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# Heat Transfer Analysis of the Boundary Layer Flow over a Vertical Exponentially Stretching Cylinder

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**Abstract-** In this article an analysis is presented to obtain the similarity solution of the steady boundary layer flow and heat transfer of a viscous fluid flowing through a vertical cylinder that is stretching exponentially along its surface. The governing partial differential equations along with the boundary conditions are reduced to into system of nonlinear ordinary differential equations by using the boundary layer approach and a suitable similarity transformation. The resulting coupled system of equations subject to the appropriate boundary conditions is solved with the help of powerful numerical technique, the Keller-box method. The effects of the involved parameters such as Reynolds numbers, Prandtl numbers and the natural convection parameter are presented through sketches. The associated physical properties on the flow and heat transfer characteristics that is the skinfriction coefficient and Nusselt numbers are presented for different parameters.

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# Heat Transfer Analysis of the Boundary Layer Flow over a Vertical Exponentially Stretching Cylinder

Abdul Rehman<sup>a</sup> & S. Nadeem<sup>σ</sup>

**Abstract-** In this article an analysis is presented to obtain the similarity solution of the steady boundary layer flow and heat transfer of a viscous fluid flowing through a vertical cylinder that is stretching exponentially along its surface. The governing partial differential equations along with the boundary conditions are reduced to into system of nonlinear ordinary differential equations by using the boundary layer approach and a suitable similarity transformation. The resulting coupled system of equations subject to the appropriate boundary conditions is solved with the help of powerful numerical technique, the Keller-box method. The effects of the involved parameters such as Reynolds numbers, Prandtl numbers and the natural convection parameter are presented through sketches. The associated physical properties on the flow and heat transfer characteristics that is the skinfriction coefficient and Nusselt numbers are presented for different parameters.

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## 1. INTRODUCTION

Fluid flow over a cylindrical shaped geometry has been an area of rigorous investment for the researchers in recent past. The notion is deliberated for Newtonian as well as non-Newtonian fluids with both steady and unsteady flows. The concept is debated with reference to boundary layer theory and stagnation points. Also, in several situations the researchers have accomplished the effects of free or natural convection heat transfer. Ahmad et al [1] have determined the impact of free convection over the problem of boundary layer flow over a horizontal cylinder having constant heat flux at the exterior. The analysis was carried for both slender and blunt orientations. In another work, Buchlin [2] has examined the experimental effects of natural and free convection heat transfer over a cylinder with slender configuration. The natural convection boundary layer flow of steady viscous fluid curving from an isothermal horizontal circular cylinder in presence of heat generation was replicated by Molla et al [3]. Further, Chang [4] has provided a difference scheme concerning the natural convection heat transfer flow of a micropolar fluid along a circular cylinder taking care of wall conduction effects. Moreover, Anwar et al [5] have analyzed the problem of boundary layer flow of a viscoelastic fluid over a horizontal circular cylinder under the influence of natural convection heat transfer. The problem of axisymmetric stagnation flow of non-Newtonian micropolar fluids, containing nanoparticles, flowing through the annular region

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between two concentric cylinders was studied by Nadeem et al [6]. The article comprised the impact of thermophoresis and Brownian motion parameters over the flow, heat and concentration profiles. Recently, Nadeem et al [7] have examined the topic of boundary layer flow of a second grade fluid in a cylinder with heat transfer analysis. Few other interesting works regarding fluid flow through cylindrical geometry are referred in [8-18].

Many authors have made successful attempts to solve the problem of flow and heat transfer over a cylinder that is stretched linearly along its axial direction. Ishak et al [19] have uncovered the effects of uniform suction for the problem of heat transfer and fluid flow due to a stretching cylinder. More recently, Wang [20] has contributed an interesting work about the natural convection heat transfer effects over a vertical stretching cylinder. To the best of author's knowledge, no successful attempt is marketed for the flow and heat transfer over a stretching cylinder that is stretched exponentially along its axial axis. Useful similarities for exponential stretching in cylindrical coordinates have been found and simplified the partial differential equations and its related boundary conditions. The solution of the problem is obtained through the powerful numeric technique the Keller-box method. The behavior of velocity and temperature profiles presented for a large range of Reynolds and Prandtl numbers. At the end the important physical quantities associated with the problem such as the skinfriction coefficient and the local Nusselt numbers are presented.

## II. FORMULATION

Consider the problem of natural convection boundary layer flow of a viscous fluid flowing over a vertical circular cylinder of radius  $a$ . The cylinder is assumed to be stretched exponentially along the axial direction with velocity  $U_w$ . The temperature at the surface of the cylinder is assumed to be  $T_w$  and the uniform ambient temperature is taken as  $T_\infty$  such that the quantity  $T_w - T_\infty > 0$  in case of the assisting flow, while  $T_w - T_\infty < 0$  in case of the opposing flow, respectively. Under these assumptions the boundary layer equations of motion and heat transfer are

$$u_r + \frac{u}{r} + w_z = 0, \quad (1)$$

$$uw_r + ww_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu(w_{rr} + \frac{1}{r} w_r) + g\beta(T - T_\infty), \quad (2)$$

$$uT_r + wT_z = \alpha(T_{rr} + \frac{1}{r} T_r), \quad (3)$$

where the velocity components along the  $(r, z)$  axes are  $(u, w)$ ,  $\rho$  is density,  $\nu$  is the kinematic viscosity,  $p$  is pressure,  $g$  is the gravitational acceleration along the  $z$ -direction,  $\beta$  is the coefficient of thermal expansion,  $T$  is the temperature,  $\alpha$  is the thermal diffusivity. The corresponding boundary conditions for the problem are

$$u(a, z) = 0, \quad w(a, z) = U_w \quad w(r, z) \rightarrow 0 \text{ as } r \rightarrow \infty, \quad (4)$$

$$T(a, z) = T_w(z), \quad T(r, z) \rightarrow T_\infty \text{ as } r \rightarrow \infty, \quad (5)$$

where  $U_w = 2ake^{z/a}$  is the fluid velocity at the surface of the cylinder.

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### III. SOLUTION OF THE PROBLEM

Introduce the following similarity transformations:

$$u = -\frac{1}{2}U_w \frac{f(\eta)}{\sqrt{\eta}}, \quad w = U_w f'(\eta), \quad (6)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{r^2}{a^2}, \quad (7)$$

where the characteristic temperature difference is calculated from the relations  $T_w - T_\infty = ce^{z/a}$ .

With the help of transformations (6) and (7), Eqs. (1) to (3) take the form

$$\eta f''' + f'' + \text{Re}(ff'' - f'^2) + \text{Re} \lambda \theta = 0, \quad (8)$$

$$\eta \theta'' + \theta' + \frac{1}{2} \text{Re Pr}(f\theta' - f'\theta) = 0, \quad (9)$$

in which  $\lambda = g\beta a(T_w - T_\infty)/U_w^2$  is the natural convection parameter,  $\text{Pr} = \nu/\alpha$  is the Prandtl number and  $\text{Re} = aU_w/4\nu$  is the Reynolds number. The boundary conditions in nondimensional form become

$$f(1) = 0, \quad f'(1) = 1, \quad f' \rightarrow 0, \quad \text{as } \eta \rightarrow \infty, \quad (10)$$

$$\theta(1) = 1, \quad \theta \rightarrow 0, \quad \text{as } \eta \rightarrow \infty. \quad (11)$$

The important physical quantities such as the shear stress at the surface  $\tau_w$ , the skinfriction coefficient  $c_f$ , the heat flux at the surface of the cylinder  $q_w$  and the local Nusselt number  $Nu$  are

$$c_f \text{Re}^{1/2} = f''(1), \quad Nu/\text{Re}^{1/2} = -\theta'(1), \quad (12)$$

The solution of the present problem is obtained by using the powerful numerical technique the Keller-box method. To develop the technique the system of differential equations (8–9) along with the boundary conditions (10–11) is converted into a first order differential system by taking

$$R = f', \quad S = R', \quad G = \theta'. \quad (13)$$

With the help of Eq.(13), Eqs.(8) and (9) take the form

$$\eta S' + S + \text{Re}(fS - R^2) + \text{Re} \lambda \theta = 0, \quad (14)$$

$$\eta G' + G + \frac{1}{2} \text{Re Pr}(fG - R\theta) = 0. \quad (15)$$

The corresponding boundary conditions are

$$f(1) = 0, \quad u(1) = 1, \quad u \rightarrow 0, \text{ as } \eta \rightarrow \infty, \quad (16)$$

$$\theta(1) = 1, \quad \theta \rightarrow 0, \text{ as } \eta \rightarrow \infty. \quad (17)$$

Further details of the numerical solution can be found in references [21-24]. The detailed discussion about the obtained numerical solutions is presented in the next section.

#### IV. RESULTS AND DISCUSSION

The problem of natural convection boundary layer flow of a viscous fluid over an exponentially stretched cylinder is studied in this article. The cylinder is assumed to be stretching exponentially along its radial direction. The exponential stretching velocity at the surface of the cylinder is assumed to be  $U_w = 2ake^{z/a}$ . The solution of the problem is obtained with the assistance of powerful second order finite difference scheme known as the Keller-box technique. The impact of the involved parameters such as the Reynolds number  $Re$ , the Prandtl number  $Pr$  and the natural convection parameter  $\lambda$  over the nondimensional velocity and temperature profiles is presented graphically and in the form of tables. *Fig.1* shows the influence of Reynolds number  $Re$  and the natural convection parameter  $\lambda$  over the velocity function  $f'$  for water's Prandtl number (that is  $Pr = 7$ ). From *Fig.1* it is observed that with increase in both Reynolds number and the natural convection parameter  $\lambda$  the velocity profile increases. This observation is consistent with the fact that enhanced natural convection parameter  $\lambda$  acquires greater density difference in fluid that asks increase in fluid velocity. The Reynolds numbers range selected in *Fig.1* is up to  $Re = 50$ . These values of  $Re$  are associated with the laminar type flow and the increasing effect of velocity profile due to increase in  $Re$  reveals the fact that high Reynolds number corresponds to enhance the rate of momentum transfer which in return demands increase in velocity profile  $f'$ . *Fig.2* compiles the influence of Prandtl number  $Pr$  and Reynolds number  $Re$  over the velocity profile  $f'$  when natural convection parameter  $\lambda = 0.5$ , the curves are provided for  $Pr = 0.72$  (air) and  $Pr = 7$  (water) for high Reynolds numbers. From *Fig.2* it is conveyed that for both laminar ( $Re = 5, 50, 500$ ) and turbulent ( $Re = 5000$ ) velocities, inflation in velocity losses dependence over increase in  $Pr$  with increasing  $Re$ . Moreover, with the increase in  $Re$ , the boundary layer thickness increases. The natural convection parameter  $\lambda$  impact over the velocity profile is sketched in *Fig.3* for different values of Prandtl number  $Pr$  when  $Re = 10$ . *Fig.3* inculcates that with increase in  $\lambda$  the velocity profile increases. The rate of increase restricts with increase in  $Pr$ . *Figs.4* and *5* telecast the influence of Prandtl numbers and Reynolds numbers over the temperature profile  $\theta$ . From *Fig.4* it is noted that with increase in Prandtl number  $Pr$  the temperature profile decreases while the thermal boundary layer is achieved much rapidly. The curves are plotted for Prandtl numbers up to  $Pr = 20000$  (such high Prandtl numbers are usually for hydrocarbon based engine oils). This decrease in temperature profile is consistent with the observation that with increase in Prandtl numbers, thermal diffusion rate decreases that also deduces the temperature function  $\theta$ . *Fig.5* indicates that with increase in Reynolds numbers, the temperature profile decreases. This signal to the fact that high Reynolds numbers corresponds to more dense fluids which corresponds to reduced temperature flow rate in the fluid and hence have low fluid temperature. *Figs.6* and *7* gathers the behaviors of Prandtl

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numbers and Reynolds numbers over the temperature profile  $\theta$  for different  $\lambda$ . From these two graphs it is obvious that with increase in  $\lambda$  temperature profile decreases for the presented ranges of Prandtl and Reynolds numbers. Figs.8 and 9 signifies the influence of small Reynolds numbers over velocity and temperature profiles  $f'$  and  $\theta$  for different pairs of Prandtl numbers and the natural convection parameter. From Fig.8 it is observed that for small  $\lambda$ , increase in Reynolds numbers causes to decrease the velocity profile but for larger  $\lambda$ , increase in Reynolds numbers results to increases the velocity profile. From Fig.9 it is clear that with increase in Reynolds numbers, a decrease in the temperature profile is observed. The behavior of skinfriction coefficient for different combinations of Prandtl numbers and  $\lambda$  is graphed in Fig.10 against Reynolds numbers. From this plot it is shown that with increase in Prandtl number and  $\lambda$ , the skinfriction coefficient decreases, while increase in Reynolds numbers enhances the skinfriction coefficient. Fig.11 contains the curves predicting the behavior of local Nusselt numbers  $Nu$  for different Reynolds numbers and  $\lambda$  plotted against Prandtl numbers  $Pr$ . It is predicted from Fig.11 that with increase in the  $Pr, Re$  and  $\lambda$ , increases the local Nusselt numbers. The stream lines pattern associated with the fluid flow are presented in Figs.12 and 13 for  $\lambda = 0.25$ , and 1.2 respectively, graphed in the  $(r, z)$  plane. From these plots the decaying pattern observed is much rapid for high  $\lambda$ . The behavior of shear stress at the surface of the cylinder  $\tau_w$  and the surface heat flux  $q_w$  is tabulated in Tables.1 and 2 for different values of the involved parameters. From these tables it is observed that Prandtl numbers and  $\lambda$  enhances the shear stress while Reynolds numbers reduces the shear stress at the surface of the cylinder, whereas local Nusselt number increases with increase in these parameters.

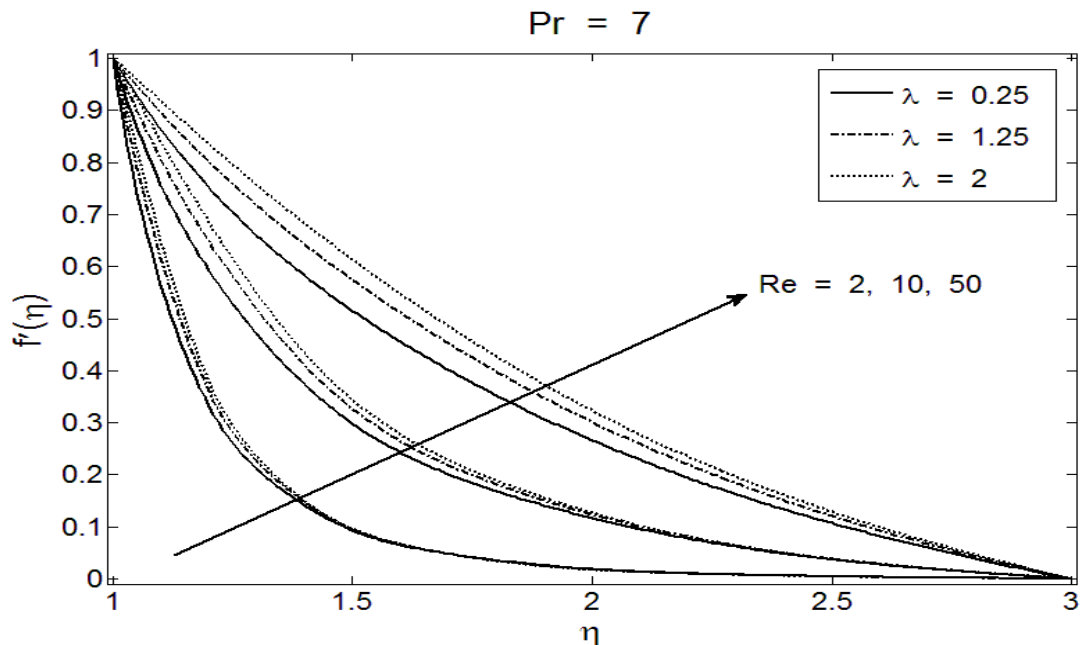


Fig.1. Influence of Reynolds numbers  $Re$  over velocity profile  $f'$  for different  $\lambda$

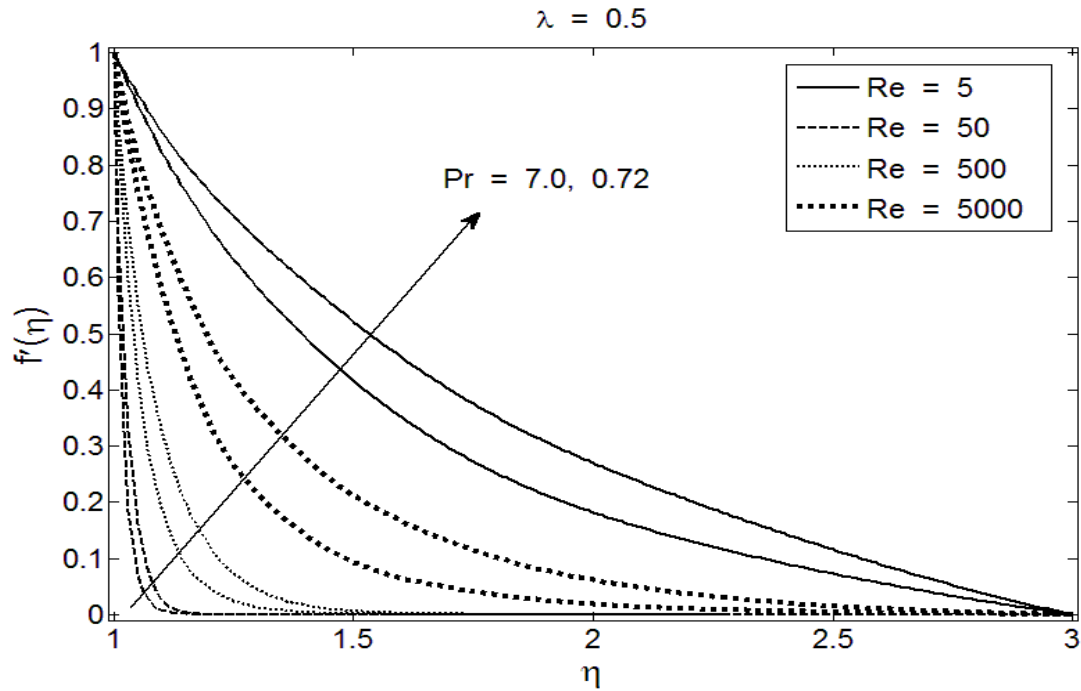


Fig.2. Influence of  $Pr$  over velocity profile  $f'$  for different  $Re$

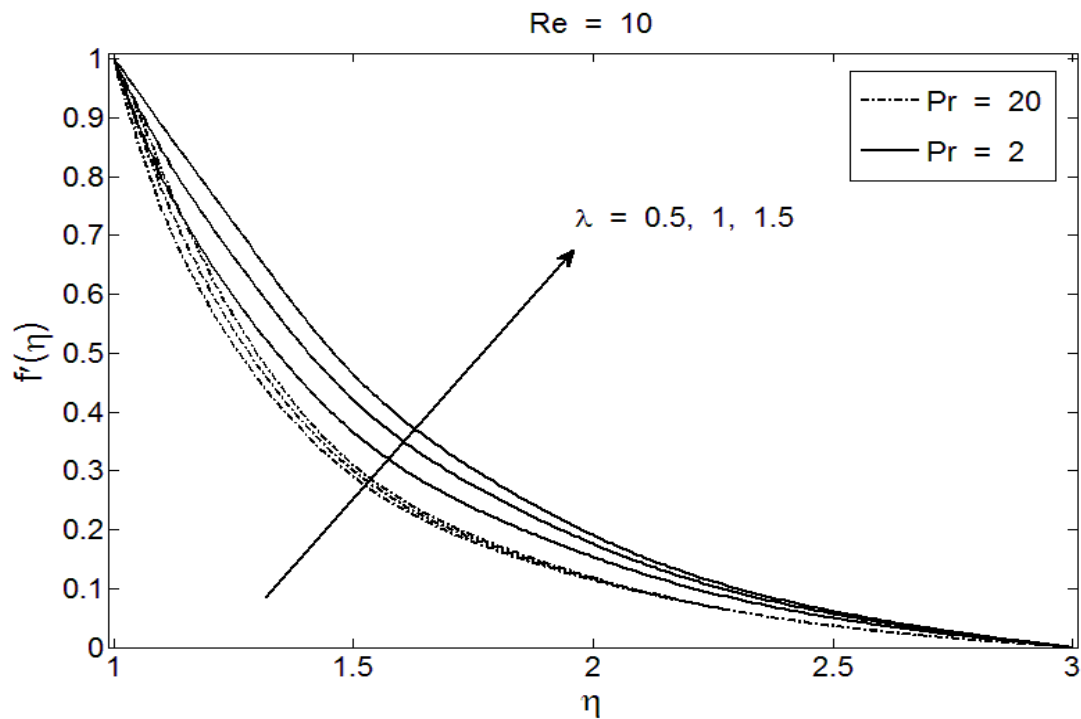
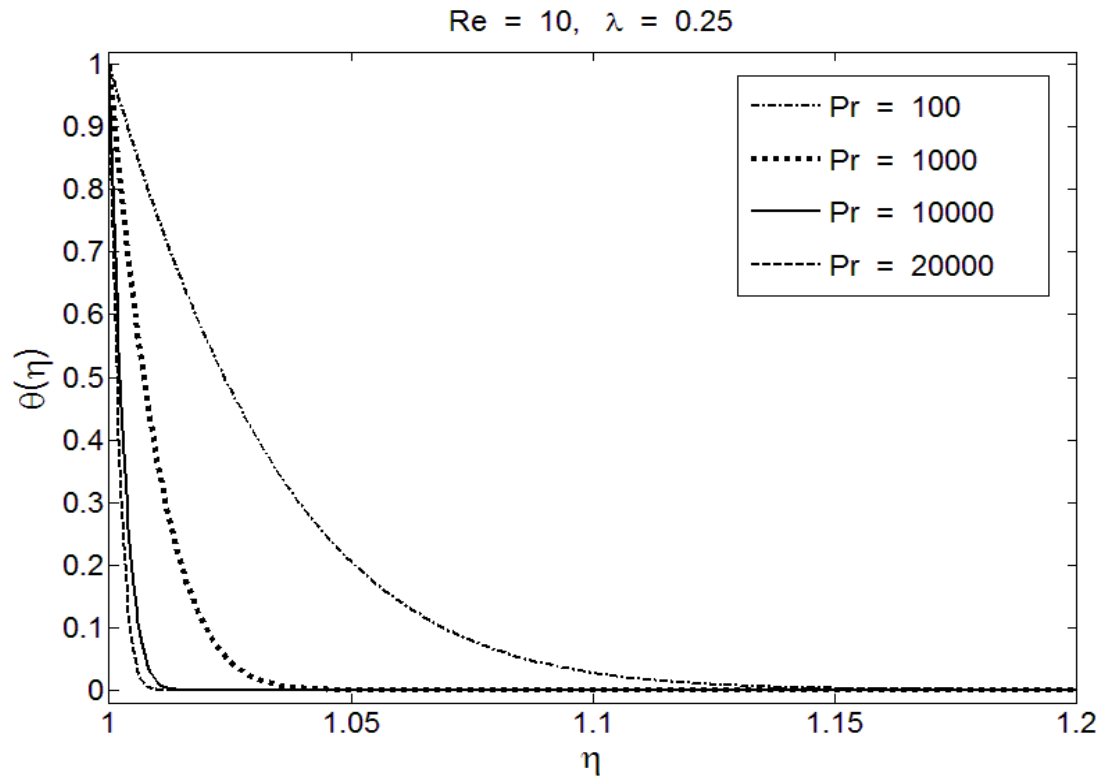
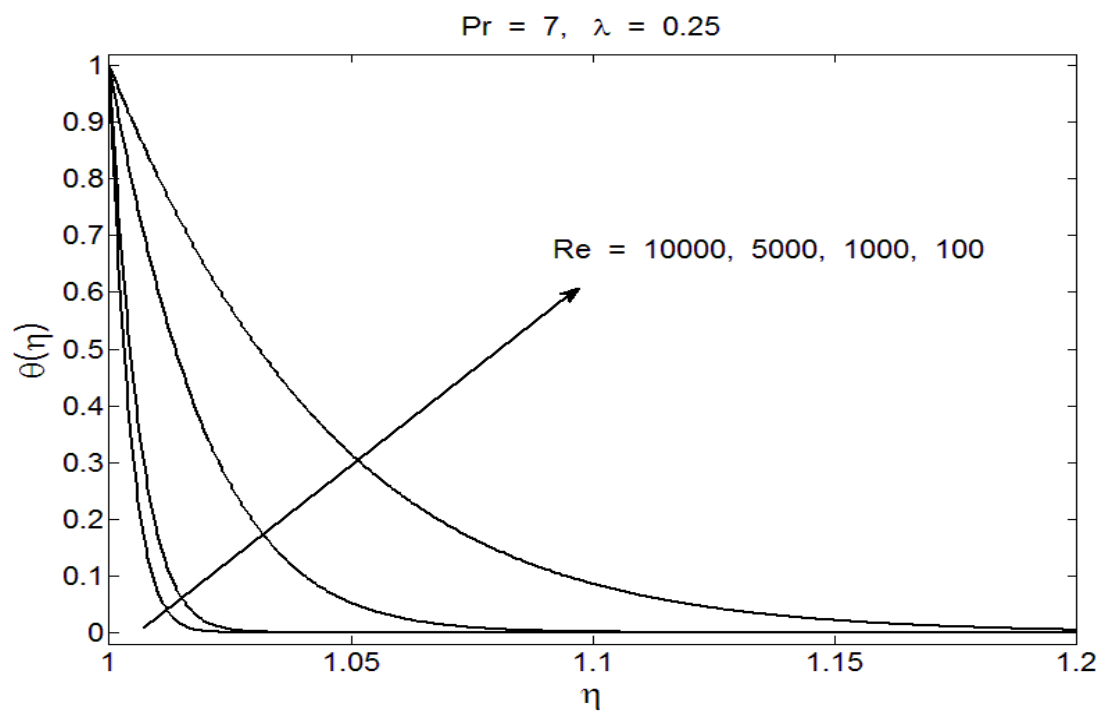


Fig.3. Influence of  $\lambda$  over velocity profile  $f'$  for different  $Pr$

Fig.4. Influence of  $Pr$  over temperature profile  $\theta$ Fig.5. Influence of  $Re$  over temperature profile  $\theta$

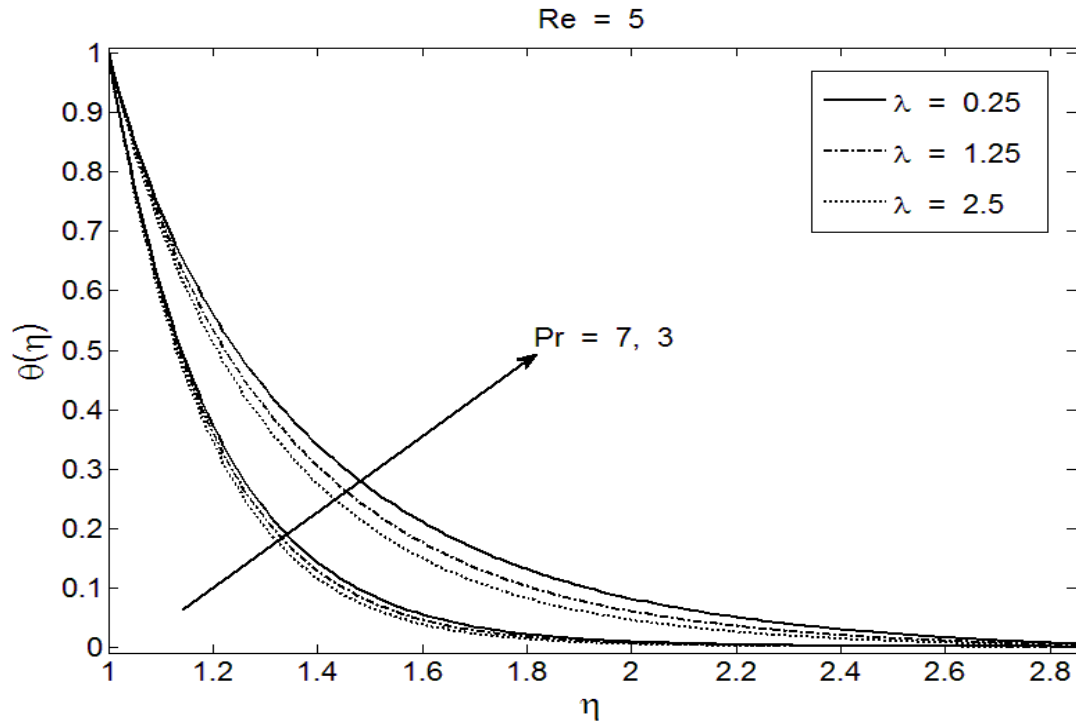


Fig.6. Influence of  $\lambda$  over temperature profile  $\theta$  for different  $Pr$

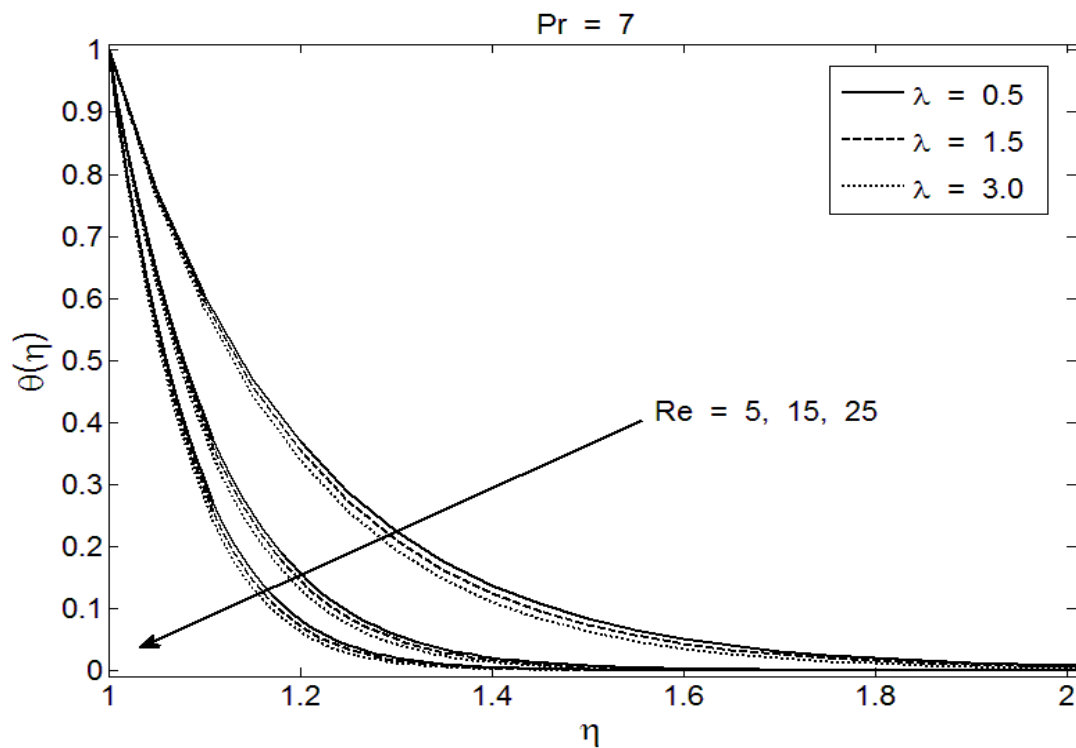


Fig.7. Influence of  $\lambda$  over temperature profile  $\theta$  for different  $Re$

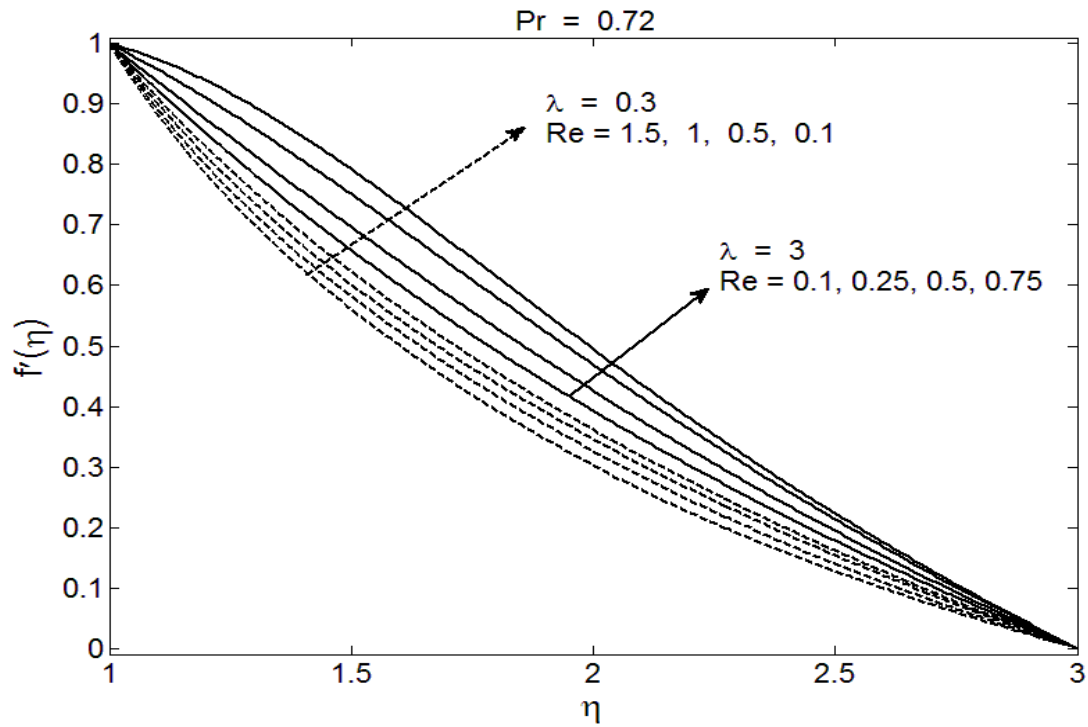


Fig.8. Behavior of small  $Re$  over the velocity profile  $f'$

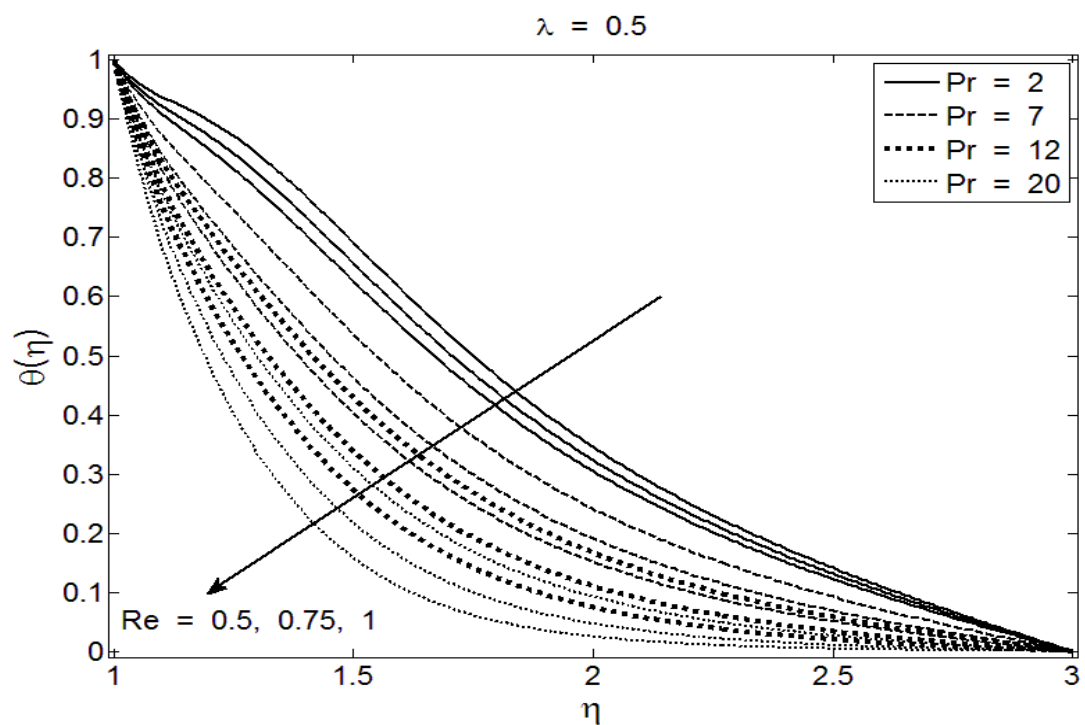


Fig.9. Behavior of small  $Re$  over the temperature profile  $\theta$

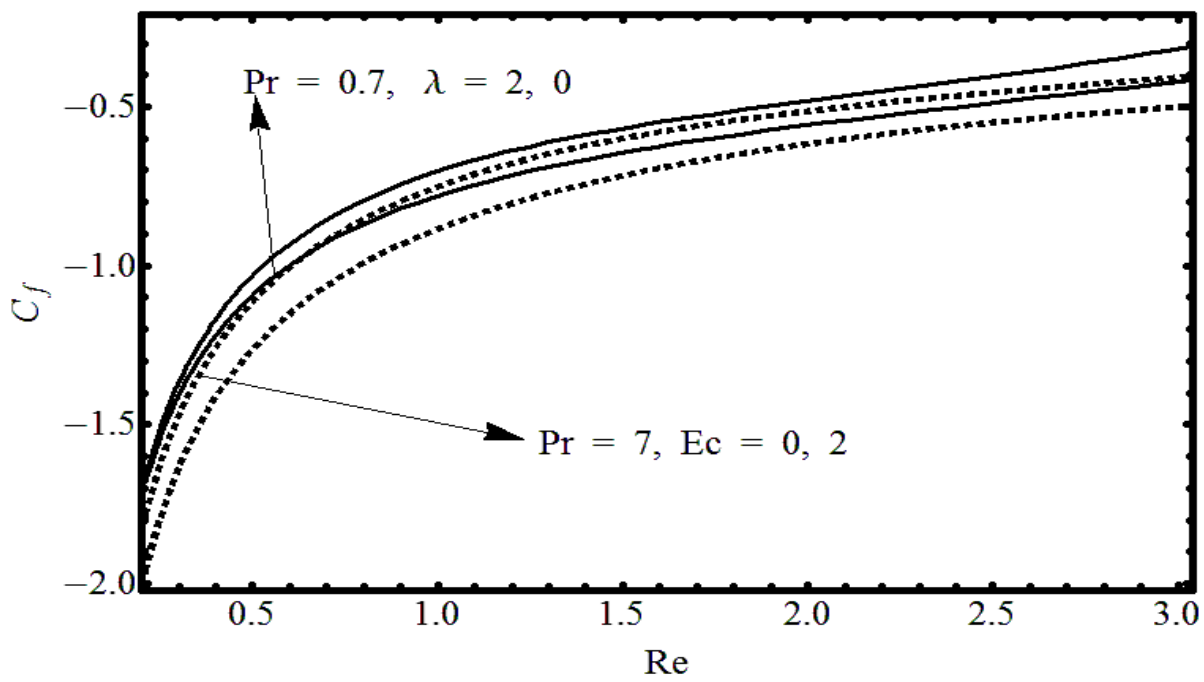


Fig.10. Influence of  $Pr$  and  $\lambda$  over  $C_f$  against  $Re$

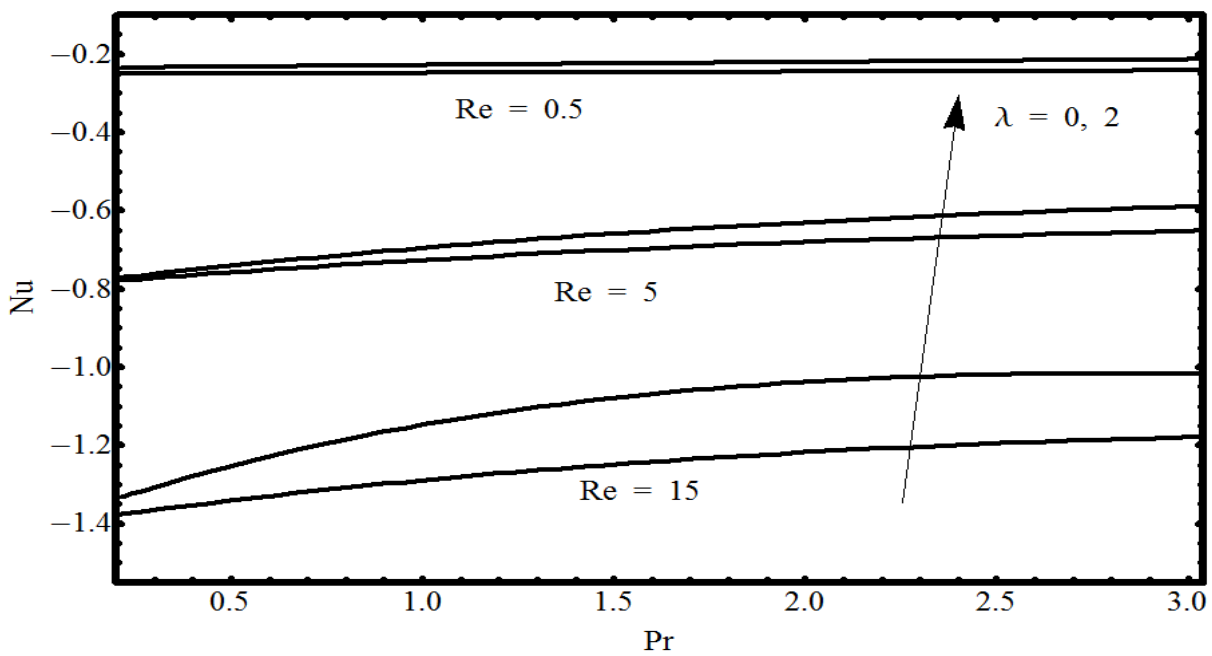


Fig.11. Influence of  $Re$  and  $\lambda$  over  $Nu$  against  $Pr$

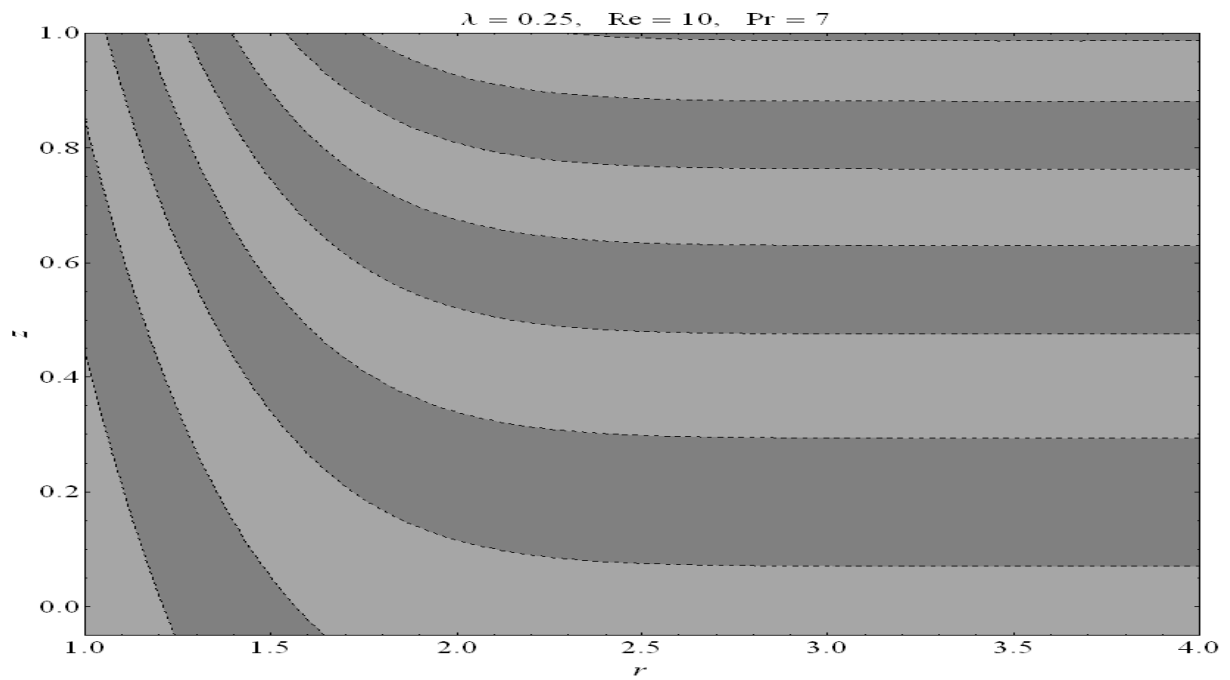


Fig.12. Stream lines pattern for  $\lambda = 0.25$

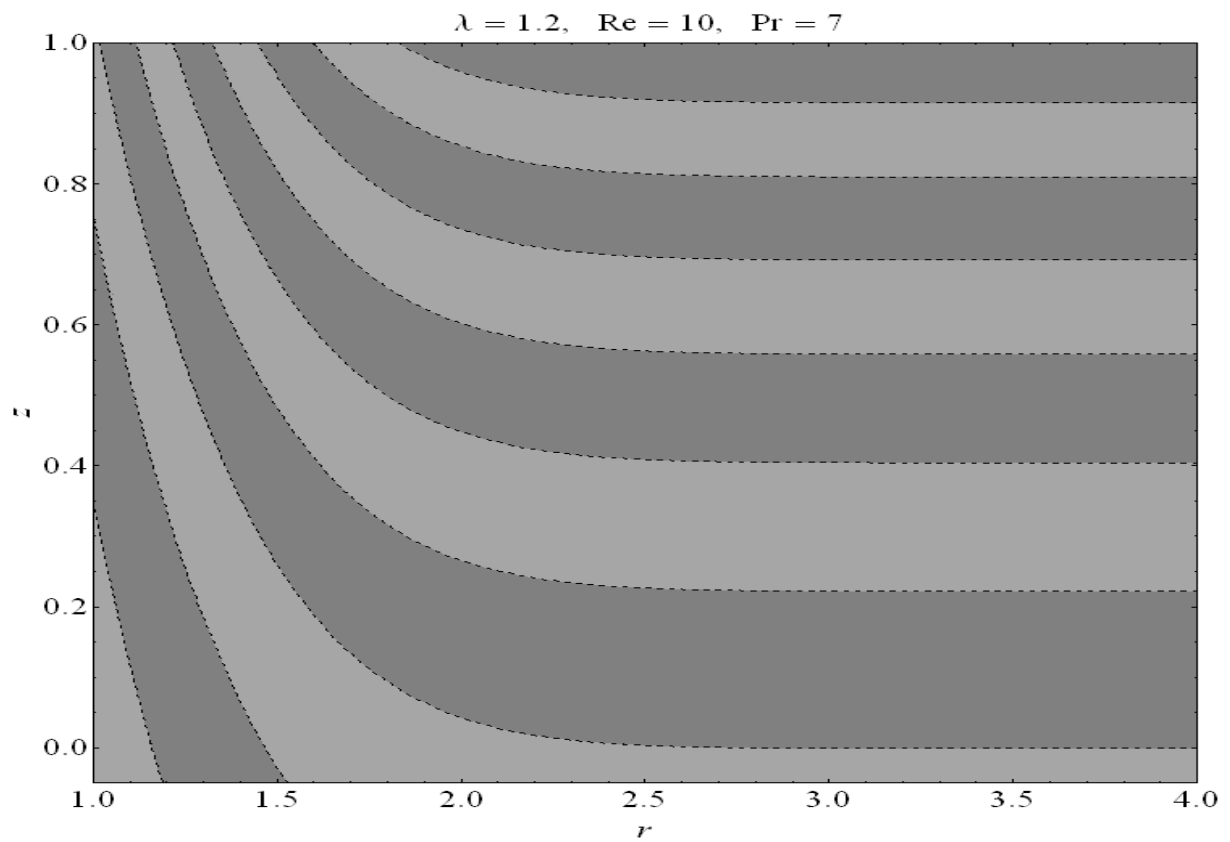


Fig.13. Stream lines pattern for  $\lambda = 1.2$

|                   | $\lambda \backslash \text{Re}$ | $f'(1)$ |        |        |        |        |         |         |
|-------------------|--------------------------------|---------|--------|--------|--------|--------|---------|---------|
|                   |                                | 0.5     | 1      | 5      | 10     | 100    | 1000    | 10000   |
| $\text{Pr} = 0.7$ | 0.25                           | 1.0691  | 1.1943 | 1.9559 | 2.6148 | 7.8386 | 24.6839 | 81.5422 |
|                   | 0.75                           | 0.9384  | 0.9608 | 1.2858 | 1.6661 | 4.9948 | 15.8302 | 53.8606 |
|                   | 1.50                           | 0.7454  | 0.6202 | 0.3844 | 0.4155 | 1.2255 | 4.0216  | 16.2816 |
|                   | 2.50                           | 0.4905  | 0.1821 | 0.7015 | 1.0833 | 3.3322 | 10.3141 | 30.0166 |
| $\text{Pr} = 7$   | 0.25                           | 1.0838  | 1.2348 | 2.1561 | 2.9292 | 8.8292 | 27.6638 | 90.0127 |
|                   | 0.75                           | 0.9836  | 1.0817 | 1.8294 | 2.4794 | 7.4529 | 23.3204 | 75.4808 |
|                   | 1.50                           | 0.8364  | 0.8600 | 1.3592 | 1.8300 | 5.4571 | 17.0109 | 54.2655 |
|                   | 2.50                           | 0.6451  | 0.5768 | 0.7610 | 1.0012 | 2.8987 | 8.9081  | 30.9185 |

Table.1. Behavior of shear stress at the surface of the cylinder for different values of the involved parameters

|                 | $\text{Re} \backslash \text{Pr}$ | $-\Theta'(1)$ |        |         |         |         |         |          |           |
|-----------------|----------------------------------|---------------|--------|---------|---------|---------|---------|----------|-----------|
|                 |                                  | 0.2           | 0.7    | 7       | 10      | 70      | 150     | 1500     | 15000     |
| $\lambda = 0$   | 1                                | 1.2281        | 1.3355 | 2.4696  | 2.8960  | 7.4317  | 10.8491 | 34.4118  | 113.4154  |
|                 | 5                                | 1.3633        | 1.7833 | 5.0510  | 6.0576  | 16.3167 | 24.0243 | 78.0879  | 297.9081  |
|                 | 10                               | 1.4905        | 2.1932 | 7.0066  | 8.4368  | 22.9968 | 33.9505 | 112.7487 | 491.7423  |
|                 | 20                               | 1.6789        | 2.7991 | 9.7683  | 11.8010 | 32.4741 | 48.0992 | 166.1533 | 861.8583  |
|                 | 50                               | 2.0891        | 4.0877 | 15.2755 | 18.5037 | 51.4576 | 76.7794 | 293.7000 | 1941.5102 |
| $\lambda = 0.5$ | 1                                | 1.2324        | 1.3530 | 2.5058  | 2.9234  | 7.4943  | 10.8660 | 34.4135  | 113.4162  |
|                 | 5                                | 1.4124        | 1.8865 | 5.1156  | 6.1103  | 16.3405 | 24.0832 | 78.0887  | 297.9041  |
|                 | 10                               | 1.6013        | 2.3794 | 7.0846  | 8.5018  | 23.0068 | 33.9665 | 112.7556 | 491.7535  |
|                 | 20                               | 1.9185        | 3.1213 | 9.8782  | 11.8927 | 32.5090 | 48.2209 | 166.1675 | 961.9010  |
|                 | 50                               | 2.6281        | 4.6405 | 15.4471 | 18.6462 | 51.5160 | 76.8235 | 293.7440 | 1941.5985 |

Table.2. Behavior of heat flux at the surface of the cylinder for different values of the involved parameters

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