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By Harun-Or-Roshid, Md. Nur Alam, Mohammad Mobarak Hossain, Mohammad Safi Ullah, Rafiqul Islam & M. Ali Akbar
Pabna University of Science and Technology, Bangladesh

Abstract- In this work, the $\exp(-\Phi(\eta))$ -expansion method is applied to solve the (2+1)-dimensional ZK-BBM equation. The traveling wave solutions are expressed in terms of the exponential functions, the hyperbolic functions, the trigonometric functions and the rational functions. The procedure is simple, direct and constructive without the help of a computer algebra system. The $\exp(-\Phi(\eta))$ –expansion method will be used in further works to establish more entirely new solutions for other kinds of nonlinear evolution equations arising in mathematical physics and engineering.

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Exact Traveling Wave Solutions for the (2+1)-Dimensional ZK-BBM Equation by Exp $(-\Phi(\eta))$ - Expansion Method

Harun-Or-Roshid [α], Md. Nur Alam [σ], Mohammad Mobarak Hossain [ρ], Mohammad Safi Ullah [ω], Rafiqul Islam [¥] & M. Ali Akbar [§]

Abstract- In this work, the $\exp(-\Phi(\eta))$ -expansion method is applied to solve the (2+1)-dimensional ZK-BBM equation. The traveling wave solutions are expressed in terms of the exponential functions, the hyperbolic functions, the trigonometric functions and the rational functions. The procedure is simple, direct and constructive without the help of a computer algebra system. The $\exp(-\Phi(\eta))$ -expansion method will be used in further works to establish more entirely new solutions for other kinds of nonlinear evolution equations arising in mathematical physics and engineering.

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I. Introduction

Nonlinear intricate physical phenomena are related to nonlinear partial differential equations (PDEs) which are involved in many fields of sciences, especially fluid mechanics, solid state physics, plasma physics, plasma wave and chemical physics, biology, chemistry, mechanics, etc. Searching for exact solutions of nonlinear PDEs plays an main role in the study of these physical phenomena and gradually becomes one of the most imperative and major farm duties. A huge deal of research work has been carried out during the past decades for the study of the nonlinear evolution equation. Powerful methods which make it possible to generate exact traveling wave solutions to nonlinear equations have emerged from the literatures in the past decades. Among them are the complex hyperbolic function method [1, 2], the Jacobi elliptic function expansion method [3, 4], the F-expansion method [5, 6], the (G′ / G) -expansion method [7-15], the Hirota's bilinear method [16], the Backlund transformation method [17], the Darboux transformation method [18], the homotopy perturbation method [19, 20], the $\exp(-\varphi(\eta))$ -expansion method [21- 25] and soon.

The objective of this article is to put into practice the $\exp(-\varphi(\eta))$ -expansion method to put up the exact solutions for nonlinear evolution equations in mathematical physics via the (2+1)-dimensional ZK-BBM equation for the first time.

The rest of the paper is organized as follows: In Section 2, we give the description of the $\exp(-\varphi(\eta))$ -expansion method. In Section 3, we apply this method to the (2+1)-dimensional ZK-BBM equation. Conclusions are given in the last section.

II. Description of the Exp$(-\Phi(\eta))$-Expansion Method

Let us consider a general nonlinear PDE in the form

$$F(u,u_t,u_x,u_{xx},u_{tt},u_{tx},\ldots), \qquad (1)$$

where $u = u(x, t)$ is an unknown function, F is a polynomial in $u(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. In the following, we give the main steps of this method:

Step 1: We combine the real variables x and t by a complex variable η

$$u(x,t)=u(\eta), \quad \eta = x \pm V\,t, \qquad (2)$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $u = u(\eta)$:

$$\Re(u,u',u'',u''',\cdots), \qquad (3)$$

where $\Re$ is a polynomial of u and its derivatives and the superscripts indicate the ordinary derivatives with respect to η .

Step 2: Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$u(\eta)=\sum_{i=0}^{N} A_i(\exp(-\Phi(\eta)))^i, \qquad (4)$$

Author α σ ¥ : Department of Mathematics, Pabna University of Science and Technology, Bangladesh. e-mail: harunorroshidmd@gmail.com
Author § : Department of Applied Mathematics, University of Rajshahi, Bangladesh.
Author ρ : Department of Mathematics, Hamdard University Bangladesh.
Author ω: Department of Mathematics, Comilla University, Bangladesh.

where A_i $(0 \le i \le N)$ are constants to be determined, such that $A_N \neq 0$ and $\Phi = \Phi(\eta)$ satisfies the following ordinary differential equation:

$$\Phi'(\eta) = \exp(-\Phi(\eta)) + \mu \exp(\Phi(\eta)) + \lambda, \tag{5}$$

Eq. (5) gives the following solutions:

Family 1: When $\mu \neq 0$, $\lambda^2 - 4\mu > 0$,

$$\Phi(\eta) = \ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)} \tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \tag{6}$$

Family 2 : When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$\Phi(\eta) = \ln\left(\frac{\sqrt{(4\mu - \lambda^2)} \tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \tag{7}$$

Family 3 : When $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$\Phi(\eta) = -\ln\left(\frac{\lambda}{\exp(\lambda(\eta + E)) - 1}\right) \tag{8}$$

Family 4 : When $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$\Phi(\eta) = \ln\left(-\frac{2(\lambda(\eta + E) + 2)}{\lambda^2(\eta + E)}\right) \tag{9}$$

Family 5 : When $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$,

$$\Phi(\eta) = \ln(\eta + E) \tag{10}$$

A_N,……,V, λ, μ are constants to be determined latter, $A_N \neq 0$, the positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function $\exp(-\Phi(\eta))$. As a result of this substitution, we get a polynomial of $\exp(-\Phi(\eta))$. We equate all the coefficients of same power of $\exp(-\Phi(\eta))$ to zero. This procedure yields a system of algebraic equations whichever can be solved to find A_N,…..V, λ, μ. Substituting the values of A_N,……V, λ, μ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

III. Applications of the Method

In this section, we will apply the $\exp(-\Phi(\eta))$ -expansion method to make the exact solutions and then the solitary wave solutions of the (2+1)-dimensional ZK-BBM equation. Let us consider the generalized form of the (2+1)-dimensional ZK-BBM equation,

$$u_t - u_x - a(u^2)_x + (bu_{xt} - ku_{yt})_x = 0\,. \tag{11}$$

where a , b and k are arbitrary constants. It arises as a description of gravity water waves in the long-wave regime.

We apply of the traveling wave variable $u(\eta) = u(x, y, t)$, $\eta = x + y - ct$, Eq. (11) is carried to an ODE

$$u'(1-c) - 2auu' + cu'''(b-k) = 0\,. \tag{12}$$

Eq. (12) is integrable, therefore, integrating twice with respect to η once yields:

$$P + u(1-c) - au^2 + cu''(b-k) = 0, \tag{13}$$

where P is an integration constant that is to be determined later.

Proceeding in a similar manner as in the above section and considering the homogeneous balance between u^2 and u'' in Eq. (13), we obtain $N = 2$. Therefore, the solution of Eq. (13) is of the form:

$$u(\eta) = A_0 + A_1(\exp(-\Phi(\eta))) + A_2(\exp(-\Phi(\eta)))^2, \tag{14}$$

Where A_0, A_1, A_2 are constants to be determined such that $A_N \neq 0$, while λ, μ are arbitrary constants. Substituting Eq. (14) into Eq. (13) and then equating the coefficients of $\exp(-\Phi(\eta))$ to zero, we obtain

$$-6cA_2k + 6cA_2b - aA_2^{\,2} = 0, \tag{15}$$

$$10cA_2\lambda b - 2aA_1A_2 - 10cA_2\lambda k + 2cA_1b - 2cA_1k = 0, \tag{16}$$

$$-3cA_1\lambda k + A_2 - 2aA_0A_2 - 8cA_2\mu k + 3cA_1\lambda b - A_2c - 4cA_2\lambda^2k - aA_1^{\,2} + 8cA_2\mu b + 4cA_2\lambda^2 b = 0, \tag{17}$$

$$-A_1c + 2cA_1\mu b - 6cA_2\mu\lambda k + A_1 + 6cA_2\mu\lambda b - cA_1\lambda^2k - 2cA_1\mu k - 2aA_0A_1 + cA_1\lambda^2 b = 0, \tag{18}$$

$$A_0 + cA_1\lambda\mu b + P + 2cA_2\mu^2 b - 2cA_2\mu^2 k - aA_0^{\,2} - cA_1\lambda\mu k - A_0c = 0, \tag{19}$$

Solving the Eqs. (15)-(19) yields

$$P = \frac{1}{4a}(-c^2 - 1 + 2c + 16c^2\lambda^2\mu bk + 16c^2\mu^2b^2 + 16c^2\mu^2k^2 + k^2\lambda^4c^2$$
$$+ b^2\lambda^4c^2 - 8c^2\lambda^2\mu b^2 - 32c^2\mu^2bk - 2b\lambda^4c^2k - 8k^2\lambda^2c^2\mu),$$

$$c = c,\quad A_0 = \frac{1}{2a}(b\lambda^2c + 8b\mu c - k\lambda^2c - c - 8k\mu c + 1),\quad A_1 = \frac{6c\lambda(b-k)}{a},\quad A_2 = \frac{6c(b-k)}{a}.$$

Where λ, μ are arbitrary constants.

Now substituting the values of V, A_o, A_1, A_2 into Eq. (14) yields

$$u(\eta) = \frac{1}{2a}(b\lambda^2c + 8b\mu c - k\lambda^2c - c - 8k\mu c + 1) + \frac{6c\lambda(b-k)}{a}(\exp(-\Phi(\eta))) + \frac{6c(b-k)}{a}(\exp(-\Phi(\eta)))^2, \tag{20}$$

Where $\eta = x - ct$.

Now substituting Eqs. (6)-(10) into Eq. (20) respectively, we get the following five traveling wave solutions of the (2+1)-dimensional ZK-BBM equation.

When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$u_1(\eta) = \frac{1}{2a}(b\lambda^2c + 8b\mu c - k\lambda^2c - c - 8k\mu c + 1) - \frac{6c\lambda(b-k)}{a}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\eta+E)\right)+\lambda}\right)$$
$$+ \frac{6c\lambda(b-k)}{a}\left(\frac{2\mu}{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\eta+E)\right)+\lambda}\right)^2.$$

where $\eta = x - ct$ and E is an arbitrary constant.

When $\mu \neq 0$, $\lambda^2 - 4\mu < 0$,

$$u_2(\eta) = \frac{1}{2a}(b\lambda^2c + 8b\mu c - k\lambda^2c - c - 8k\mu c + 1) + \frac{6c\lambda(b-k)}{a}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\eta+E)\right)-\lambda}\right)$$
$$+ \frac{6c\lambda(b-k)}{a}\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\eta+E)\right)-\lambda}\right)^2.$$

where $\eta = x - ct$ and E is an arbitrary constant.

When $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$,

$$u_3(\eta) = \frac{1}{2a}(b\lambda^2 c + 8b\mu c - k\lambda^2 c - c - 8k\mu c + 1) + \frac{6c\lambda(b-k)}{a}\left(\frac{\lambda}{\exp(\lambda(\eta+E)) - 1}\right) + \frac{6c\lambda(b-k)}{a}\left(\frac{\lambda}{\exp(\lambda(\eta+E)) - 1}\right)^2.$$

where $\eta = x - ct$ and E is an arbitrary constant.

When $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$,

$$u_4(\eta) = \frac{1}{2a}(b\lambda^2 c + 8b\mu c - k\lambda^2 c - c - 8k\mu c + 1) + \frac{6c\lambda(b-k)}{a}\left(\frac{\lambda^2(\eta+E)}{2(\lambda(\eta+E)) + 2)}\right) + \frac{6c\lambda(b-k)}{a}\left(\frac{\lambda^2(\eta+E)}{2(\lambda(\eta+E)) + 2)}\right)^2.$$

where $\eta = x - ct$ and E is an arbitrary constant.

When $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$,

$$u_5(\eta) = \frac{1}{2a}(b\lambda^2 c + 8b\mu c - k\lambda^2 c - c - 8k\mu c + 1) + \frac{6c\lambda(b-k)}{a}\frac{1}{(\eta+E)} + \frac{6c\lambda(b-k)}{a}\left(\frac{1}{(\eta+E)}\right)^2.$$

where $\eta = x - ct$ and E is an arbitrary constant.

IV. Conclusion

In this paper, the traveling wave solutions of the (2+1)-dimensional ZK-BBM equation is found successfully through the use of the $\exp(-\Phi(\eta))$ -expansion method, which includes the exponential functions solutions, the hyperbolic functions solutions, the trigonometric functions solutions and the rational functions solutions. It is shown that the $\exp(-\Phi(\eta))$ -expansion method provides a very effective and powerful mathematical tool for solving nonlinear equations in mathematical physics and engineering.

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