{aligned}
\bar{Y}_{RP}^{w(d)} =& \bar{Y}\left(1+e_0\right)\left[1-\left\{e_1-e_2-e_3+e_4-e_1^2-e_4^2+e_1e_2-e_1e_4\right.\right. \\
&\left.\left.+e_2e_4+e_1e_3-e_2e_3+e_3e_4\right\}\right]^{\alpha} \\
\bar{Y}_{RP}^{w(d)}-\bar{Y} =& \bar{Y}\left[e_0-\alpha\left(e_1-e_2-e_3+e_4-\frac{e_1^2}{2}+\frac{e_2^2}{2}+\frac{e_3^2}{2}-\frac{e_4^2}{2}+e_0e_1\right.\right. \\
&\left.-e_0e_2+e_0e_4-e_0e_3\right)+\alpha^2\left(\frac{e_1^2}{2}+\frac{e_2^2}{2}+\frac{e_3^2}{2}+\frac{e_4^2}{2}-e_1e_2\right. \\
&\left.\left.-e_1e_3+e_1e_4+e_2e_3-e_2e_4-e_3e_4\right)\right] \qquad (2)
\end{aligned}
$$

The following two cases will be considered separately.

Case I: When the second phase sample of size n is subsample of the first phase of size n_1.

Case II: when the second phase sample of size n is drawn independently of the first phase sample of size n_1.

CASE I

III. Bias, MSE and Optimum Value of $\bar{Y}_{RP}^{wd}$ in Case I

In this case, we have

$$
\begin{aligned}
&E\left(e_0\right)=E\left(e_1\right)=E\left(e_2\right)=E\left(e_3\right)=E\left(e_4\right)=0; \\
&E\left(e_0^2\right)=\left(\frac{1-f}{n}\right)C_Y^2;\quad E\left(e_1^2\right)=\left(\frac{1-f}{n}\right)C_X^2;\quad E\left(e_3^2\right)=\left(\frac{1-f}{n}\right)C_Z^2; \\
&E\left(e_2^2\right)=E\left(e_1e_2\right)=\left(\frac{1-f^*}{n}\right)C_X^2;\quad E\left(e_4^2\right)=E\left(e_3e_4\right)=\left(\frac{1-f^*}{n}\right)C_Z^2; \\
&E\left(e_0e_1\right)=\left(\frac{1-f}{n}\right)\rho_{YX}C_YC_X;\quad E\left(e_0e_2\right)=\left(\frac{1-f^*}{n}\right)\rho_{YX}C_YC_X; \\
&E\left(e_0e_3\right)=\left(\frac{1-f}{n}\right)\rho_{YZ}C_YC_Z;\quad E\left(e_0e_4\right)=\left(\frac{1-f^*}{n}\right)\rho_{YZ}C_YC_Z; \\
&E\left(e_1e_3\right)=\left(\frac{1-f}{n}\right)\rho_{XZ}C_XC_Z; \\
&E\left(e_1e_4\right)=E\left(e_2e_3\right)=E\left(e_2e_4\right)=\left(\frac{1-f^*}{n}\right)\rho_{XZ}C_XC_Z \qquad (3)
\end{aligned}
$$

where $f=\frac{n}{N}$ is the sampling fraction, $f^*=\frac{n_1}{N}$, $C_X^2=\frac{S_X^2}{\bar{X}^2}$, $C_Y^2=\frac{S_Y^2}{\bar{Y}^2}$, $C_Z^2=\frac{S_Z^2}{\bar{Z}^2}$.

Notes

Taking expectations on both the sides and using the results of (3) in (2), we get the bias of $\bar{Y}_{RP}^{w(d)}$ as

Notes

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_I = \bar{Y}\left(\frac{1-f_1}{2n}\right)\left[\alpha^2\left\{C_X^2 + C_Z^2\left(1-2K_{XZ}\right)\right\} + \alpha\left\{C_X^2\left(1-2K_{YX}\right) - C_Z^2\left(1-2K_{YZ}\right)\right\}\right] \quad (4)$$

$$= \bar{Y}\left(\frac{1-f_1}{2n}\right)\left[\alpha^2 K_3 + \alpha\left(K_1 - K_2\right)\right] \quad (5)$$

where $f_1 = \frac{n}{n_1}$, $K_{YX} = \rho_{YX}\frac{C_Y}{C_X}$, $K_1 = C_X^2\left(1-2K_{YX}\right)$, $K_2 = C_Z^2\left(1-2K_{YZ}\right)$, $K_3 = C_X^2 + C_Z^2\left(1-2K_{XZ}\right)$.

Now from equation (2), we have

$$\bar{Y}_{RP}^{w(d)} - \bar{Y} = \bar{Y}\left[e_0 - \alpha\left(e_1 - e_2 - e_3 + e_4\right)\right].$$

Squaring both the sides and taking expectations in the above equation and using the results of (3), we get the mean square error of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation as

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_I = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[\alpha^2\left\{C_X^2 + C_Z^2\left(1-2K_{XZ}\right)\right\} - 2\alpha\left(C_{YX} - C_{YZ}\right)\right]$$

$$= \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[\alpha^2 K_3 - 2\alpha S_1\right] \quad (6)$$

where $S_1 = C_{YX} - C_{YZ}$, $C_{YZ} = \rho_{YZ}C_Y C_Z$, $C_{YX} = \rho_{YX}C_Y C_X$

Differentiating $M\left(\bar{Y}_{RP}^{w(d)}\right)$ w.r.t α and equating to zero, we get

$$\alpha = \frac{S_1}{K_3}. \quad (7)$$

Now putting the optimum value of α from (7) in the proposed estimator (1), we get the asymptotically optimum estimator(AOE) as

$$\left(\bar{Y}_{RP}^{w(d)}\right)_{I(opt)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}\frac{\bar{z}}{\bar{z}_1}\right)^{I\alpha(opt)}.$$

Therefore, after putting the value of α in (4) and (6), we obtain the optimum bias and MSE of $\bar{Y}_{RP}^{w(d)}$ respectively as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)} = \bar{Y}\left(\frac{1-f_1}{2n}\right)\frac{S_1}{K_3}\left[K_1' - K_2'\right]$$

where $$K_1' = C_X^2\left(1-K_{YX}\right), \quad K_2' = C_Z^2\left(1-K_{YZ}\right).$$

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 - \bar{Y}^2\left(\frac{1-f_1}{n}\right)\frac{S_1^2}{K_3}. \tag{8}$$

Remark 1 *For $\alpha = 1$, the estimator reduces to ratio cum product estimator in double sampling. The bias and MSE of $\bar{Y}_{RP}^{(d)}$ are obtained by putting $\alpha = 1$ in relation (4) and (6) as follows*

$$B\left(\bar{Y}_{RP}^{(d)}\right)_I = \bar{Y}\left(\frac{1-f}{n}\right)\left[K_1' - C_{XZ} + C_{YZ}\right] \tag{9}$$

where $$K_1' = C_X^2\left(1-K_{YX}\right)$$

and $$MSE\left(\bar{Y}_{RP}^{(d)}\right)_I = \left(\frac{1-f}{n}\right)S_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left(K_1+K_4\right) \tag{10}$$

where $$K_4 = C_Z^2\left(1-2K_{XZ}+2K_{YZ}\right).$$

Remark 2 *For $\alpha = 1$ and when the auxiliary variate z is not used, i.e if z is non-zero constant, the proposed estimator reduces to the usual ratio estimator in two phase sampling. The bias and mean square error of $\bar{Y}_R^d$ can be obtained by putting $\alpha = 1$ and omitting the terms of z in equation (4) and (6), respectively as*

$$B\left(\bar{Y}_R^d\right)_I = \bar{Y}\left(\frac{1-f_1}{n}\right)K_1'$$

and

$$MSE\left(\bar{Y}_R^d\right)_I = \bar{Y}\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}\left(\frac{1-f_1}{n}\right)K_1. \tag{11}$$

Notes

Notes

Remark 3 *For $\alpha = 1$ and when the auxiliary variate x is not used, i.e if x is non-zero, the proposed estimator reduces to the usual product estimator in two phase sampling. The bias and mean square error of $\bar{Y}_P^d$ can be obtained by putting $\alpha = 1$ and omitting the terms of x in equation* (4) *and* (6), *respectively as*

$$B\left(\bar{Y}_P^d\right)_I = \bar{Y}\left(\frac{1-f_1}{n}\right)C_{YZ}$$

$$\text{and} \quad M\left(\bar{Y}_P^d\right)_I = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)K_5 \tag{12}$$

where $K_5 = C_X^2\left(1+2K_{YX}\right)$.

IV. Efficiency Comparisons

Compairison of the optimum proposed estimator $\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)}$

a) with sample mean per unit estimator $\bar{y}$

The MSE of sample mean $\bar{y}$ under SRSWOR sampling scheme is given by

$$V\left(\bar{y}\right) = \left(\frac{1-f}{n}\right)S_Y^2. \tag{13}$$

From equation (13) and (8), we observed

$$V\left(\bar{y}\right) - M\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha I(opt)} = \frac{S_1^2}{K_3} > 0. \tag{14}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

b) with ratio estimator in double sampling

From equation (11) and (8), we observed

$$M\left(\bar{Y}_R^d\right) - M\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left(K_1 + \frac{S_1^2}{K_3}\right) > 0. \tag{15}$$

if $K_1 > 0$, $K_3 > 0$ i.e. $K_{YX} < 1/2$, $K_{XZ} < 1/2$.

c) with product estimator in double sampling

From (12) and (8), we observed

$$M\left(\bar{Y}_P^d\right) - MSE\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[K_5 + \frac{S_1^2}{K_3}\right] > 0 \qquad (16)$$

if $K_3 > 0,\ K_5 > 0$ i.e. $K_{XZ} < 1/2$.

d) with ratio cum product estimators in double sampling

From (10) and (8), we observed

$$M\left(\bar{Y}_{RP}^d\right) - M\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left[K_1 + K_5 + \frac{S_1^2}{K_3}\right] > 0 \qquad (17)$$

if $K_1 > 0,\ K_3 > 0,\ K_5 > 0$ i.e. $K_{YX} < 1/2,\ K_{XZ} < 1/2,\ K_{XZ} - K_{YZ} < 1/2$.

Now we state the theorem

Theorem 4 *To the first degree of approximation, the proposed class of estimators* $\bar{Y}_{Rd}^{w(d)}$ *under the optimality (7) is consider to be more efficient than* $\bar{Y}_R^d,\ \bar{Y}_P^d,\ \bar{Y}_{RP}^d$ *and* $\bar{y}$ *under the given conditions* $K_1,\ K_3,\ K_4,$ *and* $K_5 > 0$, *where* $K_1 = C_X^2\left(1 - 2K_{YX}\right)$, $K_3 = C_X^2 + C_Z^2\left(1 - 2K_{XZ}\right)$, $K_4 = C_Z^2\left(1 - 2K_{XZ} + 2K_{YZ}\right)$ *and* $K_5 = C_X^2\left(1 + 2K_{YX}\right)$.

V. Bias, MSE and Optimum Value of $\bar{Y}_{RP}^{w(d)}$ in Case II

In this case, we have

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = E\left(e_4\right) = 0;$$

$$E\left(e_0^2\right) = \left(\frac{1-f}{n}\right)C_Y^2;\quad E\left(e_1^2\right) = \left(\frac{1-f}{n}\right)C_X^2;\quad E\left(e_3^2\right) = \left(\frac{1-f}{n}\right)C_Z^2;$$

$$E\left(e_2^2\right) = E\left(e_1e_2\right) = \left(\frac{1-f^*}{n}\right)C_X^2;\quad E\left(e_4^2\right) = E\left(e_3e_4\right) = \left(\frac{1-f^*}{n}\right)C_Z^2;$$

$$E\left(e_0e_1\right) = \left(\frac{1-f}{n}\right)\rho_{YX}C_YC_X;\quad E\left(e_0e_3\right) = \left(\frac{1-f}{n}\right)\rho_{YZ}C_YC_Z;$$

$$E\left(e_1e_3\right) = \left(\frac{1-f}{n}\right)\rho_{XZ}C_XC_Z;\quad E\left(e_2e_4\right) = \left(\frac{1-f^*}{n}\right)\rho_{XZ}C_XC_Z;$$

$$E\left(e_0e_2\right) = E\left(e_0e_4\right) = E\left(e_1e_2\right) = E\left(e_1e_4\right) = E\left(e_3e_2\right) = E\left(e_3e_4\right) = 0. \qquad (18)$$

Notes

Taking expectations in (2) and using the results of (18), we get the bias of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation as

Notes

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}\left[\alpha^2 N_1 K_3 + \alpha\left\{f''\left(C_X^2 - C_Z^2\right) - f' S_1\right\}\right].$$

where $\qquad f' = \frac{1-f}{n}, \quad f'' = \frac{1-f_1}{2n}, \quad N_1 = \frac{1}{2}\left(\frac{1}{n} + \frac{1}{n_1} - \frac{2}{N}\right).$

Squaring and taking expectations in both the sides of (2) and using the results of (18), we obtain the MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)$ to the first degree of approximation as

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}^2 f' C_Y^2 + \bar{Y}^2 2\left[\alpha^2 N_1 K_3 - \alpha f' S_1\right]. \tag{19}$$

Minimization of (19) is obtained with optimum value of α as

$$\alpha = \frac{f' S_1}{2N_1 K_3} = \alpha_{II(opt)}. \tag{20}$$

Substituting the value of α from (20) in (1) gives the AOE of (1) as

$$\left\{\bar{Y}_{RP}^{w(d)}\right\}_{II(opt)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}.\frac{\bar{z}}{\bar{z}_1}\right)^{\alpha_{II(opt)}}. \tag{21}$$

Thus, the resulting bias and MSE of (21) are, respectively given as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{II\alpha(opt)} = \bar{Y}\frac{f' S_1}{2N_1 K_3}\left[f'' C_1 - \frac{f' S_1}{2N_1 K_1}\right]$$

where $\qquad C_1 = C_X^2 - C_Z^2$ and

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II\alpha(opt)} = \bar{Y}^2 f' C_Y^2 - \bar{Y}^2\frac{f'^2 S_1^2}{2N_1 K_3}.$$

For simplicity, we assume that the population size N is large enough as compared to the sample sizes n and n_1 so that the finite population correction (FPC) terms $1/N$ and $2/N$ are ignored.

Ignoring the FPC in (19), the MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)_{II}$ reduces to

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}\frac{C_Y^2}{n} + \bar{Y}^2\left[\alpha^2\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \alpha\frac{S_1}{n}\right]$$

which is minimized for

$$\alpha = \frac{n_1 S_1}{(n+n_1)K_3} = \alpha^*_{II(opt)} \quad \text{(say)} \tag{22}$$

Substituting the value of α from (22) in (1), we obtained AOE of (1) as

$$\left(\bar{Y}_{RP}^{w(d)}\right)^* = \bar{Y}\left[\frac{x}{x_1}.\frac{z_1}{z}\right]^{\alpha^*_{II(opt)}}.$$

Therefore, the resulting MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)^*$ is

$$M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{C_Y^2}{n} - \bar{Y}^2\frac{n_1 S_1^2}{n(n+n_1)K_3}. \tag{23}$$

Remark 5 *For $\alpha = 1$, the proposed estimator reduces to ratio cum product estimator in double sampling and MSE is given as*

$$M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2 2\left[\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \frac{S_1}{n}\right]. \tag{24}$$

Ignoring the FPC, the variance of $\bar{y}$ under SRSWOR is given by

$$V(y)_{II} = \bar{Y}^2\frac{C_Y^2}{n}. \tag{25}$$

and the MSE of $\left(\bar{Y}_{Rd}\right)_{II}$ and $\left(\bar{Y}_{Pd}\right)_{II}$ are given by

$$M\left(\bar{Y}_{Rd}\right)_{II} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2\left[C_X^2\left(\frac{2}{n}-\frac{1}{n_1}-\frac{2}{n}K_{YX}\right)\right] \tag{26}$$

Notes

and

$$\left(\bar{Y}_{Pd}\right)_{II} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2\left[\frac{C_X^2}{n} + \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n_1}\right)C_Z^2 + \frac{2C_{YZ}}{n}\right] \tag{27}$$

respectively.

Notes

VI. Efficiency Copmparisons

Compairison of the optimum proposed estimator $\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)}$

a) with sample mean per unit estimator

From (25) and (23), we observed that

$$V\left(\bar{y}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{n_1 S_1^2}{n\left(n+n_1\right)K_3} > 0 \tag{28}$$

if $$K_3 > 0 \text{ i.e } K_{XZ} < 1/2.$$

b) with ratio estimator in double sampling

From (26) and (23), we observed that

$$M\left(\bar{Y}_{Rd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\left[\frac{n_1 S_1^2}{n\left(n+n_1\right)K_3} + C_X^2 P\right] > \; 0 \tag{29}$$

if $$K_3 > 0,\; P > 0 \text{ i.e } K_{XZ} < 1/2,\; K_{YX} < 1 - \tfrac{n}{2n_1},$$

where $$P = \tfrac{2}{n}\left(1 - K_{YX}\right) - \tfrac{1}{n_1}.$$

c) with product estimator in double sampling

From (27) and (23), we observed that

$$M\left(\bar{Y}_{Pd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\left[Q - \frac{C_Z^2}{2n_1} + \frac{n_1 S_1^2}{n\left(n+n_1\right)K_3}\right] > 0 \tag{30}$$

if $$K_3 > 0 \text{ i.e } K_{XZ} < 1/2, \text{ where } Q = C_X^2 + \tfrac{C_Z^2}{2} + 3C_{YX}.$$

d) with ratio cum product estimator in double sampling

From (24) and (23), we observed that

$$M\left(\bar{Y}_{RPd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^{*}_{II\alpha(opt)} = \bar{Y}^{2}\left[\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \frac{S_1}{2n} + \frac{n_1 S_1^2}{n\left(n+n_1\right)K_3}\right] > 0 \tag{31}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

VII. Conclusion

We have developed an efficient class of ratio-cum-product estimators in two phase sampling. The comparative study shows that the proposed estimator $\bar{Y}_{RP}^{w(d)}$ established their superiority over sample mean $\bar{Y}$, ratio estimator $\bar{Y_R^d}$, product estimator$\bar{Y_P^d}$ and ratio-cum-product estimator $\bar{Y_{RP}^d}$ in two-phase sampling under the given conditions. Hence from the resulting equation (14), (15), (16) and (17), we conclude that under the given conditions the proposed estimator is consider to be the best estimator.

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Notes

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