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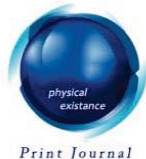
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1. Agashe, N. S and Chafle, M. R: A semi symmetric non metric connection on a Riemannian Manifold. Indian. J. Pure appl. Math. 399-409, 1992.

Lorentzian Para Sasakian Manifolds Admitting Special Semi Symmetric Recurrent Metric Connection

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Abstract - Several author as Agashe and Chafle [1], Sengupta, De. U.C, Binh [4] and many other introduced semi symmetric non metric connection in different way. In this paper we have studied LP sasakian manifold with special semi-symmetric recurrent metric connection [2] and discuss it existence in LP sasakian manifold. In section 3 we establish the relation between the Riemannian connection and special semi-symmetric recurrent metric connection on LP sasakian manifold [4]. The section 4 deals with ξ -conformally flat and ϕ concircularly flat of n dimensional LP sasakian manifold and we proved that ξ -conformally flatness with special semi-symmetric recurrent metric connection and Riemannian manifold coincide.

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1. INTRODUCTION

An n-dimensional differentiable manifold M^n is a lorentzian para-sasakian manifold if it admits tensor field of type (1,1), a contravariant vector field ξ , a covariant vector field η and a lorentzian metric g satisfying;

$$\phi^2 X = X + \eta(X)\xi \quad (1.1)$$

$$\eta(\xi) = -1 \quad (1.2)$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y) \quad (1.3)$$

$$g(X, \xi) = \eta(X) \quad (1.4)$$

$$(D_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi \quad (1.5)$$

$$D_X \xi = \phi X \quad (1.6)$$

For arbitrary vector field $and Y$, where D denotes the operator of covariant differentiation with respect to lorentzian metric g [5].

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In a LP sasakian manifold with structure (ϕ, ξ, η, g) the following relation hold.

$$(a) \quad \phi(\xi) = 0 \quad (b) \quad \eta(\phi X) = 0 \quad \text{rank} \phi = n - 1 \quad (1.7)$$

$$\text{Let us put } F(X, Y) = g(\phi X, Y) \quad (1.8)$$

$$\text{The tensor field } F \text{ is symmetric (0,2) tensor field ie } F(X, Y) = F(Y, X) \quad (1.9)$$

$$\text{And} \quad (D_X \eta)(Y) = F(X, Y) = g(\phi X, Y) \quad (1.10)$$

Also in a LP sasakian manifold the following relation holds;

$$g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y) \quad (1.11)$$

$$\text{And} \quad S(Y, \xi) = (n - 1)\eta(X) \quad (1.12)$$

For any vector field X, Y, Z where $R(X, Y)Z$ is the Riemannian curvarture tensor and S is the Ricci tensor.

Let (M^n, g) be an LP sasakian manifold with Levi-Civita connection D . we define a linear connection $\bar{D}$ on M^n by

$$\bar{D}_X Y = D_X Y - \eta(X)Y \quad (1.13)$$

Where η is 1-form associated with vector field ξ on M^n , given by

$$g(X, \xi) = \eta(X) \quad (1.14)$$

Using (1.13) the torsoin tensor $\bar{T}$ of M^n with respect to connection $\bar{D}$ is given by

$$\bar{T}(X, Y) = \bar{D}_X Y - \bar{D}_Y X - [X, Y] = \eta(Y)X - \eta(X)Y \quad (1.15)$$

A linear connection satisfying (1.15) is called semi symmetric connection. Further from (1.13), we have

$$(\bar{D}_X g)(Y, Z) = 2\eta(X)g(Y, Z) \quad (1.16)$$

A linear connection satisfying (1.16) is called semi symmetric recurrent metric connection. The word special is used to distinguish it from other connection.

II. EXISTENCE OF SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION

Let $\bar{D}$ be a linear connection in M^n , given by

$$\bar{D}_X Y = D_X Y + H(X, Y) \quad (2.1)$$

Where H is a tensor of type (1,2).

Now, we determine the tensor field H such that $\bar{D}$ satifies (1.15) and (1.16).

From (2.1), we have

$$\bar{T}(X, Y) = H(X, Y) - H(Y, X) \quad (2.2)$$

$$\text{Let } G(X, Y, Z) = (\bar{D}_X g)(Y, Z) \quad \text{then} \quad (2.3)$$

$$g(H(X, Y), Z) + g(H(X, Z), Y) - G(X, Y, Z) \quad (2.4)$$

From (2.1), (2.2) and (2.4), we have

$$\begin{aligned} & g(\bar{T}(X, Y), Z) + g(\bar{T}(Z, X), Y) + g(\bar{T}(Z, Y), X) \\ &= g(H(X, Y), Z) - g(H(Y, X), Z) + g(H(Z, X), Y) - g(H(X, Z), Y) + \\ & \quad g(H(Z, Y), X) - g(H(Y, Z), X) \\ &= 2g(H(X, Y), Z) + 2\eta(X)g(Y, Z) - 2\eta(Z)g(X, Y) + 2\eta(Y)g(X, Z) \end{aligned}$$

Or

$$H(X, Y) = \frac{1}{2} \{ \bar{T}(X, Y) + \bar{T}'(X, Y) + \bar{T}'(Y, X) \} - \eta(X)Y - \eta(Y)X + g(X, Y)\xi \quad (2.5)$$

$$\text{Where} \quad g(\bar{T}'(X, Y), Z) = g(\bar{T}(Z, X), Y) \quad (2.6)$$

Using (1.5), (2.6) we get

$$\bar{T}'(X, Y) = \eta(X)Y - g(X, Y)\xi \quad (2.7)$$

Then in view of (1.15), (2.5) and (2.7), we get

$$H(X, Y) = -\eta(X)Y$$

This implies

$$\bar{D}_X Y = D_X Y - \eta(X)Y$$

Conversely, a connection $\bar{D}$ given by (1.13), satisfies (1.15) and (1.16) show that $\bar{D}$ is a special semi symmetric recurrent metric connection.

So we state the following theorem.

Theorem 2.1: let (M^n, g) be an LP sasakian manifold with lorentzian para contact metric structure $(\emptyset, \xi, \eta, g)$ admits a special semi-symmetric connection which is given by

$$\bar{D}_X Y = D_X Y - \eta(X)Y$$

III. CURVATURE TENSOR OF M^n WITH RESPECT TO SPECIAL SEMI -SYMMETRIC RECURRENT METRIC METRIC CONNECTION $\bar{D}$

The Curvature tensor of M^n with respect to special semi - symmetric recurrent metric metric connection $\bar{D}$ is given by

$$\bar{R}(X, Y, Z) = \bar{D}_X \bar{D}_Y Z - \bar{D}_Y \bar{D}_X Z - \bar{D}_{[X, Y]} Z$$

Using (1.13) and (1.10) in above we have

$$\bar{R}(X, Y, Z) = R(X, Y, Z) \quad (3.1)$$

Hence we conclude.

Proposition 3.1: The Curvature tensor of M^n with respect to special semi-symmetric recurrent metric metric connection $\bar{D}$ coincide with the curvature tensor of connection D of Riemannian manifold.

Taking the inner product of (3.1) with W, we have

$$\bar{R}(X, Y, Z, W) = R(X, Y, Z, W) \quad (3.2)$$

Where

$$\bar{R}(X, Y, Z, W) = g(\bar{R}(X, Y, Z),$$

From (3.2), we have

$$\bar{R}(X, Y, Z, W) = -R(Y, X, Z, W) \quad (3.3)$$

$$\bar{R}(X, Y, Z, W) = -R(X, Y, W, Z) \quad (3.4)$$

Combining above two relation, we have

$$\bar{R}(X, Y, Z, W) = R(Y, X, W, Z) \quad (3.5)$$

We also have,

$$\bar{R}(X, Y, Z) + \bar{R}(Y, Z, X) + \bar{R}(Z, X, Y) = 0 \quad (3.6)$$

This is the Bianchi first identity for $\bar{D}$.

Hence we conclude that the curvature tensor of M^n with respect to special semi-symmetric recurrent metric metric connection $\bar{D}$ satisfies the first Bianchi identity.

Contracting (3.2) over X and W, we obtain

$$\bar{S}(Y, Z) = S(Y, YZ) \quad (3.7)$$

Where $\bar{S}$ and S denote the Ricci tensor of the connection $\bar{D}$ and D respectively.

From (3.7) we obtain a relation between the scalar curvature of M^n with respect to the Riemannian connection and special semi-symmetric recurrent metric connection which is given by

$$\bar{r} = r \quad (3.8)$$

So we have following

Proposition 3.2: For n dimensional LP sasakian manifold with special semi symmetric recurrent metric connection $\bar{D}$

(1) The curvature tensor $\bar{R}$ is given by (3.1)

(2) Ricci tensor $\bar{S}$ is given by (3.7)

(3) $\bar{r} = r$

IV. CONCIRCULAR CURVATURE TENSOR OF LP SASAKIAN MANIFOLD WITH RESPECT TO SPECIAL SEMI-SYMMETRIC RECURRENT METRIC CONNECTION

Analogous to the definition of concircular curvature tensor in a Riemannian manifold we define concircular curvature tensor with respect to the special semi symmetric recurrent metric connection $\bar{D}$ as

$$\bar{C}(X, Y, Z) = \bar{R}(X, Y, Z) - \frac{r}{n(n-1)}\{g(Y, Z)X - g(X, Z)Y\} \quad (4.1)$$

Using (3.1) and (3.7) in (4.1), we have

$$\bar{C}(X, Y, Z) = C(X, Y, Z) \quad (4.2)$$

So we have

Proposition 4.1: $\bar{C}(X, Y, Z) = C(X, Y, Z)$ that is manifold coincide with Riemannian Manifold.

The notion of an ξ -conformally flat contact manifold was given by Zhen, Cabrezizo and Fernander [3]. In an analogous we define an ξ -conformally flat n - dimensional LP sasakian manifold.

Definition 5.2: An n -dimensional LP sasakian manifold is called ξ -conformally flat if the condition $\bar{C}(X, Y)\xi = 0$ holds on M^n .

From (4.2) it is clear that $\bar{C}(X, Y)\xi = C(X, Y)\xi$

So we have the following theorem.

Theorem 4.1: In an n -dimensional LP Sasakian manifold, an ξ -conformally flatness with respect to special semi-symmetric recurrent metric connection and Riemannian connection coincide.

Definition 4.3: an n dimensional LP Sasakian manifold satisfying the condition

$$\phi^2 \bar{C}(\phi X, \phi Y)\phi Z = 0 \quad (4.3)$$

Is called ϕ - concircularly flat.

Let us suppose that M^n be n dimensional ϕ - concircularly flat LP sasakian manifold with respect to special semi-symmetric recurrent metric connection. It can easily be seen that $\phi^2 \bar{C}(\phi X, \phi Y)\phi Z = 0$ if and only if

$$g(\bar{C}(\phi X, \phi Y)\phi Z, \phi W) = 0 \quad (4.4)$$

For all $X, Y, Z, W \in T(M)$.

Using (4.1), ϕ - concircularly flat means

$$g(\bar{R}(\phi X, \phi Y)\phi Z, \phi W) = \frac{r}{n(n-1)}\{g(\phi Y, \phi Z)g(\phi X, \phi W) - g(\phi X, \phi Z)g(\phi Y, \phi W)\} \quad (4.5)$$

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3. Zhen, G, Cabrezizo J. L, Fernander, L.M and Fernander: on ξ conformally flat contact metric manifold. Indian J. Pure. appl. math 28, 725-734, 1997.

Let $\{e_1, e_2, \dots, \xi\}$ be a local orthogonal basis of the vector in M^n using the fact that $\{\phi e_1, \phi e_2, \dots, \xi\}$ is also a local orthogonal basis, putting $X = W = e_i$ in (4.5) and summing with respect to i , we have

$$S(\phi Y, \phi Z) = \frac{r}{n(n-1)} \{g(\phi Y, \phi Z)\} \quad (4.6)$$

Putting $Y = \phi Y, Z = \phi Z$ in (4.6) and using the fact S is symmetric, we have

$$g(\bar{R}(\phi X, \phi Y)\phi Z, \phi W) = 0$$

Hence we have

Theorem 4.2: an n -dimensional LP sasakian manifold is ϕ – concircularly flat with respect to special semi-symmetric recurrent metric connection and manifold coincide with Riemannian Manifold.

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