



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH
MATHEMATICS AND DECISION SCIENCES
Volume 13 Issue 7 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Global Attractor of the Nonlinear four Order Wave Equations

By Ling Zhao

Sichuan Normal University, China

Abstract - Global attractor of the nonlinear four order wave equations is considered in this paper. Firstly, it is proved that this system possesses an absorbing set in $L^2(\Omega) \times H^2(\Omega)$. Secondly, it is obtained that the nonlinear four order wave equations have a global attractor in $L^2(\Omega) \times H^2(\Omega)$ by using C -condition.

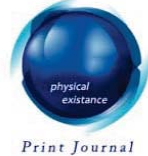
Keywords : global attractor, C -condition, four order wave equations.

GJSFR-F Classification : MSC 2010: 35B40, 35B41, 35G30, 35L75



Strictly as per the compliance and regulations of :





Global Attractor of the Nonlinear four Order Wave Equations

Ling Zhao

Abstract - Global attractor of the nonlinear four order wave equations is considered in this paper. Firstly, it is proved that this system possesses an absorbing set in $L^2(\Omega) \times H^2(\Omega)$. Secondly, it is obtained that the nonlinear four order wave equations have a global attractor in $L^2(\Omega) \times H^2(\Omega)$ by using C -condition.

Keywords : global attractor, C -condition, four order wave equations.

1. INTRODUCTION

Nonlinear four order wave equations can be applied in mechanics of elastic constructions, and have a long history([1]-[11]). But few of the studies consider attractor of the nonlinear four order wave equations. In this paper we study attractor of the following nonlinear four order wave equations

$$u_{tt} + \Delta^2 u + \alpha u_t = f(u), \quad x \in \Omega, \quad t > 0, \quad (1.1)$$

$$u = \Delta u = 0, \quad x \in \partial\Omega, \quad t > 0, \quad (1.2)$$

$$u(x, 0) = \varphi(x), \quad u_t(x, 0) = \psi(x), \quad x \in \Omega, \quad (1.3)$$

where $\alpha > 0$, Δ is the Laplacian operator, Ω denotes an open bounded set of R^n ($n = 1, 2, 3$) with smooth boundary $\partial\Omega$, and $f(s) \in C^1(R)$ satisfies the following conditions

$$F(s) = \int_{\Omega} f(s) dx \leq c, \quad (1.4)$$

$$sf(s) - F(s) \leq c, \quad (1.5)$$

and there is a $0 \leq r < \infty$, such that

$$\lim_{s \rightarrow \infty} \frac{f'(s)}{s^r} = 0. \quad (1.6)$$

As far as the theory of infinite-dimensional dynamical system is concerned, we refer to [12]-[22]. In the study of infinite dimensional dynamical system, the long-time behavior of the solution to equations is an important issue. The long-time behavior of the solution to equations can be shown by the global attractor with the finite-dimensional characteristics. Some authors have already studied the existence of the global attractor for some evolution equations. The global attractor strictly defined as ω -limit set of ball, which under additional assumptions is nonempty, compact, and invariant ([14],[16]). We investigate global attractor of the equations (1.1)-(1.3) in this article.

The paper is organized as follows. In Section 2 we recall preliminary results. In Section 3, we obtain global attractor of the equations.

II. PRELIMINARIES

Let X and X_1 be two Banach spaces, $X_1 \subset X$ a compact and dense inclusion. Consider the abstract nonlinear evolution equation defined on X , given by

$$\begin{cases} \frac{du}{dt} = Lu + G(u), \\ u(x, 0) = u_0. \end{cases} \quad (2.1)$$

where $u(t)$ is an unknown function, $L : X_1 \rightarrow X$ a linear operator, and $G : X_1 \rightarrow X$ a nonlinear operator.

A family of operators $S(t) : X \rightarrow X (t \geq 0)$ is called a semigroup generated by (2.1) if it satisfies the following properties:

- (1) $S(t) : X \rightarrow X$ is a continuous map for any $t \geq 0$,
- (2) $S(0) = id : X \rightarrow X$ is the identity,
- (3) $S(t + s) = S(t) \cdot S(s), \forall t, s \geq 0$. Then, the solution of (2.1) can be expressed as

$$u(t, u_0) = S(t)u_0.$$

Next, we introduce the concepts and definitions of invariant sets, global attractors, and ω -limit sets for the semigroup $S(t)$.

Definition 2.1 Let $S(t)$ be a semigroup defined on X . A set $\Sigma \subset X$ is called an invariant set of $S(t)$ if $S(t)\Sigma = \Sigma, \forall t \geq 0$. An invariant set Σ is an attractor of $S(t)$ if Σ is compact, and there exists a neighborhood $U \subset X$ of Σ such that for any $u_0 \in U$,

$$\inf_{v \in \Sigma} \|S(t)u_0 - v\|_X \rightarrow 0, \quad \text{as } t \rightarrow \infty.$$

In this case, we say that Σ attracts U . Especially, if Σ attracts any bounded set of X , Σ is called a global attractor of $S(t)$ in X .

For a set $D \subset X$, we define the ω -limit set of D as follows

$$\omega(D) = \bigcap_{s \geq 0} \overline{\bigcup_{t \geq s} S(t)D},$$

where the closure is taken in the X -norm. Lemma 2.2 is the classical existence theorem of global attractor by Temam [16].

Lemma 2.2 Let $S(t) : X \rightarrow X$ be the semigroup generated by (2.1). Assume the following conditions hold

- (1) $S(t)$ has a bounded absorbing set $B \subset X$, i.e., for any bounded set $A \subset X$ there exists a time $t_A \geq 0$ such that $S(t)u_0 \in B, \forall u_0 \in A$ and $t > t_A$;
- (2) $S(t)$ is uniformly compact, i.e., for any bounded set $U \subset X$ and some $T > 0$ sufficiently large, the set $\overline{\bigcup_{t \geq T} S(t)U}$ is compact in X .

Then the ω -limit set $\mathcal{A} = \omega(B)$ of B is a global attractor of (2.1), and $\mathcal{A}$ is connected providing B is connected.

Definition 2.3^[15] We say that $S(t) : X \rightarrow X$ satisfies C -condition, if for any bounded set $B \subset X$ and $\varepsilon > 0$, there exist $t_B > 0$ and a finite dimensional subspace $X_1 \subset X$, such that $\{PS(t)B\}$ is bounded, and

$$\|(I - P)S(t)u\|_X < \varepsilon, \quad \forall t \geq t_B \text{ and } u \in B,$$

where $P : X \rightarrow X_1$ is a projection.

Lemma 2.4^[15] Let $S(t) : X \rightarrow X (t \geq 0)$ be a dynamical systems. If the following condition are satisfied

- (1) there exists a bounded absorbing set $B \subset X$,
- (2) $S(t)$ satisfies C -condition,

then $S(t)$ has a global attractor in X .

We introduce the spaces

$$H = L^2(\Omega), \quad V = \{u \in H^2 | u|_{\partial\Omega} = \Delta u|_{\partial\Omega} = 0\}, \quad E = H \times V.$$

The denotations $(\cdot, \cdot), \|\cdot\|$ are the inner product and norm in H respectively, i.e., for any $u, v \in H$, $(u, v) = \int_{\Omega} uv dx$, $\|u\| = (\int_{\Omega} |u|^2 dx)^{\frac{1}{2}}$. The norm in V is $\|u\|_V = \|\Delta u\|$ and we have $\|\Delta u\|^2 \geq \beta_1 \|u\|^2$, where β_1 is the first eigenvalue of Δu with boundary condition (1.2).

Lemma 2.5 Assume the nonlinear term $f : R \rightarrow R$ satisfies (1.6). Then the nonlinear operator $f : V \rightarrow H$ is compact.

Proof. Let $\{u_n\}$ be a bounded sequence in V . By the Sobolev embedding theorem, $\{u_n\}$ is bounded in $L^k(\Omega)$ for any $1 \leq k < \infty$, and has a convergent subsequence in $L^2(\Omega)$. Without loss of generality, we assume $\{u_n\}$ converges to u_0 in H . It is sufficient to prove that $\{f(u_n)\}$ converges to $f(u_0)$ in H .

From (1.6), for any $\eta > 0$, we have

$$|f'(s)|^2 \leq \eta |s|^{2r} + c_{\eta},$$

where $c_{\eta} \rightarrow 0$ if $\eta \rightarrow 0$.

Then, for some $0 \leq \varepsilon \leq 1$, we obtain

$$\begin{aligned} \int_{\Omega} |f(u_n) - f(u_0)|^2 dx &= \int_{\Omega} |f'(u_0 + (1 - \varepsilon)(u_n - u_0))|^2 |u_n - u_0|^2 dx \\ &\leq \eta \int_{\Omega} |u_0 + (1 - \varepsilon)(u_n - u_0)|^{2r} |u_n - u_0|^2 dx + c_{\eta} \int_{\Omega} |u_n - u_0|^2 dx \\ &= \eta \int_{\Omega} |\varepsilon u_0 + (1 - \varepsilon)u_n|^{2r} |u_n - u_0|^2 dx + c_{\eta} \int_{\Omega} |u_n - u_0|^2 dx \\ &\leq 4^r \eta \int_{\Omega} (|u_0|^{2r} + |u_n|^{2r}) |u_n - u_0|^2 dx + c_{\eta} \int_{\Omega} |u_n - u_0|^2 dx \\ &\leq 4^r \eta \left[\left(\int_{\Omega} |u_0|^{2rp} dx \right)^{\frac{1}{p}} + \left(\int_{\Omega} |u_n|^{2rp} dx \right)^{\frac{1}{p}} \right] \left(\int_{\Omega} |u_n - u_0|^{2q} dx \right)^{\frac{1}{q}} + c_{\eta} \int_{\Omega} |u_n - u_0|^2 dx \end{aligned}$$

where $p > 0, q > 0$, and $\frac{1}{p} + \frac{1}{q} = 1$.

Let $n \rightarrow \infty$ and $\eta \rightarrow 0$. We have

$$\int_{\Omega} |f(u_n) - f(u_0)|^2 dx = 0.$$

Then the nonlinear operator $f : V \rightarrow H$ is compact.

III. EXISTENCE OF GLOBAL ATTRACTOR

Theorem 3.1 Let $\varphi \in H$, $\psi \in V$, and f satisfy the conditions (1.4) and (1.5). Suppose that the problem (1.1)-(1.3) has a unique weak solution and $S(t), t > 0$, defined by $S(t)(\varphi, \psi) =$

$(u(t), u_t(t))$, is the semigroup generated by the problem (1.1)-(1.3). Then $S(t)$ has a bounded absorbing ball.

Proof. Take the inner product of (1.1) in H with $v = u_t + \theta u$, $0 < \theta \leq \theta_0$, and θ_0 will be chosen later. we obtain

$$(u_{tt}, v) + (\Delta^2 u, v) + \alpha(u_t, v) = (f, u).$$

Using the condition (1.4), we have

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} (\|v\|^2 + \|\Delta u\|^2 - 2 \int_{\Omega} F(u) dx) + (\alpha - \theta) \|v\|^2 + \theta \|\Delta u\|^2 \\ & + (\theta^2 - \alpha\theta)(u, v) - \theta \int_{\Omega} f(u) u dx = 0. \end{aligned} \quad (3.1)$$

Using the Hölder inequality, Young inequality and the condition (1.5), we get

$$\begin{aligned} & (\alpha - \theta) \|v\|^2 + \theta \|\Delta u\|^2 + (\theta^2 - \alpha\theta)(u, v) - \theta \int_{\Omega} f(u) u dx \\ & \geq (\alpha - \theta) \|v\|^2 + \theta \|\Delta u\|^2 + (\theta^2 - \alpha\theta) \|u\| \|v\| - \theta \int_{\Omega} (F(u) + c) dx \\ & \geq (\alpha - \theta) \|v\|^2 + \theta \|\Delta u\|^2 - \frac{\alpha\theta}{\beta_1^{\frac{1}{2}}} \|\Delta u\| \|v\| - \theta \int_{\Omega} (F(u) + c) dx \\ & \geq (\alpha - \theta) \|v\|^2 + \theta \|\Delta u\|^2 - \frac{\theta}{2} \|\Delta u\|^2 - \theta \frac{\alpha^2}{2\beta_1} \|v\|^2 - \theta \int_{\Omega} (F(u) + c) dx \\ & \geq [\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1})] \|v\|^2 + \frac{\theta}{2} \|\Delta u\|^2 - \theta \int_{\Omega} (F(u) + c) dx. \end{aligned}$$

Choose θ_0 , such that $\theta_0(1 + \frac{\alpha^2}{2\beta_1}) = \frac{\alpha}{2}$. Hence, $\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1}) \geq \frac{\alpha}{2} \geq \theta$.

From (3.1), we have

$$\frac{1}{2} \frac{d}{dt} (\|v\|^2 + \|\Delta u\|^2 - 2 \int_{\Omega} F(u) dx) + \frac{\alpha}{2} \|v\|^2 + \frac{\theta}{2} \|\Delta u\|^2 - \theta \int_{\Omega} (F(u) + c) dx \leq 0.$$

Then, it follows that

$$\begin{aligned} & \frac{d}{dt} [\|v\|^2 + \|\Delta u\|^2 + 2 \int_{\Omega} (c - F(u)) dx] + \theta [\|v\|^2 + \|\Delta u\|^2 + 2 \int_{\Omega} (c - F(u)) dx] \\ & \leq 4c\theta |\Omega|, \end{aligned} \quad (3.2)$$

where $|\Omega|$ is measure of the Ω .

Let $y(t) = \|v\|^2 + \|\Delta u\|^2 + 2 \int_{\Omega} (c - F(u)) dx$. Then $y(t) \geq 0$ and (3.2) can be read as

$$\frac{dy}{dt} + \theta y \leq 4c\theta |\Omega|.$$

Using the Gronwall inequality, we have

$$y(t) \leq y(0)e^{-\theta t} + 4c|\Omega|(1 - e^{-\theta t}), \quad t \geq 0.$$

For any bounded set B of E , $(\varphi, \psi) \in B$ and $\int_{\Omega}[c - F(\varphi)]dx$ is bounded. Then

$$R(B) = \sup_{(\varphi, \psi) \in B} y(0) = \sup_{(\varphi, \psi) \in B} \{\|\varphi\|_V^2 + \|\psi + \theta\varphi\| + 2 \int_{\Omega} (c - F(\varphi))dx\} < \infty,$$

and

$$\lim_{t \rightarrow \infty} \sup_{(\varphi, \psi) \in B} y(t) \leq 4c|\Omega| = \mu_0^2.$$

Let $\mu_1 > \mu_0$ be fixed, and there is a $t_0 = t_0(R(B), \mu_1) = \frac{1}{\theta} \ln \frac{R(B)}{\mu_1^2 - \mu_0^2}$ such that for any $t \geq t_0$, we have $y(t) \leq \mu_1^2$. Then

$$\begin{aligned} \|u(t)\|_V^2 + \|u_t(t)\|^2 &\leq \|u(t)\|_V^2 + 2\|u_t(t) + \theta u(t)\|^2 + 2\theta^2\|u(t)\|^2 \\ &\leq \|u(t)\|_V^2 + 2\|u_t(t) + \theta u(t)\|^2 + \frac{2\theta^2}{\beta_1}\|u(t)\|_V^2 \\ &\leq 2(1 + \frac{\theta^2}{\beta_1})(\|u(t)\|_V^2 + \|u_t(t) + \theta u(t)\|^2) \\ &\leq 2(1 + \frac{\theta^2}{\beta_1})y(t) \leq 2(1 + \frac{\theta^2}{\beta_1})\mu_1^2. \end{aligned}$$

Let $\rho_0^2 = 2(1 + \frac{\theta^2}{\beta_1})\mu_1^2$. Then For all $t \geq t_0$, we have

$$\|u\|_V^2 + \|u_t\|^2 \leq \rho_0^2, \quad (3.3)$$

which implies that the ball of E , $B_0 = B_E(0, \rho_0)$, centered at 0 of radius ρ_0 , is an absorbing set.

Theorem 3.2 Assume $f : R \rightarrow R$ satisfies (1.4)-(1.6). Then the semigroup $S(t), t \geq 0$ associated with problem (1.1)-(1.3) possesses a global attractor.

Proof. The eigenvalue equation

$$\begin{cases} \Delta^2 u = \beta u, \\ u|_{\partial\Omega} = \Delta u|_{\partial\Omega} = 0, \end{cases} \quad (3.4)$$

has eigenvalues $\beta_1, \beta_2, \dots, \beta_k, \dots$ and eigenvector $\{e_k | k = 1, 2, 3, \dots\}$, and $0 < \beta_1 \leq \beta_2 \leq \dots \leq \beta_k \leq \dots$. If $k \rightarrow \infty$, $\beta_k \rightarrow \infty$. $\{e_k | k = 1, 2, 3, \dots\}$ constitutes an orthogonal base of H .

For $\forall u \in H$, we have

$$u = \sum_{k=1}^{\infty} u_k e_k, \quad \|u\|_{L^2}^2 = \sum_{k=1}^{\infty} u_k^2,$$

Introduce subspace $E_1 = \text{span}\{e_1, e_2, \dots, e_k\} \subset H$. Let E_2 be an orthogonal subspace of $E_1 \subset H$.

For $\forall (u_t, u) \in E$, we find

$$u_t = u_{1t} + u_{2t}, \quad u_{1t} = P_m u_t, \quad u_{2t} = (I - P_m)u_t,$$

and

$$u = u_1 + u_2, \quad u_1 = P_m u, \quad u_2 = (I - P_m)u,$$

where $P_m : H \rightarrow E_1$ be the orthogonal projection.

Let $P_i : E \rightarrow E_i \times E_i$ be the orthogonal projection. Thanks to Definition 2.3, we will prove that for any bounded set $B \subset E$ and $\varepsilon > 0$, there exists $t_* > 0$ such that

$$\|P_1 S(t)B\|_H \leq M, \quad \forall t > t_*, \quad M \text{ is a constant}, \quad (3.5)$$

$$\|P_2 S(t)B\|_H \leq \varepsilon, \quad \forall t > t_*, \quad \varphi \in B. \quad (3.6)$$

From Theorem 3.1, $S(t)$ has an absorbing set B_M . Then for any bounded set $B \subset E$, there exists $t_0 > 0$ such that $S(t)B \subset B_M, \forall t > t_0$, which imply (3.5).

From Lemma 2.5, $f : V \rightarrow H$ is compact, then for any $\varepsilon > 0$, there exists some $m \in \mathbb{N}$ such that

$$\|(I - P_m)f\| \leq \varepsilon.$$

Multiply (1.1) by $v_2 = u_{2t} + \theta u_2$ ($0 < \theta \leq \theta_1$), and integrate over Ω . We obtain

$$\int_{\Omega} (u_{tt} + \Delta^2 u + \alpha u_t) v_2 dx = \int_{\Omega} f v_2 dx.$$

Then, we have

$$\begin{aligned} & \frac{1}{2} \frac{d}{dt} (\|v_2\|^2 + \|\Delta u_2\|^2) + (\alpha - \theta) \|v_2\|^2 + \theta \|\Delta u_2\|^2 \\ & + (\theta^2 - \alpha\theta)(u_2, v_2) \leq (f, v_2) \leq \|(I - P_m)f\| \|v_2\| \leq \varepsilon \|v_2\| \leq \frac{\alpha}{4} \|v_2\|^2 + \frac{\varepsilon^2}{\alpha}. \end{aligned} \quad (3.7)$$

Using the Hölder inequality and Young inequality, we get

$$\begin{aligned} & (\alpha - \theta) \|v_2\|^2 + \theta \|\Delta u_2\|^2 + (\theta^2 - \alpha\theta)(u_2, v_2) \\ & \geq (\alpha - \theta) \|v_2\|^2 + \theta \|\Delta u_2\|^2 + (\theta^2 - \alpha\theta) \|u_2\| \|v_2\| \\ & \geq (\alpha - \theta) \|v_2\|^2 + \theta \|\Delta u_2\|^2 - \frac{\alpha\theta}{\beta_{m+1}^{\frac{1}{2}}} \|\Delta u_2\| \|v\| \end{aligned}$$

$$\begin{aligned} &\geq (\alpha - \theta)\|v_2\|^2 + \theta\|\Delta u_2\|^2 - \frac{\theta}{2}\|\Delta u_2\|^2 - \theta\frac{\alpha^2}{2\beta_{m+1}}\|v_2\|^2 \\ &\geq [\alpha - \theta(1 + \frac{\alpha^2}{2\beta_{m+1}})]\|v_2\|^2 + \frac{\theta}{2}\|\Delta u_2\|^2. \end{aligned}$$

Choose θ_1 , such that $\theta_1(1 + \frac{\alpha^2}{2\beta_1}) = \frac{\alpha}{2}$. Hence, $\alpha - \theta(1 + \frac{\alpha^2}{2\beta_1}) \geq \frac{\alpha}{2} \geq \theta$.

From (3.7), we have

$$\frac{1}{2} \frac{d}{dt} (\|v_2\|^2 + \|\Delta u_2\|^2) + \frac{\alpha}{4} \|v_2\|^2 + \frac{\theta}{2} \|\Delta u_2\|^2 \leq \frac{\varepsilon^2}{\alpha}.$$

Then, it follows that

$$\frac{d}{dt} (\|v_2\|^2 + \|\Delta u_2\|^2) + \theta(\|v\|^2 + \|\Delta u\|^2) \leq \frac{2\varepsilon^2}{\alpha}. \quad (3.8)$$

Using the Gronwall inequality, we have

$$\|v_2(t)\|^2 + \|\Delta u_2(t)\|^2 \leq (\|v_2(0)\|^2 + \|\Delta u_2(0)\|^2)e^{-\theta t} + \frac{2\varepsilon^2}{\alpha}(1 - e^{-\theta(t-t_0)}), \quad t \geq t_0.$$

where t_0 is given in the proof of Theorem 3.1. Then

$$\|v_2(0)\|^2 + \|\Delta u_2(0)\|^2 \leq \rho_0^2.$$

Let $t_1 - t_0 = \frac{1}{\theta} \ln \frac{\rho_0^2 \alpha}{\varepsilon^2}$. Hence

$$\|v_2(t)\|^2 + \|\Delta u_2(t)\|^2 \leq \frac{3\varepsilon^2}{\alpha}, \quad t \geq t_1.$$

Then

$$\begin{aligned} \|u_2(t)\|_V^2 + \|u_{2t}(t)\|^2 &\leq \|u_2(t)\|_V^2 + 2\|u_{2t}(t) + \theta u_2(t)\|^2 + 2\theta^2\|u_2(t)\|^2 \\ &\leq \|u_2(t)\|_V^2 + 2\|u_{2t}(t) + \theta u_2(t)\|^2 + \frac{2\theta^2}{\beta_{m+1}}\|u_2(t)\|_V^2 \\ &\leq 2(1 + \frac{\theta^2}{\beta_{m+1}})(\|u_2(t)\|_V^2 + \|u_{2t}(t) + \theta u_2(t)\|^2) \\ &\leq 2(1 + \frac{\theta^2}{\beta_{m+1}})(\|v_2(t)\|^2 + \|\Delta u_2(t)\|^2) \leq (1 + \frac{\theta^2}{\beta_{m+1}}) \frac{6\varepsilon^2}{\alpha}. \end{aligned}$$

Let $C = \frac{6}{\alpha}(1 + \frac{\theta^2}{\beta_{m+1}})$. Then for all $t \geq t_1$, we have

$$\|u\|_V^2 + \|u_t\|^2 \leq C\varepsilon^2, \quad (3.9)$$

which implies (3.6).

From Lemma 2.4, the equations(1.1)-(1.3) possess a global attractor.

REFERENCES RÉFÉRENCES REFERENCIAS

- [1] Easley JG: Nonlinear vibration of beams and rectangular plates. Z. Angew. Math. Phys. 15, 167-175 (1964)
- [2] Medeiros LA: On a new class of nonlinear wave equations. J. Math. Anal. Appl. 69, 252-262 (1979)
- [3] Fitzgibbon WE: Global existence and boundedness of solutions to the extensible beam equation. SIAM J. Math. Anal. 13, 739-745 (1982)
- [4] Kouemou Patcheu S: On a global solution and asymptotic behaviour for the generalized damped extensible beam equation. J. Diff. Equa. 135, 299-314 (1997)
- [5] Weinstock R: Calculus of variations with applications to physics and engineering. McGraw-Hill Book Company Inc., New York-Toronto-London (1952)
- [6] Agliardi R, Cicognani M: Operators of p-evolution with nonregular coefficients in the time variable. J. Diff. Equa. 202, 143-157 (2004)
- [7] Agliardi R, Zanghirati L: Cauchy problem for nonlinear p-evolution equations. Bull. Sci. Math., 133(4), 406-418 (2009)
- [8] Pausader B: Scattering and the Levandosky-Strauss conjecture for fourth-order nonlinear wave equations. J. Diff. Equa., 241: 237-278 (2007)
- [9] Tataru D, Tucsnak M: On the Cauchy problem for the full von Karman system. NoDEA Nonlinear Differential Equations Appl. 4(3): 325-340 (1997)
- [10] Koch H, Stahel A: Global existence of classical solutions to the dynamics of von Karman equations. Math. Methods Appl. Sci. 16: 581-586 (1993)
- [11] Favini A, Horn M, Lasiecka I, Tataru, D: Global existence, uniqueness and regularity of solutions to a von Karman equation with nonlinear boundary dissipation. Diff. Inte. Equa. 9: 267-294 (1996)

- [12] Babin AV, Vishik MI: Attractors of Evolution Equations, North-Holland, Amsterdam, 1992.
- [13] Ghidaglia JM: A note on the strong convergence towards attractors for damped forced Kdv equations. J. Diff. Eqn., 110: 356-359(1994)
- [14] Hale JK Asymptotic Behavior of Dissipative Systems, in: Math. Surveys Monographs, Vol. 25, American Mathematical Society, Providence, RI, 1987.
- [15] Ma QY, Wang SH, Zhong CK: Necessary and sufficient conditions for the existence of global attractor for semigroups and applications. Indiana Univ. Math. J., 51(6): 1541-1570(2002)
- [16] Temam R: Infinite-Dimensional Dynamical Systems in Mechanics and physics. Applied Mathematical Science, Vol. 68, Springer-Verlag, New York, 2nd., 1997
- [17] Carvalho AN, Cholewa JW: Attractors for strongly damped wave equations with critical nonlinearities. Pac. J. Math., 207: 287-310(2002)
- [18] Massatt P: Longtime behaviour of strongly damped nonlinear wave equations. J. Diff. Eqn., 48: 334-349 (1983)
- [19] Fan X, Zhou S: The Inflated Attractors of Non-autonomous Strongly Damped Wave Equations. Acta Math. Appl. Sinica, English Series. 20(4): 547-556(2004)
- [20] Belleri V, Pata, V: Attractors for semilinear strongly damped wave equation on R^3 . Discrete Contin. Dynam. Systems, 7: 719-735(2001)
- [21] Zhang YH, Zhong CK: Existence of global attractors for a nonlinear wave equation. Appl. Math. Lett, 18:77-84(2005)
- [22] Carvalho AN, Cholewa JW: Attractors for strongly damped wave equations with critical nonlinearities. Pacific J. Math., 207: 287-310(2002)