



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH
MATHEMATICS AND DECISION SCIENCES
Volume 13 Issue 11 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

CESARO Mean of Product Summability of Partial Differential Equations of Sequences

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Abstract- In [4], the definition of product summability method $(D, k)(C, l)$ for functions was given and some of its properties were investigated. In [2], $(D, k)(C, \alpha, \beta)$ ($k > 0$, $\alpha > 0$, $\beta > -1$) summability for functions are defined and some of its properties were investigated. In [1], the Cesàro means and Cesàro summability were discussed for sequences. In this paper, we study some results of Cesàro mean of product summability $(D, k)(C, \alpha, \beta)$ ($k > 0$, $\alpha > 0$, $\beta > -1$) of partial differential equations of sequences.

GJSFR-F Classification : MSC 2010: 32W50, 42B08



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CESÀRO Mean of Product Summability of Partial Differential Equations of Sequences

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I. INTRODUCTION

Kuttner [1], introduced the summability method for functions and investigated some of its properties. Pathak [4], defined the product summability method for functions and investigated some of its properties. Mishra and Srivastava [3], introduced the summability method for functions by generalizing summability method. Mishra and Mishra [2], introduced the summability method for functions and investigated some of its properties. In this paper, we study some results of Cesàro mean of product summability $(D, k)(C, \alpha, \beta)$ ($k > 0, \alpha > 0, \beta > -1$) of partial differential equations of sequences.

II. SOME RELATIONS AND DEFINITIONS

Let $f(x)$ be any function which is Lebesgue-measurable, and that $f : [0, +\infty) \rightarrow \mathbb{R}$, and integrable in $(0, x)$, for any finite x and which is bounded in some right hand neighbourhood of origin. Integrals of the form $\int_0^x$ are throughout to be taken as $\lim_{x \rightarrow 0^+} \int_0^x$,

$\int_0^x$ being a Lebesgue integral. For any $n > 0$, we write $a_n(x)$ for the n^{th} integral,

$$a_n(x) = \frac{1}{\Gamma(n)} \int_0^x (x-y)^{n-1} a(y) dy,$$

$$a_{-}(0)(x) = a(x)$$

The (C, α, β) transform of $a(t)$, which we denote by $\partial_{\alpha, \beta}(t)$ is given by

$$a(t) \quad (\alpha = 0)$$

$$\frac{\Gamma(\alpha + \beta + 1)}{\Gamma(\alpha)\Gamma(\beta + 1)} \frac{1}{t^{\alpha+\beta}} \int_0^x (t-u)^{\alpha-1} u^{\beta} a(y) dy, \quad (\alpha > 0, \beta > -1), \quad (2.1)$$

If, for $t > 0$, the integral defining $\partial_{\alpha,\beta}(t)$ exists and if $\partial_{\alpha,\beta}(t) \rightarrow s$ as $t \rightarrow \infty$, we say that $a(x)$ is summable (C, α, β) to s , and we write $a(x) \rightarrow s (C, \alpha, \beta)$. We write

$$g(t) = g^{(k)}(t) = kt \int_0^{\infty} \frac{x^{k-1}}{(x+t)^{k+1}} a(x) dx, \quad (k > 0) \quad (2.2) \text{ if this exists, We also write}$$

$$U_{k,\alpha,\beta}(t) = kt \int_0^{\infty} \frac{x^{k-1}}{(x+t)^{k+1}} \partial_{\alpha,\beta}(x) dx, \quad (2.3) \text{ if this exists.}$$

With the usual terminology, we say that the sequence a_n is summable,

- (I) (D, k) to the sum s , if $g(t)$ tends to a limit s as $t \rightarrow \infty$,
 (II) $(D, k)(C, \alpha, \beta)$ to s , if $U_{k,\alpha,\beta}(t)$ tends to s as $t \rightarrow \infty$. When $\beta = 0$, $(D, k)(C, \alpha, \beta)$ and $(D, k)(C, \alpha)$ denote the same method. The case $\beta = 0$ is due to Pathak[5]. We know that for any fixed $t > 0$, $k > 0$, it is necessary and sufficient for the convergence of (2.3) that $\int_1^{\infty} \frac{\partial_{\alpha,\beta}(x)}{x^2} dx$ should converge. (2.4)

If (2.4) converges, write for $x > 0$, $F_{\alpha,\beta}(x) = \int_x^{\infty} \frac{\partial_{\alpha,\beta}(t)}{t^2} dt$.

We note that $F_{\alpha,\beta}(x) = o(1)$ as $x \rightarrow \infty$. Further, (since $f(x)$ is bounded in some right hand neighbourhood of the origin) we have,

$$F_{\alpha,\beta}(x) = o\left(\frac{1}{x}\right) \text{ as } x \rightarrow 0+.$$

III. MAIN RESULTS

In this section, we have following theorems for sequences analogous to [2].

Theorem 3.1 : If $\alpha > \gamma \geq 1$, $k > 0$ then $a(x) \rightarrow s (D, k)(C, \alpha - 1, \beta)$, whenever $a(x) \rightarrow s (D, k)(C, \gamma - 1, \beta)$.

Theorem 3.2 : Let $\alpha > \gamma \geq 0$, $\beta > -1$, and suppose that $a(x)$ is summable (C, γ, β) to s and that $\int_1^{\infty} \frac{\partial_{\gamma,\beta}(x)}{x^2} dx$ converges. Then $a(x)$ is summable $(D, k)(C, \alpha, \beta)$ to s .

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