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Some Statistical Properties of Exponentiated Weighted Weibull Distribution

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GJSFR-F Classification : *MSC 2010: 97K80, 35B40*

SOMESTATISTICALPROPERTIESOFEXPONENTIATEDWEIGHTEDWEIBULLDISTRIBUTION

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Some Statistical Properties of Exponentiated Weighted Weibull Distribution

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Abstract- This article basically focused on some statistical properties of exponentiated-weighted weibull model which of course numerous authors have written one thing or the other on exponential weibull distribution and not on exponential-weighted weibull. This model is established with a view to obtaining a model that is better than both weighted weibull and weibull distribution in terms of the estimate of their characteristics and their parameters using the logit of Beta by Jones (2004). The weighted weibull distribution is proposed by Mahdy (2013) with an additional parameter called "sensitive skewness parameter". Some basic properties of the proposed model including moments and moment generating function (first and second moments about the origin even with standard deviation are derive), survival rate function, hazard rate function, asymptotic behaviours, and the estimation of parameters have been studied. The result from the new model is better representativeness in data and its flexibility and shape.

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I. Introduction

In recent time, numerous researchers had used weibull distribution as an alternative to some distribution e.g Gamma and Log-normal distribution in reliability engineering and life testing. The weibull distribution is a well known common distribution and has been a powerful probability distribution in reliability analysis, while weighted distributions are used to adjust the probabilities of the events as observed and recorded. Mahdy applied Azzalini's method to the weibull distribution that produced a new class of weighted weibull distribution as $WW(\lambda, \beta, \alpha)$ distribution with an additional parameter called "Sensitive Skewness Parameter" and the sensitive skewness parameter governs essentially the shape of the probability density function of the $WW(\lambda, \beta, \alpha)$ distribution.

The probability density and the cumulative density function (pdf and cdf) of the new class of weighted weibull distribution by Mahdy (2013) is given by

$$f_{y|\{\lambda,\beta,\alpha\}|}(y) = \frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}, \text{ for } y > 0 \tag{1}$$

and

$$F_{y|\{\lambda,\beta,\alpha\}|}(y) = \frac{[(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1]}{\alpha^{\beta}} \tag{2}$$

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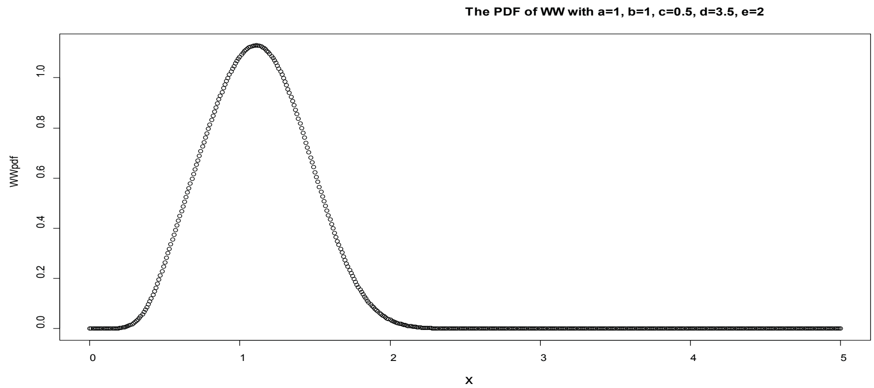

Figure 1 : The Probability Density Function of Weghted Weibull Distribution with a=1, b=1, λ =0.5, $\beta = 3.5$, $\alpha = 2$

Notes

Authors on exponenetiated-weighted weibull are very few. The aim of this article is to introduce and investigate this distribution on its statistical properties. The paper is divided as follows: In section 2, we present the proposed distribution exponentiated-weighted weibull distribution. Moments, and moment generating function is studied in section 3, section 4, discussed on the estimation of parameters mathematically and in section 5 we preset the real application to data set and section 6 concluded the research.

II. The Proposed Exponentiated-Weighted Weibull Distribution

Recently, many authors have studied the properties of exponentiated distributions. For instance, Gupta et al (2001) for exponential pareto, Nadarajah and Gupta (2007) for exponential gamma distribution, Mudholkar et al (1995) studied on exponentiated weibull distribution, Salem and Abo-Kasem (2011) based their research on estimation for the parameters of the exponentiated weibull distribution, Gupta and Kundu (2001) they put up a paper on exponentiated exponential etc. Azzalini (1985) first proposed a method of obtaining weighted and the method has been used extensively for several symmetric and non-symmetric distributions. Mahdy (2013) applied the method to study a new class of weighted weibull distribution with an additional parameter called "sensitive skewness parameter". More so, various extensions of weibull and exponential distribution have been proposed in literature. An extension of exponential distribution has been provided by Nadarajah and Kotz (2005) using the logit of Beta distribution and the logit of Beta distribution (the link function of the Beta generalized distribution) is introduced by Jones (2004). Since then extensive work has been done using the logit of beta distribution in literature. For instance, Gupta and Kundu (1999) proposed a generalized exponential distribution which provides an alternative to exponential and weibull distributions. Famoye et al (2005) also introduced the Beta-weibull distribution alongside its major properties and Cordeiro et al (2011) among others.

Now, letting y be a random variable form of the distribution with parameters and defined (1) and (2) using the logit of beta by Jones (2004), we then have

$$f_{EWW}{}^{(y)} = \frac{1}{B(a,b)}\, [F(y)]^{a-1}\ [1-F(y)]^{b-1}\ \ f(y) \qquad (3)$$

by setting b=1, we get

$$f_{EWW}{}^{(y)} = a[F(y)]^{a-1}\ \ f(y) \qquad (4)$$

Putting expressions (1) and (2) in (4) to obtain the probability density function of Exponenetiated-weighted weibull distribution

$$f_{EWW}{}^{(y)} = a\left[\frac{[(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1]}{\alpha^{\beta}}\right]^{a-1}\frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}} \quad (5)$$

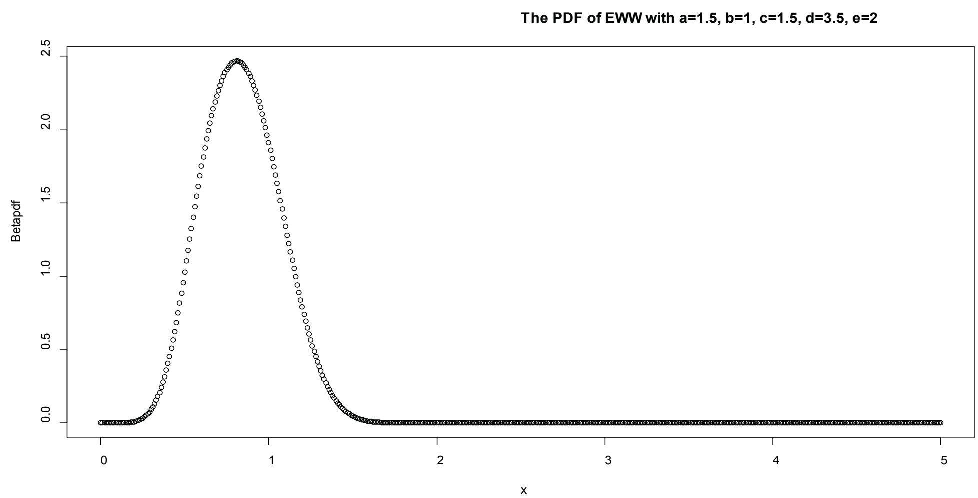

Figure 2 : The PDF of Exponentiated Weighted Weibull Distribution with values of the parameters $(a = 1.5, b = 1, c = \lambda = 1.5, d = \alpha = 3.5, e = \beta = 2.)$ and is rightly skewed

Where $a > 0, \lambda > 0, \beta > 0, \alpha > 0$ *and* $y > 0$ *such that* $Y \sim EWW(a, \lambda, \beta, \alpha)$. Equation (5) is the pdf of Exponenetiated-weighted weibull distribution.

$$\text{set } u(x) = \frac{[(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1]}{\alpha^{\beta}}$$

$$\frac{du}{dy} = \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right] \quad (6)$$

substituting dy into (5), we obtain

$$f_{EWW}{}^{(y)} = a\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right]^{a-1} du \quad (7)$$

$$u = \frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1$$

Now, (5) can becomes

$$f_{EWW}{}^{(y)} = a[U]^{a-1}\frac{du}{dy} \quad (8)$$

a) Cumulative Density Function (cdf)

The probability density function (pdf) of $EWW(a, \lambda, \beta, \alpha)$ given in (7), then expression (7) can be written as

$$F_{EWW}{}^{(y)} = P(Y \leq y) = \int_0^y f(u)du$$

$$= \int_0^y a\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right]^{a-1}\frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}\, du \quad (9)$$

$$F_{EWW}{}^{(y)} = P(Y \le y) = \int_0^y a[U]^{a-1}\,du$$

$= a\int_0^y [U]^{a-1}\,du$ and the cdf is obtained as

$$F_{EWW}{}^{(y)} = -(y)^a \tag{10}$$

b) The Survival Rate Function

The survival rate function of the Exponentiated-weighted weibull distribution is given by $S_{EWW}(y) = 1 - F_{EWW}(y) = 1 - \int_0^y f(u)du$

$$= 1 - a\int_0^y [U]^{a-1}\,du = 1 - [\,-\,(y)^a]$$

$$S_{EWW}(y) = 1 + (y)^a \tag{11}$$

c) The Hazard Rate Function

The hazard rate function of a random variable y with the pdf and cdf is defined by

$$h_{EWW}(y) = \frac{f_{EWW}(y)}{1 - F_{EWW}(y)}$$

Hence, the $EWW(a, \lambda, \beta, \alpha)$ with $f_{EWW}(y)$ and $F_{EWW}(y)$ respectively defined in (4) and (10), the hazard rate function can be expressed as:

$$= \frac{a(u)^{a-1}u'}{S_{EWW}(y)} \tag{12}$$

where U is expression in (5)

To show that $\lim_{y\to\infty} h_{EWW}(y) = 0$ *and* $\lim_{y\to 0} h_{EWW}(y) = 0$, we have the following

$$\lim_{y\to\infty} h_{EWW}(y) = \lim_{y\to\infty} \frac{a(u)^{a-1}u'}{1-(1+y)^a}$$

where, $u' = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta(1+\alpha^\beta)} - 1$

$$= \lim_{y\to\infty} \frac{a\left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1\right]^{a-1} \frac{\lambda\beta\,(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}}{1-(1+x)^a}$$

For simplification on the rigorous mathematics, we take the limit of the following:

When $y \to \infty = 0$ *and* $y \to 0 = 0$

$$= \lim_{y\to\infty} \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta} \quad = \lim_{y\to\infty} \frac{\lambda\beta(1+\alpha^\beta)\infty^{\beta-1}e^{-\lambda\infty^\beta}(1-e^{-\lambda(\alpha\infty)^\beta})}{\alpha^\beta}$$

$$= 0$$

Notes

$$= \overset{lim}{y \longrightarrow 0} \frac{\lambda\beta\,(1+\alpha^{\beta})0^{\beta-1}e^{-\lambda 0^{\beta}}\,(1-e^{-\lambda(\alpha 0)^{\beta}})}{\alpha^{\beta}} = 0$$

$As\ y \to \infty = 0\ and\ y \to 0 = 0$, expression (12) above tends to $\infty\ and\ 0$ and equal to zero.

Notes

d) Asymptotic Behaviours

Following the steps in hazard function above taken $\overset{lim}{y \longrightarrow \infty} f_{EWW}(y)\ and$ $\overset{lim}{y \longrightarrow 0} f_{EWW}(y)$ of the $EWW(a, \lambda, \beta, \alpha)$ distribution is investigated as follows. Now from expression (5),
we have

$$\overset{lim}{y \longrightarrow \infty} f_{EWW}(y) = a\left[\frac{[(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}(1+\alpha^{\beta})}}{\alpha^{\beta}}\right]^{a-1} \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}$$

taking the limit

$$= \overset{lim}{y \longrightarrow \infty} \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}} \quad = \overset{lim}{y \longrightarrow \infty} \frac{\lambda\beta(1+\alpha^{\beta})\infty^{\beta-1}e^{-\lambda\infty^{\beta}}(1-e^{-\lambda(\alpha\infty)^{\beta}})}{\alpha^{\beta}}$$

$$= 0$$

$$= \overset{lim}{y \longrightarrow 0} \frac{\lambda\beta\,(1+\alpha^{\beta})0^{\beta-1}e^{-\lambda 0^{\beta}}\,(1-e^{-\lambda(\alpha 0)^{\beta}})}{\alpha^{\beta}} = 0$$

From the above results as $y \to \infty = 0\ and\ y \to 0 = 0$, this shows that the distribution has at least a mode.

III. Moments and Moment Generating Function

Hosking (1990) described in their paper that when a random variable following a generalized beta generated distribution i.e $y \sim GBG(f, a, c)\ then\ \mu_r^{'} = E[F^{-1}K^{\frac{1}{c}}]^r$ where $K \sim B(a, 1), c$ is a constant and $F^{-1}(y)$ is the inverse of CDF of the weighted weibull distribution, since $EWW(a, \lambda, \beta, \alpha)$ distribution is a special form when a=c=1.We then derive the moment generating function (mgf) of the proposed distribution $m(t) = E(e^{ty})$ and the general rth moment of a beta generated distribution is defined by

$$\mu_r^{'} = \frac{1}{B(a,1)}\int_0^1 [F^{-1}(y)]^r\ [y]^{a-1}\, du \tag{13}$$

Also, using the taylor series expansion around the point $E(y_f) = \mu_f$ to obtain

$$\mu_r^{'} = \sum_{u=0}^{r}\binom{r}{k}[F^{-1}(\mu)]^{r-k}\ [F^{-1(1)}(\mu_f)]^k \sum_{k=0}^{n}(-1)^i \binom{r}{i} \tag{14}$$

Cordeiro et al (2011) gave an alternative series expansion for $\mu_r^{'}$ in terms of $r(r, U) = E(U^r\ F(U)^{Y})$ where k follows the parent distribution then for u = 0, 1, …

$$\mu_r^{'} = \frac{1}{B(a,1)}\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\, r(r, i-1)$$

They further described another mgf of y for generated beta distribution as

$$M(t) = \frac{1}{B(a,1)} \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \rho(t, i-1) \quad (15)$$

Where, $$\rho(t,r) = \int_{-\infty}^{\infty} e^{ty} [F(y)]^m f(y)\, dy$$

Therefore, $$M_y^{(t)} = \frac{1}{B(a,1)} \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \int_{-\infty}^{\infty} e^{ty} [F(y)]^{(i+1)-1} f(y)\, dy \quad (16)$$

Substituting both probability density and cumulative density function of the weighted weibull distribution into (16), we obtain

$$M_{EWW(y)}^{(t)} = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \int e^{ty} \left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta (1+\alpha^\beta)} - 1\right]^{(i+1)-1} \frac{\lambda\beta(1+\alpha^\beta) y^{\beta-1} e^{-\lambda y^\beta} (1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}\, dy \quad (17)$$

Equation (17) becomes the mgf of Exponentiated-weighted weibull distribution. Then, setting a=1 and i = 0, the same expression (17) is reduced to becomes the parent distribution. To obtain the rth moment of $EWW(a,\lambda,\beta,\alpha)$, the weighted weibull distribution by Mahdy (2013) and is given by

$$M_{(y)}^{(t)} = \int_0^{\infty} e^{ty} \frac{\lambda\beta(1+\alpha^\beta) y^{\beta-1} e^{-\lambda y^\beta} \left(1-e^{-\lambda(\alpha y)^\beta}\right)}{\alpha^\beta}\, dy$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!\alpha^\beta} \{\lambda^{-\frac{j}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{j+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{j+\beta}{\beta}\right)}\} \quad (18)$$

Equation (17) can be re-written as

$$M_{EWW(y)}^{(t)} = a \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{t^j}{j!\alpha^\beta} \{\lambda^{-\frac{j}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{j+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{j+\beta}{\beta}\right)}\} \quad (19)$$

The rth moment of the $EWW(a,\lambda,\beta,\alpha)$ distribution can also be written from equation (19) as

$$\mu_r' = E(Y^r) = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{t^r}{r!\alpha^\beta} \{\lambda^{-\frac{r}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{r+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \quad (20)$$

Again, putting a = 1 in expression (20) leads to the rth moment of the weighted weibull model by Mahdy (2013) and is given by

$$\mu_r' = E(Y^r) = \frac{1}{\alpha^\beta} \{\lambda^{-\frac{r}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{r+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \quad (21)$$

$$\mu_{EWW(r)}' = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{r}{\alpha^\beta} \{\lambda^{-\frac{1}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{r+\beta}{\beta}) (1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \quad (22)$$

Notes

From (22), it is easy to obtain the first and the second mean about the origin e.g when r = 1and the second moment when r = 2, etc.

The first moment of $EWW(a,\lambda,\beta,\alpha)$ is obtain

$$\mu'_{EWW(1)} = a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{1}{\beta}}(1+\alpha^\beta)\Gamma(\frac{1+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{1+\beta}{\beta}\right)}\} \quad (23)$$

Notes

The second moment can also be obtained as follows:

$$\mu'_2 = V(a,1,\lambda,\beta,\alpha)^{(y)} = V_1 - V_2 \quad (24)$$

where,

$$V_1 = E(a,\lambda,\beta,\alpha)^{(Y^2)} = a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{2}{\beta}}(1+\alpha^\beta)\Gamma(\frac{2+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{2+\beta}{\beta}\right)}\}$$

$$V_2 = E(a,\lambda,\beta,\alpha)^{(Y)} = (a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{2}{\beta}}(1+\alpha^\beta)\Gamma\left(\frac{1+\beta}{\beta}\right)\left(1-(1+\alpha^\beta)\right)^{-\left(\frac{1+\beta}{\beta}\right)}\})^2$$

Likewise, the standard deviation is given by

$$SD_{EWW}.(a,\lambda,\beta,\alpha)^{(Y)} = \sqrt{V_1 - V_2}$$

IV. Estimation of Parameter

We show the maximum likelihood estimate (MLEs) of the parameter of $EWW(a,\lambda,\beta,\alpha)$ distribution mathematically following Cordeiro et al (2011) and Shittu and Adepoju (2013) studied on the log-likelihood function for $\omega = (a,c,\varphi)$, where $\varphi = (\lambda,\beta,\alpha)$ and setting ω to be a vector of parameter and is given by

$$L(\omega) = nlogc - nlog[B(a,1)] + \sum_{i=1}^{n}\log[f(y;\varphi)] + (a-1)\sum_{i=1}^{n}\log[F(y;\varphi)] \quad (25)$$

Note that, b = c = 1 (24) becomes $\theta = (a,\varphi)$

$$L(\omega) = -nlog(a,1) + \sum_{i=1}^{n}\log[f(y;\varphi)] + (a-1)\sum_{i=1}^{n}\log[F(y;\varphi)] \quad (26)$$

where, $f(y;\varphi) = \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}$ and

$$F(y;\varphi) = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta(1+\alpha^\beta)} - 1$$

$$L_{EWW}^{(\omega)} = -nlog(a,1) + \sum_{i=1}^{n} \log\left[\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}\right] + (a-1)\sum_{i=1}^{n} \log\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}(1+\alpha^{\beta})} - 1\right] \quad (27)$$

For determining the MLE of $a, \lambda, \beta, \alpha$, we take the partial derivative of the (27) with respect to $(a, \lambda, \beta, \alpha)$ as follows:

$$\frac{L_{EWW}^{(\omega)}}{\partial a} = -nlog(a,1) + (a-1)\sum_{y=1}^{n} \log\left[\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}(1+\alpha^{\beta})} - 1\right] \quad (28)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial \lambda} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\lambda}\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}{\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}\right] +$$

$$\text{(a-1)}\sum_{y=1}^{n} \log\left[\frac{\frac{\partial}{\partial\lambda}(\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1)}{\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1}\right] \quad (29)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial \beta} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\beta}\frac{\lambda\beta\,(1+\alpha^{\beta})y\,e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}{\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}\right] +$$

$$\text{(a-1)}\sum_{y=1}^{n} \log\left[\frac{\frac{\partial}{\partial\beta}(1-\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1)}{\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1}\right] \quad (30)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial \alpha} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\alpha}\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}{\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}\right] +$$

$$\text{(a-1)}\sum_{y=1}^{n} \log\left[\frac{\frac{\partial}{\partial\alpha}(1-\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1)}{\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1}\right] \quad (31)$$

The equations derive above can also be solved using iteration method (Newton Raphson) to obtain the $\hat{a}, \hat{\lambda}, \hat{\beta}, \hat{\alpha}$ the MLE of $(a, \lambda, \beta, \alpha)$ respectively.

Taking second derivatives of the said equations 28, 29, 30 and 31 with respect to the parameters above, it is possible to derive the interval estimate and hypothesis tests on the model parameter. This may be shown in further research.

V. Real Data Set

The data used in this section was studied by Lemonte in his BJPS Accepted Manuscript on the remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang (2003) to compare and contrast the Exponentiated Weighted Weibull and Weighted Weibull distribution.

Notes

Notes

R (code) software is used to determine the maximum likelihood estimates and the log-likelihood for the Exponentiated Weighted Weibull distribution are: $\hat{a} = 8.95941$, $\hat{\lambda} = 8.17089$, $\hat{\beta} = 8.36111$, $\hat{\alpha} = 7.91621$ *and* $log_{EWW} = 448.5781$ while the maximum likelihood estimates and the log-likelihood for the Weighted Weibull distribution are: $\hat{\lambda} = 7.90434$, $\hat{\beta} = 8.44282$, $\hat{\alpha} = 8.70033$ and $log_{WW} = 448.4913$, where lg_{LWW} *and* lg_{WW} denote log-likelihood of both Exponentiated Weighted Weibull distribution and Weighted Weibull distribution.

$$\begin{pmatrix} 0.802169648 & -0.02880174 & 0.002221849 & -0.02674447 \\ -0.028801742 & 0.65433442 & -0.025345533 & -0.02069611 \\ 0.002221849 & -0.02534553 & 0.705642669 & -0.02353514 \\ -0.026744474 & -0.02069611 & -0.023535137 & 0.60907454 \end{pmatrix}$$

VI. Conclusion

We investigated on the statistical properties of the proposed distribution e.g moments, moment generating function, estimation of parameters using R (Code) software for data analysis presented in this article. We also upgraded with an additional parameter to the existing three parameters in the weighted weibull distribution and the results from the estimated parameters show that the Exponentiated Weighted Weibull distribution has a better representation of data than weighted weibull distribution.

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