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Con-s-k-EP Generalized Inverses

By G. Ramesh & B. K. N. Muthugobal

Govt. Arts College(Autonomous), India

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Con-s-k-EP Generalized Inverses

G. Ramesh^α & B. K. N. Muthugobal^σ

Abstract- In this paper, equivalent conditions for various generalized inverses of a con-s-k-EP_r matrix to be con-s-k-EP_r are determined. Generalized inverses belonging to the sets A{1,2}, A{1,2,3} and A{1,2,4} of a con-s-k-EP_r matrix A are characterized.

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I. INTRODUCTION

Let $c_{n \times n}$ be the space of $n \times n$ complex matrices of order n . Let C_n be the space of all complex n tuples. For $A \in c_{n \times n}$. Let $\bar{A}, A^T, A^*, A^S, \bar{A}^S, A^\dagger, R(A), N(A)$ and $\rho(A)$ denote the conjugate, transpose, conjugate transpose, secondary transpose, conjugate secondary transpose, Moore Penrose inverse range space, null space and rank of A respectively. A solution X of the equation $AXA = A$ is called generalized inverse of A and is denoted by A^- . If $A \in c_{n \times n}$ then the unique solution of the equations $AXA = A, XAX = X, [AX]^* = AX, (XA)^* = XA$ [2] is called the Moore-Penrose inverse of A and is denoted by $A^\dagger$. A matrix A is called con-s-k-EP_r if $(A) = r$ and $N(A) = N(A^T V K)$ (or) $R(A) = R(K V A^T)$. Throughout this paper let " k " be the fixed product of disjoint transposition in $S_n = \{1, 2, \dots, n\}$ and k be the associated permutation matrix.

Let us define the function $k(x) = (x_{k(1)}, x_{k(2)}, \dots, x_{k(n)})$. A matrix $A = (a_{ij}) \in c_{n \times n}$ is s-k-symmetric if $a_{ij} = a_{n-k(j)+1, n-k(i)+1}$ for $i, j = 1, 2, \dots, n$. A matrix $A \in c_{n \times n}$

Author α : Associate Professor, Ramanujan Research Centre, Department of Mathematics, Govt. Arts College(Autonomous), Kumbakonam-612 001, India.

Author σ : Research Scholar, Ramanujan Research Centre, Department of Mathematics, Govt. Arts College(Autonomous), Kumbakonam-612 001, India. e-mail: bkn.math@gmail.com

is said to be Con-s-k-EP if it satisfies the condition $A_x = 0 \Leftrightarrow A^s \neq (x) = 0$ or equivalently $N(A) = N(A^T VK)$. In addition to that A is con-s-k-EP $\Leftrightarrow KVA$ is con-EP or AVK is con-EP and A is con-s-k-EP $\Leftrightarrow A^T$ is con-s-k-EP_r moreover A is said to be Con-s-k-EP_r if A is con-s-k-EP and of rank r . For further properties of con-s-k-EP matrices one may refer [1].

In **Theorem (2.11)** [1], it is shown that A is con-s-k-EP_r, if and only if A^+ is con-s-k-EP_r. Thus the con-s-k-EP_r property of complex matrices is preserved for its Moore-Penrose inverses. However, all other generalized inverses of a con-s-k-EP_r

matrix need not be con-s-k-EP_r. For instance, let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ for $k=(1,2)(3)$, the

associated permutation matrix be $K = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $V = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Here, A is

con-s-k-EP₁. But $A^- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ is a inverse of A which is not con-s-k-EP₁.

A generalized inverse $A^- \in A\{1,2\}$ is shown to be con-s-k-EP_r whenever A is con-s-k-EP_r under certain conditions in the following way.

Theorem 2.1

Let $A \in C_{n \times n}$, $X \in A\{1,2\}$ and AX, XA are con-s-k-EP_r matrices. Then A is con-s-k-EP_r $\Leftrightarrow X$ is con-s-k-EP_r.

Proof

Since AX and XA are con-s-k-EP_r, by **Theorem (2.11)** [1], we have

$R(AX) = R(KV(AX)^T)$ and $R(KV(XA)^T) = R(XA)$. Since $X \in A\{1,2\}$ we have

$AXA = A, XAX = X$.

R_{ref}

1. Krishnamoorthy, S., Gunasekaran, K. and Muthugobal, B.K.N., "Con-s-k-EP matrices", *Journal of Mathematical Sciences and Engineering Applications*, Vol. 5, No.1, 2011, 353 – 364.

Now,

$$\begin{aligned} R(A) &= R(AX) \\ &= R(KV(AX)^T) \\ &= R(KVX^T A^T) \\ &= R(KVX^T) \end{aligned}$$

$$\begin{aligned} R(KVA^T) &= R(KVA^T X^T) \\ &= R(KV(XA)^T) \\ &= R(XA) \\ &= R(X) \end{aligned}$$

Now, A is con-s-k-EP_r $\Leftrightarrow R(A) = R(KVA^T)$ and $(A) = r$
 $\Leftrightarrow R(KVX^T) = R(X)$ and $(A) = (X) = r \Leftrightarrow X$ is con-s-k-EP_r.

Remark 2.2

In the above Theorem, the conditions that both AX and XA to be

con-s-k-EP_r are essential. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ for $k=(1,2)(3)$, the associated

permutation matrix $K = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $V = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. A is con-s-k-EP₁.

$X = A^- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in A_{\{1,2\}}$ $AX = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$; $XA = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. AX and XA

are not con-s-k-EP₁. Also X is not con-s-k-EP₁.

Now, we show that generalized inverses belonging to the sets $A_{\{1,2,3\}}$ and $A_{\{1,2,4\}}$ of a con-s-k-EP_r matrix A is also con-s-k-EP_r, under certain conditions in the following Theorems.

Theorem 2.3

Let $A \in C_{n \times n}$, $X \in A\{1, 2, 3\}$, $R(X) = R(A^T)$. Then A is con-s-k-EP_r $\Leftrightarrow X$ is con-s-k-EP_r.

Proof

Since $X \in A\{1, 2, 3\}$, we have $AXA = A$, $XAX = X$, $(AX)^T = AX$. Therefore,
 $R(A) = R(AX) = R((AX)^T) = R(X^T)$

$$\begin{aligned}
 R(X) = R(A^T) &\Rightarrow XX^\dagger = A^T(A^T)^\dagger \\
 &\Rightarrow XX^\dagger = A^T(A^\dagger)^T \\
 &\Rightarrow XX^\dagger = (A^\dagger A)^T \\
 &\Rightarrow XX^\dagger = A^\dagger A \\
 &\Rightarrow KVXX^\dagger VK = KVA^\dagger AVK \\
 &\Rightarrow (KVX)(KVX)^\dagger = (AVK)^\dagger AVK \\
 &\Rightarrow R(KVX) = R((AVK)^T) \\
 &\Rightarrow R(KVX) = R(KVA^T)
 \end{aligned}$$

A is con-s-k-EP_r $\Leftrightarrow R(A) = R(KVA^T)$ and $(A) = r \Leftrightarrow R(X^T) = R(KVX)$ and
 $(A) = (X) = r \Leftrightarrow X$ is con-s-k-EP_r.

Theorem 2.4

Let $A \in C_{n \times n}$, $X \in A\{1, 2, 4\}$, $R(A) = R(X^T)$. Then A is con-s-k-EP_r $\Leftrightarrow X$ is con-s-k-EP_r.

Proof

Since $X \in A\{1, 2, 4\}$, we have $AXA = A$, $XAX = X$, $(XA)^T = XA$ and
 $R(A) = R(X^T)$.

$$\begin{aligned}
 \text{Now, } R(KVA^T) &= R(KVA^T X^T) \\
 &= R(KV(XA)^T) \\
 &= R(KVXA) \\
 &= R(KVX)
 \end{aligned}$$

$\mathbf{A}$ is con-s-k-EP_r $\Leftrightarrow \mathbf{R}(\mathbf{A}) = \mathbf{R}(\mathbf{KVA}^T)$ and $(\mathbf{A}) = \mathbf{r} \Leftrightarrow \mathbf{R}(\mathbf{X}^T) = \mathbf{R}(\mathbf{KVX})$
 and $(\mathbf{A}) = (\mathbf{X}) = \mathbf{r} \Leftrightarrow \mathbf{X}$ is con-s-k-EP_r.

Remark 2.5

In particular, if $\mathbf{X} = \mathbf{A}^\dagger$ then $\mathbf{R}(\mathbf{A}^\dagger) = \mathbf{R}(\mathbf{A}^T)$ holds. Hence $\mathbf{A}$ is con-s-k-EP_r is equivalent to $\mathbf{A}^\dagger$ is con-s-k-EP_r.

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