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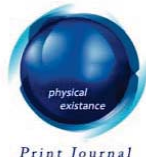
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Some Results on a Lorentzian Para Sasakian Manifolds

Mobin Ahmad^α, Archana Srivastava^σ & Amit Prakash^ρ

Abstract - The object of this paper is to study a type of non-flat differentiable manifold called Generalized Pseudo Symmetric, Generalized Pseudo Ricci symmetric, Generalized Ricci Recurrent, Semi pseudo symmetric and Semi pseudo Ricci symmetric manifold in a Lorentzian para-Sasakian manifold.

Keywords and phrases : *ip-sasakian manifold, generalized pseudo symmetric, generalized pseudo ricci symmetric, generalized ricci recurrent, semi pseudo symmetric.*

I. INTRODUCTION

A non -flat differentiable manifold (M^n, g) ($n > 3$), is called generalized pseudo symmetric $G(PS)_n$, if there exists a vector field P and 1- form A, B, C, D on M Such that

$$(D_X S)(Y, Z) = 2A(X)S(Y, Z) + B(R(X, Y), Z) + C(Y)S(X, Z) + D(Z)S(X, Y) + p(R(X, Z)Y). \quad (1.1)$$

In 1993, Chaki and Koley [6] introduced another type of non-flat differentiable manifold (M^n, g) ($n > 2$), satisfies the condition

$$(D_X S)(Y, Z) = 2A(X)S(Y, Z) + B(Y)S(X, Z) + C(Z)S(X, Y), \quad (1.2)$$

where A, B, C are three non-zero 1-form and D denotes the operator of covariant differentiation with respect to g . Such a manifold were called by them a generalized pseudo Ricci-symmetric manifold and an n - dimensional manifold of this kind were denoted by $G(PRS)_n$.

De, Guha and Kamilya [7] introduced and studied a type of differentiable manifold (M^n, g) ($n > 2$), whose Ricci tensors of type $(0, 2)$ satisfies the condition

$$(D_X S)(Y, Z) = A(X)S(Y, Z) + B(X)g(Y, Z), \quad (1.3)$$

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where A and B are two non-zero 1-forms, P, Q are two vector fields such that

$$g(X, P) = A(X),$$

$$g(X, Q) = B(X),$$

such a manifold were called by them a generalized Ricci-recurrent manifold and an n -dimensional manifold of this kind were denoted by $(GR)_n$.

Tarafdar and Jawarneh [4] introduced a type of non-flat differentiable manifold (M^n, g) ($n > 3$), whose curvature tensor R satisfies the condition

$$(D_X R)(Y, Z)W = 2A(X)R(Y, Z)W + A(Y)R(X, Z)W + A(Z)R(Y, X)W \\ + A(W)R(Y, Z)X + A(W)R(Y, Z)X, \quad (1.4)$$

where A is a non-zero 1-form and $g(X, P) = A(X)$,

Such a manifold were called by them a Semi pseudo symmetric and an n -dimensional manifold of this kind is denoted by $(SPS)_n$.

Tarafdar and Jawarneh [5] introduced another type of non-flat Riemannian manifold (M^n, g) ($n > 3$), Whose Ricci-tensor of type $(0, 2)$ satisfies the condition

$$(D_X S)(Y, Z) = A(Y)S(X, Z) + A(Z)S(X, Y), \quad (1.5)$$

where A is a non zero 1-form, such a manifold were called by them Semi pseudo Ricci-symmetric and an n -dimensional manifold of this kind is denoted by $(SPRS)_n$.

II. PRELIMINARIES

An n -dimensional differentiable manifold M^n is a Lorentzian para-Sasakian (LP -Sasakian) manifold if it admits a $(1, 1)$ -tensor field ϕ , a contravariant vector field ξ , a covariant vector field η and a Lorentzian metric g which satisfy ([2], [3])

$$\phi^2 X = X + \eta(X)\xi, \quad (2.1)$$

$$\eta(\xi) = -1, \quad (2.2)$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad (2.3)$$

$$g(X, \xi) = \eta(X), \quad (2.4)$$

$$(D_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi, \quad (2.5)$$

and

$$D_X \xi = \phi X, \quad (2.6)$$

for any vector fields X and Y , where D denotes covariant differentiation with respect to g ([1], [2], [3]).

In an LP -Sasakian manifold M^n with structure (ϕ, ξ, η, g) , it is easily seen that

$$(a) \phi \xi = 0 \quad (b) \eta(\phi X) = 0 \quad (c) \text{rank}(\phi) = n - 1. \quad (2.7)$$

R_{ref.}

1. Mihai, I and Rosca, R.: On Lorentzian P -Sasakian manifolds, Classical Analysis, World scientific Publ. Singapore (1992), 155-169.

Let us put

$$F(X, Y) = g(\phi X, Y). \quad (2.8)$$

Then the tensor field F is symmetric $(0, 2)$ tensor field i.e.,

$$F(X, Y) = F(Y, X), \quad (2.9)$$

and

$$F(X, Y) = (D_X \eta)(Y). \quad (2.10)$$

Also in an LP-Sasakian manifold, the following relation holds:

$$R(X, Y, Z, \xi) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \quad (2.11)$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \quad (2.12)$$

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \quad (2.13)$$

$$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y), \quad (2.14)$$

and

$$S(X, \xi) = (n-1)\eta(X), \quad (2.15)$$

for any vector fields X, Y, Z , Where $R(X, Y)Z$ is the Riemannian curvature tensor, S is the Ricci tensor.

III. GENERALIZED PSEUDO SYMMETRIC LP- SASAKIAN MANIFOLD

Suppose that M^n is a generalized pseudo symmetric LP-sasakian manifold Putting $Z = \xi$ in (1.1), we get

$$\begin{aligned} (D_X S)(Y, \xi) &= 2A(X)S(Y, \xi) + B(R(X, Y)\xi) + C(Y)S(X, \xi) \\ &\quad + D(\xi)S(X, Y) + p(R(X, \xi)Y). \end{aligned} \quad (3.1)$$

Using (2.2), (2.12) and (2.15) in (3.1), we get

$$\begin{aligned} (D_X S)(Y, \xi) &= 2(n-1)A(X)\eta(Y) + \eta(Y)B(X) - \eta(X)B(Y) + (n-1)C(Y)\eta(X) \\ &\quad + D(\xi)S(X, Y) + \eta(Y)p(X) - g(X, Y)p(\xi). \end{aligned} \quad (3.2)$$

We know that

$$(D_X S)(Y, \xi) = (n-1)g(\phi X, Y) - S(Y, \phi X), \quad (3.3)$$

Therefore, from (3.2) and (3.3), we get

$$2(n-1)A(X)\eta(Y) + \eta(Y)B(X) - \eta(X)B(Y) + (n-1)C(Y)\eta(X) + D(\xi)S(X, Y)$$

$$+\eta(Y)p(X) - g(X,Y)p(\xi)=(n-1)g(\phi X,Y)-S(Y,\Phi X). \quad (3.4)$$

Therefore, putting $X=Y=\xi$ in (3.4) and using (2.1) and (2.15), we get

$$(n-1)[2A(\xi)+C(\xi)+D(\xi)]=0.$$

Since $n \geq 3$, we obtain $2A(\xi)+C(\xi)+D(\xi)=0. \quad (3.5)$

So, the vanishing of the 1-form $2A+C+D$ over the vector field ξ is necessary in order that M be an LP-Sasakian manifold.

Now we will show that $2A+C+D=0$ holds for all vector fields on M .

Putting $Y=\xi$ in (1.1), we get

$$\begin{aligned} (D_X S)(\xi, Z) &= 2(n-1)A(X)\eta(Z)+\eta(Z)B(X)-g(X,Z)B(\xi)+C(\xi)S(X,Z) \\ &+ (n-1)D(Z)\eta(X)+\eta(Z)p(X)-\eta(X)p(Z). \end{aligned} \quad (3.6)$$

Also, We know that

$$(D_X S)(\xi, Z)=(n-1)g(\phi X,Z)-S(\phi X,Z). \quad (3.7)$$

From (3.6) and (3.7), We get

$$\begin{aligned} 2(n-1)A(X)\eta(Z)+\eta(Z)B(X)-g(X,Z)B(\xi)+C(\xi)S(X,Z)+(n-1)D(Z)\eta(X) \\ +\eta(Z)p(X)-\eta(X)p(Z) = (n-1)g(\phi X,Z)-S(\phi X,Z). \end{aligned} \quad (3.8)$$

Putting $Z=\xi$, in (3.8), we get

$$\begin{aligned} -2(n-1)A(X)-B(X)-\eta(X)B(\xi)+(n-1)C(\xi)\eta(X)+(n-1)D(\xi)\eta(X) \\ -p(X)-\eta(X)p(\xi)=0. \end{aligned} \quad (3.9)$$

Putting $X=\xi$ in (3.8), we get

$$2(n-1)A(\xi)\eta(X)+(n-1)C(\xi)\eta(X)-(n-1)D(X)+p(X)+\eta(X)p(\xi)=0. \quad (3.10)$$

Adding (3.9) and (3.10) and using (3.5), we get

$$-2(n-1)A(X)-\eta(X)B(\xi)-B(X)+(n-1)C(\xi)\eta(X)-(n-1)D(X)=0. \quad (3.11)$$

Now, Putting $X=\xi$ in (3.4), and replacing Z with X , we get

$$2(n-1)A(\xi)\eta(X)+B(\xi)\eta(X)+B(X)-(n-1)C(X)+(n-1)D(\xi)\eta(X)=0. \quad (3.12)$$

Adding (3.11) and (3.12) and using (3.5), we get

$$(n-1)[2A(X)+C(X)+D(X)]=0$$

Since $n \geq 3$, Hence we get

$$2A(X)+C(X)+D(X)=0, \text{ for all } X$$

This implies $2A+C+D=0$.

Hence, we can state the following theorem:

Theorem 3.1 .There exists no generalized pseudo symmetric LP - Sasakian manifold (M^n, g) , $n \geq 3$, if $2A+C+D$ is not every where zero.

IV. GENERALIZED PSEUDO RICCI SYMMETRIC LP - SASAKIAN MANIFOLD

Suppose that M^n is a generalized pseudo Ricci symmetric LP - Sasakian manifold then

$$(D_X S)(Y, Z) = 2A(X)S(Y, Z) + B(Y)S(X, Z) + C(Z)S(X, Y). \quad (4.1)$$

Putting $Z = \xi$, in (4.1) and using (2.15), we get

$$(D_X S)(Y, \xi) = 2(n-1)A(X)\eta(Y) + (n-1)B(Y)\eta(X) + C(\xi)S(X, Y). \quad (4.2)$$

Also, we know that

$$(D_X S)(Y, Z) = (n-1)g(\phi X, Y) - S(Y, \phi X). \quad (4.3)$$

From (4.2) and (4.3), we get

$$2(n-1)A(X)\eta(Y) + (n-1)B(Y)\eta(X) + C(\xi)S(X, Y) = (n-1)g(\phi X, Y) - S(Y, \phi X). \quad (4.4)$$

Putting $X = Y = \xi$ in (4.4), we get

$$(n-1)[2A(\xi) + B(\xi) + C(\xi)] = 0.$$

Which gives (since $n \geq 3$),

$$[2A(\xi) + B(\xi) + C(\xi)] = 0. \quad (4.5)$$

Putting $X = \xi$ in (4.4), we have

$$(n-1)\eta(Y)[2A(\xi) + C(\xi)] - (n-1)B(Y) = 0.$$

So, By virtue of (4.5) this yields

$$(n-1)[\eta(Y)B(\xi) + B(Y)] = 0,$$

$$\text{Which gives us (since } n \geq 3 \text{),} \quad B(Y) = -\eta(Y)B(\xi). \quad (4.6)$$

Similarly, Taking $Y = \xi$ in (4.4), we get

$$-2A(X) + \eta(X)[B(\xi) + C(\xi)] = 0.$$

Hence applying (4.5), we get

$$A(X) = -\eta(X)A(\xi). \quad (4.7)$$

Since $(\nabla_{\xi}S)(\xi, X) = 0$, then from (1.2), we obtain

$$(n-1)\eta(X)[2A(\xi) + B(\xi)] - (n-1)C(X) = 0. \quad (4.8)$$

So, by making the use of (4.5), the equation (4.8) reduces to

$$C(X) = -C(\xi)\eta(X) \quad (4.9)$$

Adding equation (4.6), (4.7) and (4.9), we get

$$2A(X) + B(X) + C(X) = -[2A(\xi) + B(\xi) + C(\xi)]\eta(X) \quad (4.10)$$

And then, from (4.5), it follows that

$$2A(X) + B(X) + C(X) = 0, \text{ for all } X,$$

$$\text{Thus } 2A + B + C = 0.$$

Hence, we can state the following theorem:

Theorem 4.1. There exists no generalized Pseudo Ricci-Symmetric LP-Sasakian manifold (M^n, g) , $n \geq 3$ if $2A + B + C$ is not everywhere zero.

V. GENERALIZED RICCI RECURRENT LP- SASAKIAN MANIFOLD ADMITTING CODAZZI TYPE RICCI TENSOR

We know that

$$(D_X S)(Y, Z) = D_X S(Y, Z) - S(D_X Y, Z) - S(Y, D_X Z). \quad (5.1)$$

Then, from (5.1) and (1.3), we get

$$A(X)S(Y, Z) + B(X)g(Y, Z) = D_X S(Y, Z) - S(Y, D_X Z). \quad (5.2)$$

Putting $Z = \xi$ in above relation, we get

$$(n-1)A(X)\eta(Y) + B(X)\eta(Y) = (n-1)g(\emptyset X, Y) - S(Y, \emptyset X). \quad (5.3)$$

Putting $Y = \xi$ in (5.3), we have

$$(n-1)A(X) + B(X) = 0. \quad (5.4)$$

Here we assume that a generalized Ricci recurrent manifold admits Codazzi type Ricci tensor

$$\text{i.e. } (D_X S)(Y, Z) = (D_Y S)(X, Z). \quad (5.5)$$

Then by virtue of (1.3), it follows from (5.5) that

$$A(X)S(Y, Z) + B(X)g(Y, Z) = A(Y)S(X, Z) + B(Y)g(X, Z). \quad (5.6)$$

Putting $X = \xi$ in above, we get

$$A(\xi)S(Y, Z) + B(\xi)g(Y, Z) = [(n-1)A(Y) + B(Y)]\eta(Z). \quad (5.7)$$

In view of (5.4), (5.7) yields

$$S(Y, Z) = \lambda g(Y, Z), \text{ where } \lambda = -\frac{B(\xi)}{A(\xi)}.$$

i.e. M^n is an Einstein manifold.

Hence, we can state the following theorem:

Theorem 5.1. If a generalized Ricci recurrent LP - Sasakian manifold admits a Codazzi type Ricci tensor, then it is an Einstein manifold with constant $\lambda = -\frac{B(\xi)}{A(\xi)}$.

VI. SEMI PSEUDO SYMMETRIC LP - SASAKIAN MANIFOLD (SPS)_N (M^n, g), $n > 3$

Suppose that M^n is a Semi pseudo symmetric LP-Sasakian manifold then from (1.4), we obtain

$$(D_X S)(Y, \xi) = (2n - 1)A(X)\eta(Y) + (n - 2)A(Y)\eta(X) + A(\xi)S(X, Y). \text{ Since } g(X, P) = A(X) \Rightarrow A(\xi) = g(\xi, P) = \eta(P) \quad (6.1)$$

$$\text{This implies } (D_X S)(Y, \xi) = (2n - 1)A(X)\eta(Y) + (n - 2)\eta(X)A(Y) + \eta(P)S(X, Y). \quad (6.2)$$

We also know that

$$(D_X S)(Y, \xi) = (n - 1)g(\phi X, Y) - S(Y, \phi X). \quad (6.3)$$

From equation (6.2) and (6.3), we get

$$(2n - 1)A(X)\eta(Y) + (n - 2)\eta(X)A(Y) + \eta(P)S(X, Y) = (n - 1)g(\phi X, Y) - S(Y, \phi X). \quad (6.4)$$

Putting $X = \xi$ in (6.4), we get

$$(3n - 2)\eta(P)\eta(Y) - (n - 2)A(Y) = 0. \quad (6.5)$$

Putting $Y = \xi$ in (6.5), we get

$$\eta(P) = 0 \quad (6.6)$$

Hence, from (6.5) and (6.6), we get

$$A(Y) = 0.$$

Which is inadmissible by the definition of (SPS)_n.

Hence, we can state the following theorem:

Theorem 6.1. A (SPS)_n ($n > 3$), cannot be an LP sasakian manifold.

VII. SEMI PSEUDO RICCI SYMMETRIC $(SPRS)_n$ $(M^n, g), n > 3$ LP -SASAKIAN MANIFOLD

In this section we assume that a $(SPRS)_n$ is an LP sasakian manifold then from (1.5), we obtain

$$(D_X S)(Y, \xi) = (n-1)A(Y)\eta(X) + A(\xi)S(X, Y). \quad (7.1)$$

Also we know that

$$(D_X S)(Y, \xi) = (n-1)g(\phi X, Y) - S(Y, \phi X). \quad (7.2)$$

From (7.1) and (7.2), we get

$$(n-1)A(Y)\eta(X) + A(\xi)S(X, Y) = (n-1)g(\phi X, Y) - S(\phi X, Y). \quad (7.3)$$

Putting $X = \xi$ in (7.3), we get

$$A(Y) = A(\xi)\eta(Y). \quad (7.4)$$

Putting $Y = \xi$ in (7.4), we get

$$A(\xi) = 0. \quad (7.5)$$

Using (7.5) in (7.4), we get

$$A(Y) = 0.$$

Which is inadmissible by the definition of $(SPRS)_n$.

Hence, we can state the following theorem .:

Theorem 7.1. A $(SPRS)_n$ $(n > 3)$ can not be an LP sasakian manifold.

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