



Unsteady Couette Flow with Transpiration in a Rotating System

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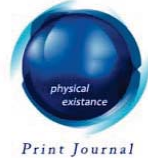
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Unsteady Couette Flow with Transpiration in a Rotating System

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Abstract - The unsteady Couette flow with transpiration of a viscous fluid in a rotating system has been considered. An exact solution of the governing equations has been obtained by using Laplace Transform Technique. Solutions for velocity distributions and the shear stresses have been obtained for small time $\tau = 0.05$ as well as large time $\tau = 10.0$. It is found that for small times the primary velocity profile increases with decrease in K^2 with constant Re while the secondary velocity profile decreases with decrease in K^2 . It is also found that for large times, the primary flow increases with increase in K^2 , the secondary velocity behaves in an oscillatory manner near the moving plate and increases near the stationary plate. There exists a back flow in the region $0.0 \ll \varphi \ll 1.0$. The shear stress due to primary flow decreases with increase in K^2 . On the other hand, it increases due to secondary flow with increase in rotation parameter with constant Re for small times. It is also observed that the shear stress for large time with constant Re shows layers of separation in both primary and secondary flow due to high rotation.

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NOMENCLATURE

Re	Reynolds number
τ	Time
Ω	Uniform angular velocity
d	Distance between the two parallel plates
U	Uniform velocity
u	Velocity component along x – direction
v	Velocity component along y – direction
K^2	Rotation parameter which is the reciprocal of Ekman number
u_1	Velocity distribution due to primary flow
v_1	Velocity distribution due to secondary flow
$q(\varphi, \tau)$	Velocity distribution
τ_x	Non – dimensional shear stress due to primary flow for small time τ
τ_y	Non – dimensional shear stress due to secondary flow for small time τ
τ_{x_0}	Non – dimensional shear stress due to primary flow for large time τ
τ_{y_0}	Non – dimensional shear stress due to secondary flow for large time τ

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$\nu = \frac{\mu}{\rho}$	Kinematic coefficient of viscosity
w_0	Transpiration parameter
φ	Non-dimensional variable

I. INTRODUCTION

The study of the flow of a viscous incompressible fluid in rotating frame of reference has drawn considerable interest in recent years due to its wide applications in designing thermo siphon tubes, in cooling turbine blades, etc. Also Hydrodynamic coquette flow in a rotating system is a challenging approach to atmospheric Science that exerts its influence of rotation to help in understanding the behavior of Oceanic circulation and the formation of galaxies in taking into account the flow of electron is continuously liberated from the sun what is called Solar Wind. Several investigations have been carried out on various types of flow in a rotating frame of reference. Couette flow in a rotating systems leads to a start up process implying thereby a viscous layer boundary is suddenly set into motion and the rate of rotation becomes important in the application of geophysics and fluid engineering as already stated in such a way that the problem is to be analysed by the effect of rotation for small as well as large time τ . However, Vidyanidhi and Nigam (1967) expedite the situation of a secondary flow in a rotating channel by taking into account of the flow of a viscous fluid between two parallel plates in a rotating system with uniform angular velocity about an axis perpendicular to their planes under the influence of a constant pressure gradient. Barik, et al (2012) made an attempt to study the effects of heat and mass transfer on the flow over stretching sheet in the presence of a heat source. Seth et al (2011) studied MHD couette flow of class II in a rotating system. Jana and Datta (1977) have studied the steady Couette flow and heat transfer in a rotating system. Mazumder (1991) investigated an exact solution of an oscillatory Couette flow in a rotating system. Khaled (2012) also studied the transient magnetohydrodynamic in mixed double convection along a vertical plate embedded in a non – Darcy porous medium. Al – Odat (2012) studied transient non – Darcy mixed convection during a vertical surface in porous medium with such suction or injection. Seth et al. (1982) have studied unsteady Couette Flow in a rotating system. Our present problem is to study the unsteady Couette flow with transpiration confined between two plates, rotating with the same angular velocity Ω about an axis perpendicular to their planes and the flow is induced due to the motion of the upper plate. When the upper plate is not moving with uniform velocity, we have the problem considered by Berker (1979). It is important to note that when the upper plate is moving there is no reason to expect only the axially symmetric solution. In this context, it may be noted that Berker [4] established the existence of non – axisymmetric solution for the flow of an incompressible viscous fluid between two disk rotating about a common axis with d same angular velocity. In relating to the fact that Rajagopal [17] has studied, the now of viscoelastic fluid between two rotating disks is a decisive importance to an asymmetric solution of this problem. Singh, Gorla and Rajhans [20] investigated a periodic solution of oscillatory couette flow through a porous medium in a rotating system. Parter and Rajagopal [16] investigated the problem of flow of an incompressible viscous fluid between two parallel disks about a common axis with different angular speeds. They rigorously proved that the problem admits of solution that lacks axial symmetry. With regard to the existence theorem of Parter and Rajagopal it is pointed out that an extensive numerical computation were carried out by Lai et al. [13] with their study of non – axisymmetric

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4. Berker R. (1979), Arch. Mech. Stosow. 31, 265

flow between two parallel rotating disks. It becomes a significant approach to a viscous flow above a single rotating disk which gives a similar result as studied by Lai et al. [14]. Erdogan [6] has considered the unsteady flow induced by the rotation about non – coincident axes while both of the disks are initially rotating with the same angular velocity about a common axis. An extension of this problem of the unsteady flow produced by a sudden coincidence of the axes while two disks are initially rotating with the same angular velocity about non – coincident axes was studied by Ersoy [7]. Greenspan [9] devoted to the theory of unsteady hydrodynamic Couette flow in a rotating system.

The aim and objective of this our present work deals with the time development of a start up Couette flow with transpiration in a rotating frame of reference for small time τ and large time τ where the frictional layer of the upper plate is suddenly set into motion with uniform velocity U with the transparency of a rigid body rotation. A remarkable feature of this problem is to discuss the physical insight into the flow pattern for small time τ by applying Laplace transform technique. The velocity distributions and the shear stresses due to the primary and secondary flows are obtained for small as well as large time τ .

II. FORMULATION OF THE PROBLEM

Consider the unsteady flow of a viscous incompressible fluid confined between parallel plates at a distance d apart, rotating with uniform angular velocity Ω about an axis normal to the plate with transpiration w_0 . We choose the coordinate system in such a way that the x – axis is along the plate (stationary plate) and z – axis is perpendicular to it and y – axis normal to the xy – plane. The flow is induced by motion of the upper plate while the lower plate is kept fixed. The upper upper plate moves with uniform velocity U ($t > 0$) in the x – direction. Since the plates are infinite along x and y – directions, all physical quantities will be the functions of z and t only.

Denoting velocity u and v along x and y – directions, respectively, the Navier Stoke equations in a rotating frame of reference are

$$\frac{\partial u}{\partial t} - w_0 \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} + 2\Omega v \quad (2.1)$$

$$\frac{\partial v}{\partial t} - w_0 \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - 2\Omega u \quad (2.2)$$

Where $\nu = \frac{\mu}{\rho}$ is the kinematic coefficient of viscosity and w_0 is the transpiration parameter.

The initial and boundary conditions are

$$u = v = 0 \text{ for } t \leq 0, 0 \leq z \leq d \quad (2.3)$$

$$u = v = 0 \text{ at } z = 0, t > 0$$

$$u = U, v = 0 \text{ at } z = d, t > 0 \quad (2.4)$$

Let us define the dimensionless quantities as

$$u_1 = \frac{u}{U}, v_1 = \frac{v}{U}, \varphi = \frac{z}{d}, \tau = \frac{vt}{d^2}, w_0 = \frac{Rev}{d} \quad (2.5)$$

Therefore, as a consequence of equation (2.5), equations (2.1) and (2.2) become

$$\frac{\partial u_1}{\partial \tau} - w_0 \frac{\partial u_1}{\partial \varphi} = \nu \frac{\partial^2 u_1}{\partial \varphi^2} + 2K^2 v_1 \quad (2.6)$$

$$\frac{\partial v_1}{\partial \tau} - w_0 \frac{\partial v_1}{\partial \varphi} = \nu \frac{\partial^2 v_1}{\partial \varphi^2} - 2K^2 u_1 \quad (2.7)$$

Where $K^2 = \frac{\Omega d^2}{\nu}$ is the rotating parameter which is the reciprocal of Eckman number.

Combining equations (2.6) and (2.7), we get

$$\frac{\partial q}{\partial \tau} - Re \frac{\partial q}{\partial \varphi} = \nu \frac{\partial^2 q}{\partial \varphi^2} - 2iK^2 q \quad (2.8)$$

$$q = u_1 + iv_1, i = \sqrt{-1} \quad (2.9)$$

The initial and boundary conditions (2.3) and (2.4) become

$$q = 0 \text{ for } \tau \leq 0, 0 \leq \varphi \leq 1 \quad (2.10)$$

$$q = 0 \text{ at } \varphi = 0, \tau > 0$$

$$q = 1 \text{ at } \varphi = 1, \tau > 0 \quad (2.11)$$

III. THE LAPLACE TRANSFORM TECHNIQUE SOLUTION

In this section, the Laplace transforms technique which leads to the solution of equation (2.8) together with the initial and boundary condition (2.10) and (2.11) is described.

Substituting,

$$q(\varphi, \tau) = F(\varphi, \tau)e^{-2iK^2\tau} \quad (3.1)$$

Equation (2.8) becomes

$$\frac{\partial F}{\partial \tau} - Re \frac{\partial F}{\partial \varphi} = \frac{\partial^2 F}{\partial \varphi^2} \quad (3.2)$$

The initial and boundary conditions (2.10) and (2.11) become

$$F(0, \tau) = 0, F(0, \tau) = 0 \text{ and } F(1, \tau) = e^{2iK^2\tau} \quad (3.3)$$

Using the Laplace transform technique, equation (3.2) becomes

$$s\bar{F} = \frac{\partial^2 \bar{F}}{\partial \varphi^2} + Re \frac{\partial \bar{F}}{\partial \varphi} \quad (3.4)$$

Where

$$\bar{F} = \int_0^\infty F(\varphi, \tau) d\tau \quad (3.5)$$

Finally, the boundary conditions (3.3) take the form

$$\bar{F}(0) = 0 \text{ and } \bar{F}(1) = \frac{1}{s-2iK^2} \quad (3.6)$$

The solution of equation (3.4) together with conditions (3.6) is

$$\bar{F}(\varphi, s) = \frac{\sinh\left(\frac{\sqrt{Re^2 + 4s - Re}}{2}\right)\varphi}{s - 2iK^2 \sinh\left(\frac{\sqrt{Re^2 + 4s - Re}}{2}\right)} \quad (3.7)$$

Equation (3.7) is the solution of the problem. Since the flow is unsteady, we shall discuss the following cases:

Case 1: Solutions for small time τ . In this case the method used by Ersoy [7] is used because it converges rapidly for small time ($\tau \ll 1$) corresponding to large $s (\gg 1)$ and thus equation (3.7) can be written using Lorent's expansion as

$$\bar{F}(\varphi, s) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(2iK^2)^n}{s^{n+1}} \left[e^{-(2m+1-\varphi)\left(\frac{\sqrt{Re^2 + 4s - Re}}{2}\right)} - e^{-(2m+1+\varphi)\left(\frac{\sqrt{Re^2 + 4s - Re}}{2}\right)} \right] \quad (3.8)$$

$$F(\varphi, \tau) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (2iK^2)^n (4\tau)^n e^{-\frac{Re}{2}} \left\{ i^{2n} \frac{1}{2} \left[\exp\left(Re \left(\frac{2m+1-\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1-\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau}\right) + \exp\left(-Re \left(\frac{2m+1-\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1-\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau}\right) \right] - i^{2n} \frac{1}{2} \left[\exp\left(Re \left(\frac{2m+1+\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1+\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau}\right) + \exp\left(-Re \left(\frac{2m+1+\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1+\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau}\right) \right] \right\} \quad (3.9)$$

Where

$$\begin{aligned} i^n \operatorname{erfc}(x) &= \int_x^{\infty} i^{n-1} \operatorname{erfc}(\zeta) d\zeta \\ i \operatorname{erfc}(x) &= \int_x^{\infty} \operatorname{erfc}(\zeta) d\zeta \\ i^0 \operatorname{erfc}(x) &= \operatorname{erfc}(x) \end{aligned} \quad (3.10)$$

The solution (3.9) can be written as

$$F(\varphi, \tau) = \sum_{n=0}^{\infty} (2K^2)^n i^n (4\tau)^n T_r \quad r = 0, 2, 4, 6 \dots \quad (3.11)$$

Where

$$\begin{aligned} T_r &= \sum_{m=0}^{\infty} e^{-\frac{Re}{2}} \left\{ i^{2n} \frac{1}{2} \left[\exp\left(Re \left(\frac{2m+1-\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1-\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau}\right) + \exp\left(-Re \left(\frac{2m+1-\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1-\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau}\right) \right] - i^{2n} \frac{1}{2} \left[\exp\left(Re \left(\frac{2m+1+\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1+\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau}\right) + \exp\left(-Re \left(\frac{2m+1+\varphi}{2}\right)\right) \operatorname{erfc}\left(\frac{2m+1+\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau}\right) \right] \right\} \quad r = 0, 2, 4, 6, 8, \dots \quad (3.12) \end{aligned}$$

Ref

5. Dash S, Maji S, L, Guria M, Jana R. N.(2009), Unsteady Couette Flow in a Rotating System, Mathematical and Comp. Modeling, 50, 1211 – 1217.

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On separating real and imaginary parts, we get the velocity distributions for primary and secondary flow respectively as follows

$$u_1 = e^{-\frac{Re}{2}} \{ [T_0 - (2K^2)^2(4\tau)^2T_4 + (2K^2)^4(4\tau)^4T_8 + (2K^2)^6(4\tau)^6T_{12} \dots] \cos 2K^2\tau + [(2K^2)(4\tau)T_2 - (2K^2)^3(4\tau)^3T_6 + (2K^2)^5(4\tau)^5T_{10} + (2K^2)^7(4\tau)^7T_{14} \dots] \sin 2K^2\tau \} \quad (3.13)$$

$$v_1 = e^{-\frac{Re}{2}} \{ [(2K^2)(4\tau)T_2 - (2K^2)^3(4\tau)^3T_6 + (2K^2)^5(4\tau)^5T_{10} + (2K^2)^7(4\tau)^7T_{14} \dots] \cos 2K^2\tau - [T_0 - (2K^2)^2(4\tau)^2T_4 + (2K^2)^4(4\tau)^4T_8 + (2K^2)^6(4\tau)^6T_{12} \dots] \sin 2K^2\tau \} \quad (3.14)$$

Differentiating equation (3.12) with respect to φ , we get

$$\begin{aligned} \frac{dT_r}{d\varphi} = & e^{-\frac{Re}{2}} \left\{ i^{2n} \frac{1}{2} \left[\exp \left(Re \left(\frac{2m+1-\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1-\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) + \right. \right. \\ & \left. \exp \left(-Re \left(\frac{2m+1-\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1-\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) \right] - \\ & i^{2n} \frac{1}{2} \left[\exp \left(Re \left(\frac{2m+1+\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1+\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) + \right. \\ & \left. \left. \exp \left(-Re \left(\frac{2m+1-\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1-\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) \right] \right\} \quad (3.15) \end{aligned}$$

This gives

$$\frac{dT_r}{d\varphi} = Y_{r-1} \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) e^{-\frac{Re}{2}}, r = 0, 1, 2, 3, \dots \quad (3.16)$$

Where

$$\begin{aligned} Y_{r-1} = & \left\{ i^{2n} \frac{1}{2} \left[\exp \left(Re \left(\frac{2m+1-\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1-\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) + \right. \right. \\ & \left. \exp \left(-Re \left(\frac{2m+1-\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1-\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) \right] - \\ & i^{2n} \frac{1}{2} \left[\exp \left(Re \left(\frac{2m+1+\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1+\varphi}{8\sqrt{\tau}} + 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) + \right. \\ & \left. \left. \exp \left(-Re \left(\frac{2m+1-\varphi}{2} \right) \right) \operatorname{erfc} \left(\frac{2m+1-\varphi}{8\sqrt{\tau}} - 2Re\sqrt{\tau} \right) \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) \right] \right\}, r = 0, 1, 2, 3, \dots \quad (3.17) \end{aligned}$$

The non-dimensional shear stress can be obtained as

$$\frac{dq}{d\varphi} = \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) \sum_{n=0}^{\infty} (2K^2)^n i^n (4\tau)^n Y_{n-1} e^{-2iK^2 - \frac{Re}{2}} \quad (3.18)$$

The non-dimensional shear stresses due to primary flows and secondary flows at the plate $\varphi = 0$ can be respectively written as

$$\begin{aligned} \tau_x = & \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) e^{-\frac{Re}{2}} \{ [Y_{-1} - (2K^2)^2(4\tau)^2Y_3 + (2K^2)^4(4\tau)^4Y_7 + (2K^2)^6(4\tau)^6Y_{11} \dots] \cos 2K^2\tau + \\ & [(2K^2)(4\tau)Y_1 - (2K^2)^3(4\tau)^3Y_5 + (2K^2)^5(4\tau)^5Y_9 + (2K^2)^7(4\tau)^7Y_{13} \dots] \sin 2K^2\tau \}_{\varphi=0} \quad (3.19) \end{aligned}$$

$$\tau_y = \left(\frac{1}{8\sqrt{\tau}} - \frac{Re}{2} \right) e^{-\frac{Re}{2}} \{ [(2K^2)(4\tau)Y_1 - (2K^2)^3(4\tau)^3Y_5 + (2K^2)^5(4\tau)^5Y_9 + (2K^2)^7(4\tau)^7Y_{13} \dots] \cos 2K^2\tau - [Y_{-1} - (2K^2)^2(4\tau)^2Y_3 + (2K^2)^4(4\tau)^4Y_7 + (2K^2)^6(4\tau)^6Y_{11} \dots] \sin 2K^2\tau \}_{\varphi=0} \quad (3.20)$$

Case 2: Solution for large time τ . For large time the method given in Batchelor[3] can be used. The solution of equation (2.8) subject to the conditions (2.10) and (2.11), can be written in the form

$$q(\varphi, s) = \frac{\sinh\left(\frac{\sqrt{Re^2+4s-Re}}{2}\right)\varphi}{s-2iK^2\sinh\left(\frac{\sqrt{Re^2+4s-Re}}{2}\right)} + F_1(\varphi, \tau) \quad (3.21)$$

Where the first term on the right hand side is the steady solution and $F_1(\varphi, \tau)$ shows the departure from steady state. $F_1(\varphi, \tau)$ Satisfies the differential equation

$$\frac{\partial F}{\partial \tau} + 2iK^2F_1 = \frac{\partial^2 F}{\partial \varphi^2} + Re \frac{\partial F}{\partial \varphi} \quad (3.22)$$

With

$$F_1(0, \tau) = 0, F_1(1, \tau) = 0, F_1(0, \tau) = -\frac{\sinh\left(\frac{\sqrt{Re^2+4s-Re}}{2}\right)\varphi}{\sinh\left(\frac{\sqrt{Re^2+4s-Re}}{2}\right)} \quad (3.23)$$

The solution of equation (3.22) may be written in the form

$$F_1(\varphi, \tau) = A_n \sin n\pi\varphi e^{-\lambda_n^2\tau - \frac{Re}{2}} \quad (3.24)$$

Where

$$\lambda_n^2 = \frac{\pi^2 n^2 + 8iK^2}{4} \quad (3.25)$$

The coefficient A_n can be determined from the initial conditions which is

$$\sum_{n=0}^{\infty} A_n \sin n\pi\varphi = -\frac{\sinh\left(\frac{\sqrt{Re^2+4s-Re}}{2}\right)\varphi}{\sinh\left(\frac{\sqrt{Re^2+4s-Re}}{2}\right)} \quad (3.26)$$

The velocity distribution $q(\varphi, s)$ is given by

$$q(\varphi, s) = \frac{\sinh(\sqrt{Re^2+8iK^2-Re})\varphi}{\sinh(\sqrt{Re^2+8iK^2-Re})} + 4 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2\tau - \frac{Re}{2}}}{n^2\pi^2 + 8iK^2} \sin n\pi\varphi \quad (3.27)$$

On separating the real and imaginary parts, we get

$$u_1 = \frac{S(4KRe\varphi)S(4KRe)+C(4KRe\varphi)C(4KRe)}{S^2(4KRe)+C^2(4KRe)} + 4 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2\tau - \frac{Re}{2}}}{(n^2\pi^2)^2+(8iK^2)^2} \sin n\pi\varphi \times (n^2\pi^2 \cos 2K^2 - 8K^2 \sin 2K^2) \quad (3.28)$$

$$v_1 = \frac{C(4KRe\varphi)C(4KRe)-S(4KRe\varphi)S(4KRe)}{S^2(4KRe)+C^2(4KRe)} + 4 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2\tau - \frac{Re}{2}}}{(n^2\pi^2)^2+(8iK^2)^2} \sin n\pi\varphi \times (8K^2 \cos 2K^2 + n^2\pi^2 \sin 2K^2) \quad (3.29)$$

Where,

$$\begin{aligned}
 S(4KRe\varphi) &= \sinh(4KRe\varphi)\cos(4KRe) \\
 C(4KRe\varphi) &= \cosh(4KRe\varphi)\sin(4KRe) \\
 S(4KRe) &= \sinh(4KRe)\cos(4KRe) \\
 C(4KRe) &= \cosh(4KRe)\sin(4KRe)
 \end{aligned} \tag{3.30}$$

The second term of equations (3.28) and (3.29) represent the initial oscillations of the fluid velocity which decay experimentally with time and the frequency of the oscillation is $2K^2$.

The non – dimensional shear stresses due to primary and secondary flows at the stationary plate $\varphi = 0$ are given by

$$\left(\frac{\partial q}{\partial \varphi}\right)_{\varphi=0} = \frac{\sqrt{Re^2+8iK^2}-Re}{\sinh(\sqrt{Re^2+8iK^2}-Re)} + 2 \sum_{n=1}^{\infty} \frac{n^2\pi^2(-1)^n e^{-\left(\frac{n^2\pi^2+8iK^2}{4}\right)\tau}}{n^2\pi^2+8iK^2} \tag{3.31}$$

On separating the real and imaginary parts, we get the shear stresses components of due to the primary and the secondary flows respectively as

$$\tau_{x_0} = \frac{4KRe[S(4KRe)+C(4KRe)]}{S^2(4KRe)+C^2(4KRe)} + 2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2\tau-\frac{Re}{2}}}{(n^2\pi^2)^2+(8iK^2)^2} \sin n\pi\varphi \times (n^2\pi^2\cos 2K^2 - 8K^2\sin 2K^2) \tag{3.32}$$

$$\tau_{y_0} = \frac{4KRe[S(4KRe)+C(4KRe)]}{S^2(4KRe)+C^2(4KRe)} + 2 \sum_{n=1}^{\infty} \frac{n\pi(-1)^n e^{-\lambda_n^2\tau-\frac{Re}{2}}}{(n^2\pi^2)^2+(8iK^2)^2} \sin n\pi\varphi \times (+8K^2\sin 2K^2 n^2\pi^2\cos 2K^2) \tag{3.33}$$

Where

$S(4KRe)$ and $C(4KRe)$ are defined in (3.30).

IV. DISCUSSION OF RESULTS

To study the effect of rotation with transpiration, the stream wise velocity profiles for primary and the secondary flows are plotted graphically against φ for different values of K^2 and constant Value for Re with small time ($\tau \ll 1$) and large time ($\tau \gg 1$). Taking $\tau = 0.05$ and $\tau = 10.0$

Figures 1 and 2 show the variation of primary and secondary velocities u_1 and v_1 respectively against φ for different values of rotation parameter K^2 with $= 0.05$. Here Re is assumed a value so that the transpiration of the flow in the system is laminar. It is evident from figure 1 that the primary velocity u_1 increases with decrease in K^2 . Figure 2 shows that the secondary velocity v_1 increases with increase in K^2 . It is also observed from figure 2 that the velocity profiles are skewed near the moving plate while the maximum peak occurs near the stationary plate this is due to the transpiration from the system.

Figures 3 and 4 are plotted such that they show the variation of primary and secondary velocities u_1 and v_1 respectively against φ for different values of rotation

parameter K^2 with $\tau = 10.0$. it is observed from figure3 that the primary velocity u_1 increases with increase in K^2 . It is seen from figure 4 that v_1 behaves in an oscillatory manner near the moving plate due to the transpiration while it increases near the stationary plate with increase in K^2 . It is stated that there exists a back flow which lies in the region $0.0 < \varphi < 1.0$. It comes to a conclusion that the start up flow will lead to an occurrence of a back flow for large time when the rate of rotation is high with constant transpiration.

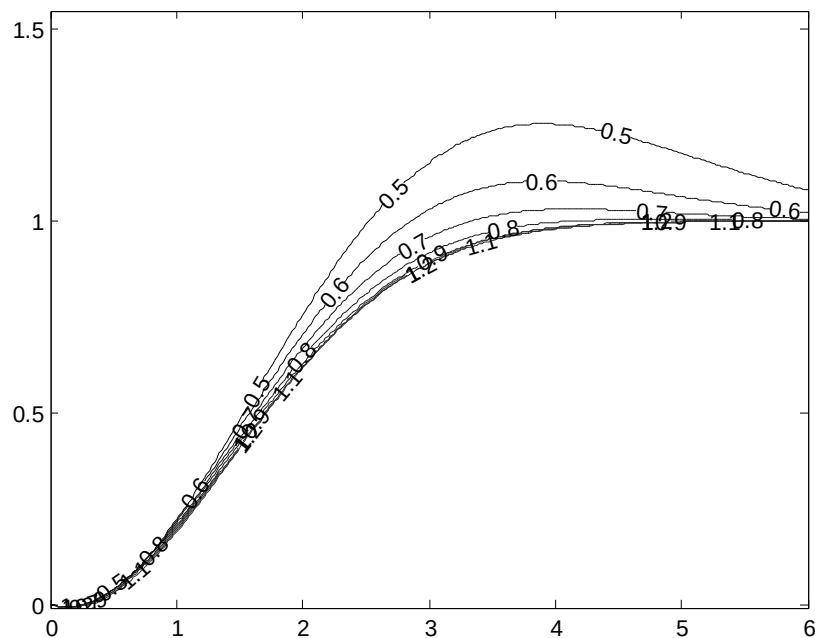


Figure 1 : Primary velocity profile u_1 against φ for different values of K^2 with $\tau = 0.05$

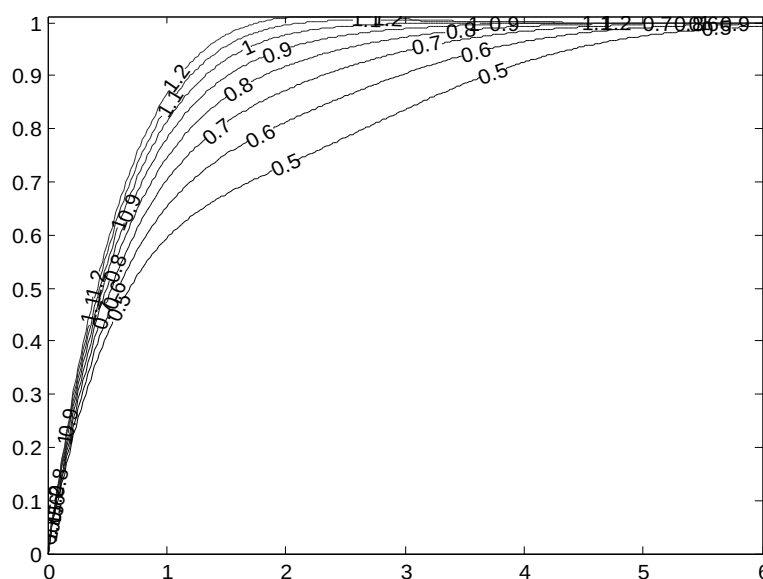


Figure 2 : Secondary velocity profile v_1 against φ for different values of K^2 with $\tau = 0.05$

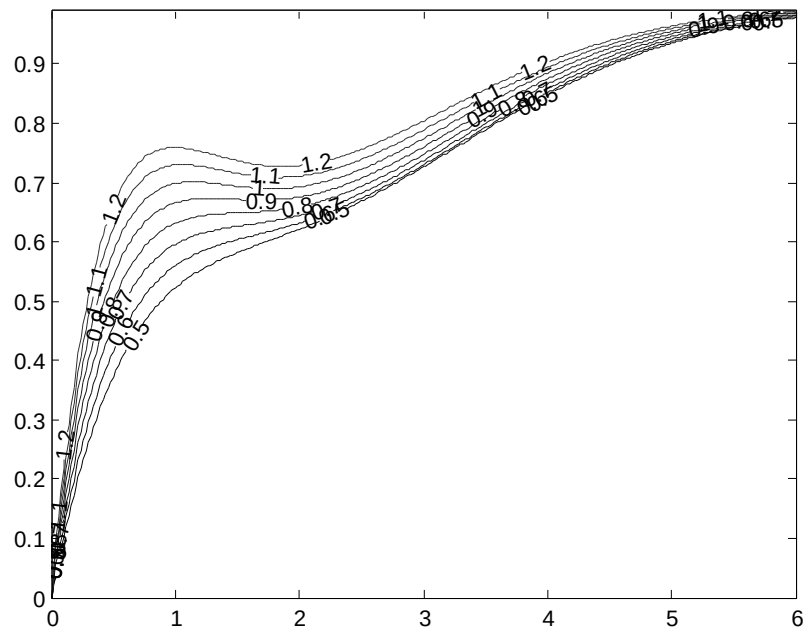


Figure 3 : Primary velocity profile v_1 against φ for different values of K^2 with $\tau = 10.0$

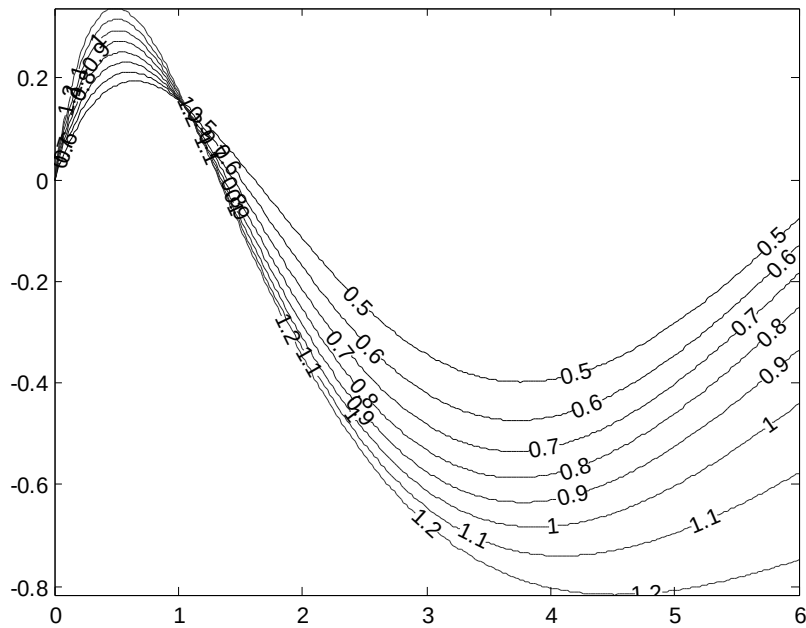


Figure 4 : Secondary velocity profile v_1 against φ for different values of K^2 with $\tau = 10.0$

Table 1 : Shear Stresses at the plate $\varphi = 0$ for small time $\tau = 0.05$ and $Re = 10.00$

K^2	Shear Stress due to Primary Flow (τ_x)	Shear Stress due to Secondary Flow (τ_y)
2.0	0.064991990	0.015195550
4.0	0.057874710	0.027697940
6.0	0.047290120	0.035139770
8.0	0.035079770	0.035767860
10.0	0.023563680	0.028661410
12.0	0.015325670	0.013854810

Table 2 : Shear Stresses at the plate $\varphi = 0$ for Large time $\tau = 10.0$ and $Re = 10.0$

K	Shear Stress due to Primary Flow (τ_{x_0})	Shear Stress due to Secondary Flow (τ_{y_0})
1.0	9.2544900	-3.1758700
1.5	6.7748620	-5.9575870
2.0	2.7346490	-7.0528330
2.5	0.79611640	-5.7605270
3.0	-2.5394700	-3.3887110
3.5	-2.7236030	-1.2372250
4.0	-2.0665060	0.15183220
4.5	-1.1879270	0.76656270
5.0	-0.4549140	0.83720510

Tables 1 and 2 presented the Shear stresses at the stationary plate $\varphi = 0$ due to the primary and secondary flows for small time $\tau = 0.05$ and large time $\tau = 10.0$ and $Re = 10.0$.

Table 1 shows that the shear stress due to primary flow (τ_x) decreases with increase in K^2 for small time τ . On the other hand, it increases due to secondary flow (τ_y) with increase in K^2 . It is noticed from Table 2 that the shear stress due primary flow (τ_{x_0}) decreases from $K = 1.0$ to 2.5 and increases from $K = 3.0$ to 5.0. Similarly the shear stress due to secondary flow (τ_{y_0}) decreases from $K = 1.0$ to 2.0 and increases from $K = 2.5$ to 5.0.

V. CONCLUSION

An analysis and investigation has been made on unsteady Couette flow with transpiration in a rotating system for small time as well as large time τ . Laplace transform technique is applied for the solution of velocity distributions and shear stresses are analyzed for small time as well as large time τ .

Unsteady coquette flow with transpiration in a rotating system leads to a start up flow with occurrence of back flow for large time τ when the rate of rotation is high and the transpiration is constant. The significant of a study of the flow pattern is in such a way that the shear stresses due to primary and secondary flow for large time $\tau = 10.0$ show layer of separation when a back flow occurs.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Al – odat M. Q. (2012), Transient MHD in Mixed Double Convection Along Vertical Surface in Porous Medium with Suction or Injection. J. Applied Math. And Comp. Sci. 156, 679 – 694.
2. Barik R. N, Dash G. C and Rath K. (2012), Effects of Heat and Mass Transfer on the Flow over a Stretching Surface with Heat Source. Journ. of Mathematics and Modeling, 2, 7.
3. Batchelor G. K. (1967), An introduction to fluid Dynamics, Cambridge press, Cambridge.
4. Berker R. (1979), Arch. Mech. Stosow. 31, 265
5. Dash S, Maji S, L, Guria M, Jana R. N.(2009), Unsteady Couette Flow in a Rotating System, Mathematical and Comp. Modeling, 50, 1211 – 1217.
6. Erdogan M. E. (1995), Unsteady Flow Induced by Rotation about Non – Coincident axes, Int. J. Non – Linear Mech. 30, 711.
7. Ersoy H. V. (2003), Unsteady Flow Produced by the Rotation of a Coincidence of Axes, Meccanica 38, 325.
8. Guria M, Jana R.N, Ghosh S.K. (2006), Unsteady Couette Flow in a Rotating System, Int. J. Non – Linear Mech. 41, 838 – 843.
9. Greenspan H. P. (1990), The theory of Rotating Fluids Cambridge University press, Cambridge.
10. Jana R.N, Datta N. (1977), Couette Flow and Heat Transfer in a Rotating System, Acta Mech, 26, 301.
11. Jana R. N, Datta N, Mazumder B.S., 1977), MHD Couette Flow and Heat Transfer in a Rotating Systems, J. Phys. Soc. Japan, 42, 1034 – 1039.
12. Khaled K. Jaber (1984), Transient MHD Mixed Double Diffusive Convection along a Vertical Plate Embedded in a Non - Darcy Porous Medium with Suction or Injection, J. Math. Stat. 8, (1), 15 – 23.
13. Lai C. Y, Rajagopal K. R. (1984), Non – asymmetric Flow between Two Parallel Rotating Disks., J. Fluid Mech. 146, 203.
14. Lai C. Y, Rajagopal K. R. (1985), Viscous Flow above a Single Rotating Disk, J. Fluid Mech. 157, 471.
15. Mazumber B. S. (1991), An Exact Solution of an Oscillatory Couette Flow in a Rotating Systems, Trans. ASME J. Appl. Mech. 58, 1104 – 1107.
16. Parter S. V, Rajagopal K.R. (1984), Flow of an incompressible Viscous Fluid between Two Parallel Disks, Arch. Ration. Mech. Anal. 86, 305
17. Rajagopal K. R. (1992), Viscouselastic Fluid between Two Rotating Disks, Theor. Comput. Fluid Dyn. 3, (4), 185.
18. Seth G. S, Hussain S. M and Singh J. K. (2011), MHD Couette Flow of Class II in a Rotating System, J. Appl. Math. Bioinformatics, 1, (1), 31 – 35.
19. Seth. G. S, Jana R. N, Maiti M. K. (1982), Unsteady Couette Flow between in a Rotating Systems, Int. J. Eng. Sci. 20, 989.
20. Singh K. D, Gork G, Rajhans. (2005), Periodic Of Oscillatory Couette Flow through a Porous Medium in a Rotating Systems, Indian J. Pure Appl. Math. 36, (3), 151 – 159.
21. Vidyanidhi V, Nigan S.D. (1967), Secondary Flow in a Rotating Channel J. Math. Phys. Sci. 1, 85.