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A Study on New Sequence of Functions Involving the Generalized Contour Integral

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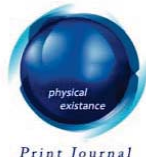
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Ref.

1. Chak, A. M., (1956) A class of polynomials and generalization of stirling numbers, Duke J. Math., 23, 45-55.

A Study on New Sequence of Functions Involving the Generalized Contour Integral

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Abstract - The aim of the present paper is an attempt to introduce a new sequence of functions $\left\{V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) / n = 0, 1, 2, \dots\right\}$ in equation (1.15), which involving with the generalized contour integral $I_{P,Q;R}^{M,N}[x]$. By using operational technique, some interesting generating relations and finite summation formulae are obtained in sections 2 and 3. These generating relations and finite summation formulae are unified in nature and act as key formulae from which, we can obtain as their special cases. For the sake of illustration we record here some special cases of our main formulas in section 4, which are also new and known.

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1. INTRODUCTION

Operational techniques have drawn the attention of several researchers in the study of sequences of functions and polynomials.

In 1956, Chak [1] defined a class of polynomials as,

$$G_{n,k}^{(\alpha)}(x) = x^{-\alpha-kn+n} e^x (x^k D)^n [x^\alpha e^{-x}] \quad (1.1)$$

Where k is constant and $n = 0, 1, 2, \dots, D \equiv \frac{d}{dx}$.

Gould and Hopper [5] introduced generalized Hermite polynomials in 1962 as,

$$H_n^r(x, a, p) = (-1)^n x^{-a} \exp(px^r) D^n [x^a \exp(-px^r)] \quad (1.2)$$

Chatterjea [4] studied as a class of polynomials for generalized Laguerre polynomials in 1964,

$$T_m^{(\alpha)}(x, p) = \frac{1}{n!} x^{-\alpha-n-1} \exp(px^r) (x^2 D)^n [x^{\alpha+1} \exp(-px^r)] \quad (1.3)$$

In 1968, Singh [16] studied the generalized Truesdell polynomials defined as,

$$T_n^{(\alpha)}(x, r, p) = x^{-\alpha} \exp(px^r) (xD)^n [x^\alpha \exp(-px^r)] \quad (1.4)$$

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Srivastava and Singh [19] introduced a general class of polynomials in 1971 as,

$$G_n^{(\alpha)}(x, r, p, k) = \frac{1}{n!} x^{-\alpha-kn} \exp(px^r) (x^{k+1}D)^n [x^\alpha \exp(-px^r)] \quad (1.5)$$

In 1971, the Rodrigues formula for generalized Laguerre polynomials is given by Mittal [8] as,

$$T_{kn}^{(\alpha)}(x) = \frac{1}{n!} x^{-\alpha} \exp(p_k(x)) D^n [x^{\alpha+n} \exp(-p_k(x))] \quad (1.6)$$

Where $p_k(x)$ is a polynomial in x of degree k .

Mittal [9] also proved following relation for (1.6) as,

$$T_{kn}^{(\alpha+s-1)}(x) = \frac{1}{n!} x^{-\alpha-n} \exp(p_k(x)) T_s^n [x^\alpha \exp(-p_k(x))] \quad (1.7)$$

Where s is constant and $T_s \equiv x(s + xD)$.

Recently, Shukla and Prajapati [15] obtained several properties of (1.7).

Chandel [2] also studied a class of polynomials defined in 1973 as,

$$T_n^{\alpha,k}(x, r, p) = x^{-\alpha} \exp(px^r) (x^k D)^n [x^\alpha \exp(-px^r)] \quad (1.8)$$

In 1974, Chandel [3] established a generalization of polynomial system as,

$$G_n(h, g(x), k) = e^{-hg(x)} (x^k D)^n [e^{-hg(x)}] \quad (1.9)$$

Where $g(x)$ is a function of x , h and k are constants.

In the same year, Srivastava [14] discussed some operational formulas generalized function

$F_n^{(r,m)}(x, a, k, p)$ in the form,

$$F_n^{(r,m)}(x, a, k, p) = x^{-a} \exp(px^r) (x^k D)^n [x^{a+km} \exp(-px^r)] \quad (1.10)$$

In 1975, Joshi and Parjapat [6] introduced the a class of polynomial,

$$M_{vn}^{(\alpha)}(x, a, k) = \frac{x^{-\alpha-nk}}{n!} \exp(p_v(x)) [x^k (a + xD)]^n [x^\alpha \exp(-p_v(x))] \quad (1.11)$$

Subsequently in 1975, Patil and Thakare [11] have obtained several formulae and generating relation for

$$P_n^{(\alpha)}(x, r, s, a, k, \lambda) = x^{-\alpha} \exp(px^r) [x^k (\lambda + xD)]^n [x^{\alpha+sn} \exp(-px^r)] \quad (1.12)$$

R_{ef.}

8. Mittal, H. B., (1971) A generalization of Laguerre polynomial, Publ. Math. Debrecen, 18, 53-58.

In 1979, Srivastava and Singh [17] studied a sequence of functions $V_n^{(\alpha)}(x; a, k, s)$ defined as

$$V_n^{(\alpha)}(x; a, k, s) = \frac{x^{-\alpha}}{n!} \exp\{p_k(x)\} \theta^n [x^\alpha \exp\{-p_k(x)\}] \quad (1.13)$$

By employing the operator $\theta \equiv x^a(s + xD)$, where s is constant and $p_k(x)$ is a polynomial in x of degree k .

J.C. Parjapati and N.K. Ahudia [12] introduced the sequence of function defined as,

$$V_n^{(\alpha, \beta, \delta)}(x; a, k, s) = \frac{1}{n!} x^{-\beta} W(\alpha, \delta; p_k(x)) (T_x^{a, s})^n [x^\beta W(\alpha, \delta; -p_k(x))] \quad (1.14)$$

A new sequence of function $\{V_n^{(M, N; P, Q; \alpha)}(x; a, k, s) / n = 0, 1, 2, \dots\}$ is introduced in this paper as,

$$V_n^{(M, N; P, Q; R; \alpha)}(x; a, k, s) = \frac{1}{n!} x^{-\alpha} I_{P, Q; R}^{M, N} [p_k(x)] (T_x^{a, s})^n [x^\alpha I_{P, Q; R}^{M, N} [-p_k(x)]] \quad (1.15)$$

Where $T_x^{a, s} \equiv x^a(s + xD)$, $D \equiv \frac{d}{dx}$, a and s are constants, $\beta \geq 0$, k is finite and non-negative integer, $p_k(x)$ is a polynomial in x of degree k and $I_{P, Q; R}^{M, N}[x]$ is I-function, defined and represented in the following manner [13]:

$$I_{P, Q; R}^{M, N} \left[z \left| \begin{matrix} (a_j, \alpha_j)_{1, N} \\ (b_j, \beta_j)_{1, M} \end{matrix} \right. \begin{matrix} (a_{ji}, \alpha_{ji})_{N+1, P_i} \\ (b_{ji}, \beta_{ji})_{M+1, Q_i} \end{matrix} \right] = \frac{1}{2\pi i} \int_L \phi(\xi) z^\xi d\xi \quad (1.16)$$

$$\phi(\xi) = \frac{\prod_{j=1}^M \Gamma(b_j - \beta_j \xi) \prod_{j=1}^N \Gamma(1 - a_j + \alpha_j \xi)}{\sum_{i=1}^R \left[\prod_{j=M+1}^{Q_i} \Gamma(1 - b_{ji} + \beta_{ji} \xi) \prod_{j=N+1}^{P_i} \Gamma(a_{ji} - \alpha_{ji} \xi) \right]} \quad (1.17)$$

and M, N, P_i, Q_i are integers satisfy $0 \leq N \leq P_i, 1 \leq M \leq Q_i (i = 1, \dots, R)$, R is finite, $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are positive numbers and a_j, b_j, a_{ji}, b_{ji} are complex numbers. I-function which is a generalized form of the well known Fox's H-function [18]. In the sequel the I-function will be studied under the following conditions of existence:

$$(I) A_i > 0, |\arg z| < \frac{A_i \pi}{2} \quad (1.18)$$

$$(II) A_i \geq 0, |\arg z| \leq \frac{A_i \pi}{2} \text{ and } \operatorname{Re}(B+1) < 0 \quad (1.19)$$

$$\text{Where } A_i = \sum_{j=1}^N \alpha_j - \sum_{j=N+1}^{P_i} \alpha_{ji} + \sum_{j=1}^M \beta_j - \sum_{j=M+1}^{Q_i} \beta_{ji}, \forall i = (1, 2, \dots, R) \quad (1.20)$$

$$\text{And } B = \sum_{j=1}^M b_j + \sum_{j=M+1}^{Q_i} b_{ji} - \sum_{j=1}^N a_j - \sum_{j=N+1}^{P_i} a_{ji} + \frac{1}{2}(P_i - Q_i), \forall i = (1, 2, \dots, R) \quad (1.21)$$

Some generating relations and finite summation formulae of class of polynomials or sequence of function have been obtained by using the properties of the differential operators. $T_x^{a,s} \equiv x^a (s + xD)$, $T_x^{a,1} \equiv x^a (1 + xD)$, where $D \equiv \frac{d}{dx}$, is based on the work of Mittal [10], Patil and Thakare [11], Srivastava and Singh [17]. Some useful Operational Techniques are given below:

$$\exp(t T_x^{a,s}) (x^\beta f(x)) = x^\beta (1 - ax^a t)^{-\left(\frac{\beta+s}{a}\right)} f\left(x(1 - ax^a t)^{-1/a}\right) \quad (1.22)$$

$$\exp(t T_x^{a,s}) (x^{\alpha-an} f(x)) = x^\alpha (1 + at)^{-1+\left(\frac{\alpha+s}{a}\right)} f\left(x(1 + at)^{1/a}\right) \quad (1.23)$$

$$(T_x^{a,s})^n (xuv) = x \sum_{m=0}^{\infty} \binom{n}{m} (T_x^{a,s})^{n-m} (v) (T_x^{a,1})^m (u) \quad (1.24)$$

$$(1 + xD)(1 + a + xD)(1 + 2a + xD)(1 + 3a + xD) \dots (1 + (m-1)a + xD) x^{\beta-1} = a^m \left(\frac{\beta}{a}\right)_m x^{\beta-1} \quad (1.25)$$

$$(1 - at)^{-\frac{\alpha}{a}} = (1 - at)^{-\frac{\beta}{a}} \sum_{m=0}^{\infty} \left(\frac{\alpha - \beta}{a}\right)_m \frac{(at)^m}{m!} \quad (1.26)$$

II. GENERATING RELATIONS

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) x^{-an} t^n = (1 - at)^{-\left(\frac{\alpha+s}{a}\right)} I_{P,Q;R}^{M,N}(p_k(x)) I_{P,Q;R}^{M,N}\left(-p_k\left(x(1 - at)^{-1/a}\right)\right) \quad (2.1)$$

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,R;\alpha-an)}(x; a, k, s) x^{-an} t^n = (1 + at)^{-1+\left(\frac{\alpha+s}{a}\right)} I_{P,Q;R}^{M,N}(p_k(x)) I_{P,Q;R}^{M,N}\left(-p_k\left(x(1 + at)^{1/a}\right)\right) \quad (2.2)$$

$$\begin{aligned} & \sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) t^n \\ &= (1 - at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{I_{P,Q;R}^{M,N}(p_k(x))}{I_{P,Q;R}^{M,N}\left(p_k\left(x(1 - at)^{-1/a}\right)\right)} V_n^{(M,N;P,Q,R;\alpha)}\left(x(1 - at)^{-1/a}; a, k, s\right) \end{aligned} \quad (2.3)$$

Ref.

17. Srivastava, A. N. and Singh, S. N., (1979) Some generating relations connected with a function defined by a Generalized Rodrigues formula, Indian J. Pure Appl. Math., 10(10), 1312-1317.

Proof of (2.1)

From (1.15), we consider

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) t^n = x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) \exp(t T_x^{a,s}) \left[x^{\alpha} I_{P,Q;R}^{M,N}(-p_k(x)) \right] \quad (2.4)$$

Using operational technique (1.22), above equation (2.4) reduces to:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) t^n \\ &= x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) x^{\alpha} (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} I_{P,Q;R}^{M,N}\left(-p_k\left(x(1 - ax^a t)^{-1/a}\right)\right) \\ &= (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} I_{P,Q;R}^{M,N}(p_k(x)) I_{P,Q;R}^{M,N}\left(-p_k\left(x(1 - ax^a t)^{-1/a}\right)\right) \end{aligned} \quad (2.5)$$

And replacing t by tx^{-a} , this gives (2.1).

Proof of (2.2)

Again from (1.15), we have

$$\sum_{n=0}^{\infty} x^{-an} V_n^{(M,N;P,Q,R;\alpha-an)}(x; a, k, s) t^n = x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) \exp(t T_x^{a,s}) \left[x^{\alpha-an} I_{P,Q;R}^{M,N}(-p_k(x)) \right] \quad (2.6)$$

applying the operational technique (1.23), we get

$$\begin{aligned} & \sum_{n=0}^{\infty} x^{-an} V_n^{(M,N;P,Q,R;\alpha-an)}(x; a, k, s) t^n \\ &= x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) x^{\alpha} (1 + at)^{\frac{\alpha+s}{a}-1} I_{P,Q;R}^{M,N}\left[-p_k\left(x(1 + at)^{1/a}\right)\right] \\ &= (1 + at)^{\frac{\alpha+s}{a}-1} I_{P,Q;R}^{M,N}(p_k(x)) I_{P,Q;R}^{M,N}\left[-p_k\left(x(1 + at)^{1/a}\right)\right] \end{aligned} \quad (2.7)$$

This proves (2.2).

Proof of (2.3)

We can write (1.15) as,

$$\left(T_x^{a,s}\right)^n \left[x^{\alpha} I_{P,Q;R}^{M,N}(-p_k(x)) \right] = n! x^{\alpha} \frac{1}{I_{P,Q;R}^{M,N}(p_k(x))} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) \quad (2.8)$$

Or

$$\exp\left(t\left(T_x^{a,s}\right)\right) \left\{ \left(T_x^{a,s}\right)^n \left[x^{\alpha} I_{P,Q;R}^{M,N}(-p_k(x)) \right] \right\}$$

$$\begin{aligned}
&= n! \exp(t T_x^{a,\alpha}) \left[x^\alpha \frac{1}{I_{P,Q;R}^{M,N}(p_k(x))} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) \right] \\
&\quad \sum_{m=0}^{\infty} \frac{t^m}{m!} (T_x^{a,s})^{m+n} \left[x^\alpha I_{P,Q;R}^{M,N}(-p_k(x)) \right] \\
&= n! \exp(T_x^{a,s}) \left[x^\alpha \frac{1}{I_{P,Q;R}^{M,N}(p_k(x))} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) \right]
\end{aligned} \tag{2.9}$$

Using the operational technique (1.20), above equation can be written as:

$$\begin{aligned}
&\sum_{m=0}^{\infty} \frac{t^m}{m!} (T_x^{a,s})^{m+n} \left[x^\alpha I_{P,Q;R}^{M,N}(-p_k(x)) \right] \\
&= n! x^\alpha (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{1}{I_{P,Q;R}^{M,N}\left(p_k\left(x(1 - ax^a t)^{-1/a}\right)\right)} V_n^{(M,N;P,Q,R;\alpha)}\left(x(1 - ax^a t)^{-1/a}; a, k, s\right)
\end{aligned} \tag{2.10}$$

use of (2.8) gives

$$\begin{aligned}
&\sum_{m=0}^{\infty} \frac{t^m (m+n)!}{m! n!} x^\alpha \frac{1}{I_{P,Q;R}^{M,N}(p_k(x))} V_{m+n}^{(M,N;P,Q,R;\alpha)}(x; a, k, s) \\
&= x^\alpha (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{1}{I_{P,Q;R}^{M,N}\left[p_k\left(x(1 - ax^a t)^{-1/a}\right)\right]} V_n^{(M,N;P,Q,R;\alpha)}\left(x(1 - ax^a t)^{-1/a}; a, k, s\right)
\end{aligned} \tag{2.11}$$

Therefore

$$\begin{aligned}
&\sum_{m=0}^{\infty} \binom{m+n}{n} V_{m+n}^{(M,N;P,Q,R;\alpha)}(x; a, k, s) t^m \\
&= (1 - ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} \frac{I_{P,Q;R}^{M,N}(p_k(x))}{I_{P,Q;R}^{M,N}\left(p_k\left(x(1 - ax^a t)^{-1/a}\right)\right)} V_n^{(M,N;P,Q,R;\alpha)}\left(x(1 - ax^a t)^{-1/a}; a, k, s\right)
\end{aligned} \tag{2.12}$$

And replacing t by tx^{-a} , this gives the result (2.3).

III. FINITE SUMMATION FORMULAE

$$V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) = \sum_{m=0}^{\infty} \frac{1}{m!} \left(a \frac{x}{x}\right)^m \left(\frac{\alpha}{a}\right) V_{n-m}^{(M,N;P,Q,R;0)}(x; a, k, s) \tag{3.1}$$

$$V_n^{(M,N;P,Q,R;\alpha)}(x;a,k,s) = \sum_{m=0}^{\infty} \frac{1}{m!} (a \, x)^m \left(\frac{\alpha - \beta}{a} \right)_m V_{n-m}^{(M,N;P,Q,R;\beta)}(x;a,k,s) \quad (3.2)$$

Proof of (3.1)

From equation (1.15), we have:

$$V_n^{(M,N;P,Q,R;\alpha)}(x;a,k,s) = \frac{1}{n!} x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) (T_x^{(a,s)})^n [x x^{\alpha-1} I_{P,Q;R}^{M,N}(-p_k(x))] \quad (3.3)$$

Using the operational technique (1.24), we have:

$$\begin{aligned} & V_n^{(M,N;P,Q,R;\alpha)}(x;a,k,s) \\ &= \frac{1}{n!} x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) x \sum_{m=0}^n \binom{n}{m} (T_x^{(a,s)})^{n-m} [I_{P,Q;R}^{M,N}(-p_k(x))] (T_x^{a,1})^m (x^{\alpha-1}) \\ &= \frac{1}{n!} x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) x \sum_{m=0}^n \frac{n!}{m!(n-m)!} x^{a(n-m)} \times \\ & \quad \left[(s + xD)(s + a + xD)(s + 2a + xD) \dots (s + (n-m-1)a + xD) \right] \times \\ & \quad \left[I_{P,Q;R}^{M,N}(-p_k(x)) \right] x^{am} \left[(1 + xD)(1 + a + xD)(1 + 2a + xD) \dots (1 + (m-1)a + xD) \right] (x^{\alpha-1}) \end{aligned} \quad (3.4)$$

Using the result (1.24), we have:

$$= I_{P,Q;R}^{M,N}(p_k(x)) \sum_{m=0}^n \frac{1}{m!(n-m)!} x^a \prod_{i=0}^{n-m-1} (s + ia + xD) [I_{P,Q;R}^{M,N}(-p_k(x))] a^m \left(\frac{\alpha}{a} \right)_m \quad (3.5)$$

Put $\alpha = 0$ and replacing n by $n-m$ in (3.3), we get:

$$\begin{aligned} & V_{n-m}^{(M,N;P,Q,R;0)}(x;a,k,s) = \frac{1}{(n-m)!} I_{P,Q;R}^{M,N}(p_k(x)) (T_x^{a,s})^{n-m} [I_{P,Q;R}^{M,N}(-p_k(x))] \\ & \Rightarrow \frac{1}{(n-m)!} (T_x^{a,s})^{n-m} [I_{P,Q;R}^{M,N}(-p_k(x))] = \frac{1}{I_{P,Q;R}^{M,N}(p_k(x))} V_{n-m}^{(M,N;P,Q,R;0)}(x;a,k,s) \end{aligned} \quad (3.6)$$

This gives

$$\frac{1}{(n-m)!} \prod_{i=0}^{n-m-1} (s + ia + xD) [I_{P,Q;R}^{M,N}(-p_k(x))] = x^{a(n-m)} \frac{1}{I_{P,Q;R}^{M,N}(p_k(x))} V_{n-m}^{(M,N;P,Q,R;0)}(x;a,k,s) \quad (3.7)$$

From equation (3.5) and (3.7) we have the main the result.

Proof of (3.2)

Equation (1.15) can be written as:

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) t^n = x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) \exp(T_x^{(a,s)}) [x^{\beta} I_{P,Q;R}^{M,N}(-p_k(x))] \quad (3.8)$$

Applying the (1.21) in equation (3.8), we have:

$$\begin{aligned} & \sum_{n=0}^{\infty} V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) t^n \\ &= x^{-\alpha} I_{P,Q;R}^{M,N}(p_k(x)) x^{\alpha} (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} I_{P,Q;R}^{M,N}\left(-p_k\left(x(1-ax^a t)^{-1/a}\right)\right) \\ &= (1-ax^a t)^{-\left(\frac{\alpha+s}{a}\right)} I_{P,Q;R}^{M,N}(p_k(x)) I_{P,Q;R}^{M,N}\left(-p_k\left(x(1-ax^a t)^{-1/a}\right)\right) \end{aligned} \quad (3.9)$$

Applying the result from equation (1.26); equation (3.9) reduces to:

$$\begin{aligned} &= (1-ax^a t)^{-\left(\frac{\beta+s}{a}\right)} \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} I_{P,Q;R}^{M,N}(p_k(x)) I_{P,Q;R}^{M,N}\left(-p_k\left(x(1-ax^a t)^{-1/a}\right)\right) \\ &= \sum_{m=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a t)^m}{m!} x^{-\beta} I_{P,Q;R}^{M,N}(p_k(x)) \exp(T_x^{a,s}) [x^{\beta} I_{P,Q;R}^{M,N}(-p_k(x))] \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m t^{n+m}}{m! n!} x^{-\beta} I_{P,Q;R}^{M,N}(p_k(x)) (T_x^{a,s})^n [x^{\beta} I_{P,Q;R}^{M,N}(-p_k(x))] \\ &= \sum_{n=0}^{\infty} \sum_{m=0}^n \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m t^n}{m! (n-m)!} x^{-\beta} I_{P,Q;R}^{M,N}(p_k(x)) (T_x^{a,s})^{n-m} [x^{\beta} I_{P,Q;R}^{M,N}(-p_k(x))] \end{aligned} \quad (3.10)$$

Now equating the coefficient of t^n , we get:

$$\begin{aligned} & V_n^{(M,N;P,Q,R;\alpha)}(x; a, k, s) \\ &= \sum_{m=0}^n \left(\frac{\alpha-\beta}{a}\right)_m \frac{(ax^a)^m}{m! (n-m)!} x^{-\beta} I_{P,Q;R}^{M,N}(p_k(x)) (T_x^{a,s})^{n-m} [x^{\beta} I_{P,Q;R}^{M,N}(-p_k(x))] \end{aligned} \quad (3.11)$$

Using the equation (1.15) in (3.11) we have the result (3.2).

IV. SPECIAL CASES

- a) If we put $R=1$, I-function reduces to Fox's H-function [18, p. 10, Eqn. (2.1.1)], then the equation (2.1), (2.2) and (2.3) takes the following form:

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,1;\alpha)}(x; a, k, s) x^{-an} t^n$$

$$= (1-at)^{-1+\left(\frac{\alpha+s}{a}\right)} H_{P,Q}^{M,N}(p_k(x)) H_{P,Q}^{M,N}\left(-p_k\left(x(1-at)^{-1/a}\right)\right)$$
(4.1)

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,1;\alpha-an)}(x; a, k, s) x^{-an} t^n$$

$$= (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} H_{P,Q}^{M,N}(p_k(x)) H_{P,Q}^{M,N}\left(-p_k\left(x(1+at)^{1/a}\right)\right)$$
(4.2)

$$\sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(M,N;P,Q,1;\alpha)}(x; a, k, s) t^n$$

$$= (1-at)^{-1+\left(\frac{\alpha+s}{a}\right)} \frac{H_{P,Q}^{M,N}(p_k(x))}{H_{P,Q}^{M,N}\left(p_k\left(x(1-at)^{-1/a}\right)\right)} V_n^{(M,N;P,Q,1;\alpha)}\left(x(1-at)^{-1/a}; a, k, s\right)$$
(4.3)

- b) If we put $R=1; \alpha_j = \beta_j = 1$, then the I-function reduces to general type of G-function

$$[7] \text{ i.e. } I_{P,Q;R}^{M,N} \left[z \left| \begin{matrix} (a_j, 1)_{1,N} \\ (b_j, 1)_{1,M} \end{matrix} \right. \begin{matrix} (a_j, 1)_{N+1,P} \\ (b_j, 1)_{M+1,Q} \end{matrix} \right] = G_{P,Q}^{M,N} \left[z \left| \begin{matrix} (a_j, 1)_{1,p} \\ (b_j, 1)_{1,q} \end{matrix} \right. \right], \text{ the equation (2.1), (2.2) and}$$

(2.3) takes the following form:

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,1;\alpha)}(x; a, k, s) x^{-an} t^n$$

$$= (1-at)^{-1+\left(\frac{\alpha+s}{a}\right)} G_{P,Q}^{M,N}(p_k(x)) G_{P,Q}^{M,N}\left(-p_k\left(x(1-at)^{-1/a}\right)\right)$$
(4.4)

$$\sum_{n=0}^{\infty} V_n^{(M,N;P,Q,1;\alpha-an)}(x; a, k, s) x^{-an} t^n$$

$$= (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} G_{P,Q}^{M,N}(p_k(x)) G_{P,Q}^{M,N}\left(-p_k\left(x(1+at)^{1/a}\right)\right)$$
(4.5)

Ref.

$$\sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(M,N;P,Q,1;\alpha)}(x; a, k, s) t^n \quad (4.6)$$

$$= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{G_{P,Q}^{M,N}(p_k(x))}{G_{P,Q}^{M,N}(p_k(x(1-at)^{-1/a}))} V_n^{(M,N;P,Q,1;\alpha)}(x(1-at)^{-1/a}; a, k, s)$$

- c) If we put $R=1, N=P_i=P, M=1, Q_i=Q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_{ji}=1-b_j, \beta_{ji}=\beta_j$, then I-function reduces to the generalized wright hypergeometric function [20, p.287],

$$\text{i.e. } I_{p,q+1;1}^{1,p} \left[z \left| \begin{matrix} (1-a_j, \alpha_j)_{1,p} \\ (0,1), (1-b_j, \beta_j)_{1,q} \end{matrix} \right. \right] = {}_p\psi_q \left[\begin{matrix} (a_j, \alpha_j)_{1,p} \\ (b_j, \beta_j)_{1,q} \end{matrix} ; -z \right], \text{ the equation (2.1), (2.2) and}$$

(2.3) takes the following form:

$$\sum_{n=0}^{\infty} V_n^{(P,Q;\alpha)}(x; a, k, s) x^{-an} t^n \quad (4.7)$$

$$= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} {}_p\psi_q(p_k(x)) {}_p\psi_q(-p_k(x(1-at)^{-1/a}))$$

$$\sum_{n=0}^{\infty} V_n^{(P,Q;\alpha-an)}(x; a, k, s) x^{-an} t^n \quad (4.8)$$

$$= (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} {}_p\psi_q(p_k(x)) {}_p\psi_q(-p_k(x(1+at)^{1/a}))$$

$$\sum_{n=0}^{\infty} \binom{m+n}{m} V_n^{(P,Q;\alpha)}(x; a, k, s) t^n \quad (4.9)$$

$$= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{{}_p\psi_q(p_k(x))}{{}_p\psi_q(p_k(x(1-at)^{-1/a}))} V_n^{(P,Q;\alpha)}(x(1-at)^{-1/a}; a, k, s)$$

- d) The interesting special cases/relationships between (1.15) and class of polynomials (1.1)-(1.14) can also be obtained for appropriate values of parameters α, a, k, s and those of involving in I-function.

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