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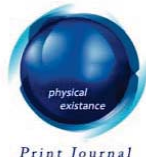
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Uniform Strict Practical Stability Criteria for Impulsive Functional Differential Equations

Dilbaj Singh^α & Sanjay Kumar Srivastava^σ

Abstract - Strict stability is the kind of stability that can give us some information about the rate of decay of the solution. There are some results about strict stability of functional differential equations. On other hand, in the study of stability, an interesting set of problems deal with bringing sets close to a certain state, rather than the equilibrium state. The desired state of a system may be mathematically unstable and yet the system may oscillate sufficiently near this state that its performance is acceptable. Many problems fall into this category. Such considerations led to the notion of practical stability which is neither weaker nor stronger than stability. In this paper, strict practical stability of Impulsive functional differential equations in which the state variables on the impulses are related to time delay is considered. By using Lyapunov functions and Razumikhin technique, some criteria for strict practical stability for functional differential equations, in which the state variables on the impulses are related to the time delay, are provided.

Keywords : *impulsive functional differential equation, strict stability, lyapunov function, razumikhin technique, time delay.*

1. INTRODUCTION

The impulsive differential equations represent a more natural framework for mathematical modeling of many real world phenomena than ordinary differential equations. In recent years, significant progress has been made in the theory of impulsive differential Impulses can make unstable systems stable, so it has been widely used in many fields such as physics, chemistry, biology, population dynamics, industrial robotics and so on. The impulsive differential equations represent a more natural framework for mathematical modeling of many real world phenomena than ordinary differential equations. In recent years, significant progress has been made in the theory of impulsive differential equations [3-14]. In addition to that, functional differential equations have a wide application in our society. So it is important to study them. There are some results on impulsive functional differential equations [5,6,7,8,11,13,14].

We can easily see that in the previous works about impulsive functional differential equations the authors always suppose that the state variables on the impulses are only related to the present state. But in most cases, it is more applicable that the state variables on the impulses are also related to the former state. But there are rare results about impulsive functional differential equations in which state variable on the impulses are related to the time delay. In addition to that strict stability is analogous to

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Lyapunov's uniform asymptotic stability. It gives us some information about the rate of decay of the solutions. In [1], the authors have explored further the definitions of strict stability of differential equations and have gotten some results. In [13] authors have gotten some results about the strict stability of impulsive functional differential equations in which the state variables on the impulses are not related to the time delay. Moreover, in the study of Lyapunov stability, an interesting set of problems deal with bringing sets close to a certain state, rather than the equilibrium state. The desired state of a system may be mathematically unstable and yet the system may oscillate sufficiently near this state that its performance is acceptable. Many problems fall into this category including the travel of a space vehicle between two points, an aircraft or a missile which may oscillate around a mathematically unstable course yet its performance may be acceptable, the problem in a chemical process of keeping the temperature within certain bounds, etc. Such considerations led to the notion of practical stability which is neither weaker nor stronger than Lyapunov stability. In [11] authors have gotten some results about the practical stability of impulsive functional differential equations in Equations in terms of two measures. In [2] authors have gotten some results about strict practical stability of delay differential equations. In this paper, strict practical stability of impulsive functional differential equations in which the state variables on the impulses are related to the time delay is considered.

This paper is organized as follows. In Section II, we introduce some basic definitions and notations. In Section III, some criteria in the form of theorem for strict practical stability of impulsive functional differential equations is obtained in which state variables on the impulses are related to the time delay is investigated. Finally, concluding remarks are given in Section IV.

II. PRELIMINARIES

Consider the following Impulsive functional differential equation in which the state variables on the impulses are related to time delay.

$$\begin{aligned} x'(t) &= f(t, x_t), \quad t \geq t_0, \quad t \neq \tau_k \\ x(\tau_k) &= I_k(x(\tau_k^-)) + J_k(x(\tau_k^- - \tau)), \quad k=1, 2, 3, \dots, \end{aligned} \quad (1)$$

Where $x \in R^n$, $f \in C[R^+ \times D, R^n]$, $I_k, J_k \in C[R^n, R^n]$, D is an open set in $PC([- \tau, 0], R^n)$, where $\tau = \text{constant} > 0$,

$PC([- \tau, 0], R^n) = \{\phi : [- \tau, 0] \rightarrow R^n, \phi(t) \text{ is continuous everywhere except a finite number of points } \hat{t} \text{ at which } \phi(\hat{t}^+) \text{ and } \phi(\hat{t}^-) \text{ exist and } \phi(\hat{t}^+) = \phi(\hat{t}^-)\}$, $f(t, 0) = 0$, for all $t \in R$, $I_k(0) \equiv 0$, $J_k \equiv 0$, $0 = \tau_0 < \tau_1 < \tau_2 < \tau_3 < \dots < \tau_k < \dots, \tau_k \rightarrow \infty$, for $k \rightarrow \infty$ and $x(t^+) = \lim_{s \rightarrow t^+} x(s)$, $x(t^-) = \lim_{s \rightarrow t^-} x(s)$.

For each $t \geq t_0$, $x_t \in D$ is defined by $x_t(s) = x(t+s)$, $-\tau \leq s \leq 0$. For $\phi \in PC([- \tau, 0], R^n)$ $\|\phi\|_1$ is defined by $\|\phi\|_1 = \sup_{-\tau \leq s \leq 0} \|\phi\|$, $\|\phi\|_2$ is defined by $\|\phi\|_2 = \inf_{-\tau \leq s \leq 0} \|\phi\|$, where $\|\cdot\|$ denotes the norm of a vector R^n . We can see that $x(t) \equiv 0$ is a solution of (1) which we call the zero solution.

A function $x(t)$ is called a solution of (1) with the initial condition $x_\sigma = \phi$. Where $\sigma \geq t_0$ and $\phi \in PC([- \tau, 0], R^n)$, the initial value problems of equation (1) is

Ref.

2. Lakshmikantham V. and Zhang Y., "Strict practical stability of delay differential equation", Applied Mathematics and Computation, vol. 118, pp. 275-285, 2001.

$$\begin{aligned}
 x'(t) &= f(t, x_t), \quad t \geq t_0 \quad t \neq \tau_k \\
 x(\tau_k) &= I_k(x(\tau_k^-)) + J_k(x(\tau_k^- - \tau)), \quad k=1, 2, 3, \dots, \quad x_\sigma = \varphi
 \end{aligned} \tag{2}$$

Throughout this paper we let the following hypothesis hold.

(H₁) For $t \in [\sigma - \tau, \sigma]$, the solution $x(t; \sigma, \varphi)$ coincides with the function $\varphi(t - \sigma)$.

(H₂) For each function $x(s) : [\sigma - \tau, \infty) \rightarrow R^n$, which is continuous everywhere except at the point $\{\tau_k\}$ at which $x(\tau_k^+), (\tau_k^-)$ exist and $x(\tau_k^+) = x(\tau_k^-)$, $f(t, x_t)$ is continuous for almost all $t \in [\sigma, \infty)$ and at the discontinuous points f is right continuous.

(H₃) $f(t, \phi)$ is Lipschitzian in ϕ in each compact set in $PC([-\tau, 0], R^n)$.

(H₄) The functions $I_k, J_k, k=1, 2, \dots$, are such that if $x \in D, I_k \neq 0$ and $J_k \neq 0$, then $I_k(x) + J_k(x(t - \tau)) \in D$.

Under the hypothesis (H₁)–(H₂), there is a unique solution of problems (2) through-out (σ, φ) .

We are using the following notations:

$$S(\rho) = \{x \in R^n : \|x\| < \rho\},$$

$$K = \{a \in C[R^+, R^+] : a(t) \text{ is monotone strictly increasing and } a(0) = 0\},$$

We have following definitions.

Definition 2.1 : The trivial solution of (2) is said to be

- 1) Strictly practical stable, if for any $\sigma \geq t_0$ there exist (λ_1, A_1) , $\lambda_1 \leq A_1$, we have $\|\varphi\| \leq \lambda_1$ implies $\|x(t)\| \leq A_1$ for some $t_0 \in R^+$; and for every $\lambda_2 \leq \lambda_1$ there exist $A_2 \leq \lambda_2$, such that $\|\varphi\| > \lambda_2$ implies $\|x(t)\| > A_2$ for all $t \geq \sigma$.
- 2) Uniformly strictly practical stable if (1) holds for all $t_0 \in R^+$.

Definition 2.2 : The function $V(x) : [t_0, \infty) \times S(\rho) \rightarrow R^+$ belongs to class V_0 if:

- 1) The function V is continuous on each of the sets $[\tau_{k-1}, \tau_k) \times S(\rho)$ and for all $t \geq t_0$, $V(t, 0) \equiv 0$;
- 2) $V(t, x)$ is locally Lipschitzian in $x \in S(\rho)$;
- 3) For each $k = 1, 2, \dots$, there exist finite limits

$$\lim_{(t, y) \rightarrow (\tau_k^-, x)} V(t, y) = V(\tau_k^-, x)$$

With $V(\tau_k^+, x) = V(t, x)$ satisfied.

Definition 2.3 : Let $V \in V_0$, for $(t, x) \in [\tau_{k-1}, \tau_k) \times S(\rho)$, D^+V is defined as

$$D^+V(t, x(t)) = \lim_{\delta \rightarrow 0^+} \sup \frac{1}{\delta} \{V(t + \delta, x(t + \delta)) - V(t, x(t))\}$$

III. MAIN RESULT

Now we consider the uniformly strict practical stability of the impulsive functional differential equation in which the state variables on the impulses are related to the time delay. We have the following theorem about the uniform practical stability of the system (1).

Theorem 3.1 Assume that

- (i) There exist $0 < \lambda_1 \leq A_1$, $a_1, b_1 \in k$, $V_{\lambda_1}(t, x(t)) \in v_0$. Such that

$$b_1(\|x(t)\|) \leq V_{\lambda_1}(t, x(t)) \leq a_1(\|x(t)\|).$$

- (ii) For any solution of (1), $V_{\lambda_1}(t+s, x(t+s)) \leq V_{\lambda_1}(t, x(t))$ for $s \in [-\tau, 0]$ and $D^+V_{\lambda_1}(t, x(t)) \leq 0$.

- (iii) For all $k \in Z^+$, $x \in S(\rho)$.

$$V_{\lambda_1}(\tau_k, I_k(x(\tau_k^-)) + J_k(x(\tau_k^- - \tau))) \leq \frac{1+c_k}{2} [V_{\lambda_1}(\tau_k^-, x(\tau_k^-)) + V_{\lambda_1}(\tau_k^- - \tau, x(\tau_k^- - \tau))].$$

Where

$$c_k \geq 0 \text{ and } \sum_{k=1}^{\infty} c_k < \infty.$$

- (iv) For any $0 < \lambda_2 \leq \lambda_1$, there exist $a_2, b_2 \in k$, $V_{\lambda_2}(t, x(t)) \in v_0$. Such that

$$b_2(\|x(t)\|) \leq V_{\lambda_2}(t, x(t)) \leq a_2(\|x(t)\|).$$

- (v) For any solution of (1), $V_{\lambda_2}(t+s, x(t+s)) \geq V_{\lambda_2}(t, x(t))$ for $s \in [-\tau, 0]$ and

$$D^+V_{\lambda_2}(t, x(t)) \geq 0.$$

- (vi) For all $k \in Z^+$, $x \in S(\rho)$.

$$\begin{aligned} & V_{\lambda_2}(\tau_k, I_k(x(\tau_k^-)) + J_k(x(\tau_k^- - \tau))) \\ & \geq \frac{1-d_k}{2} [V_{\lambda_2}(\tau_k^-, x(\tau_k^-)) + V_{\lambda_2}(\tau_k^- - \tau, x(\tau_k^- - \tau))]. \end{aligned}$$

$$\text{Where } 0 \leq d_k < 1 \text{ and } \sum_{k=1}^{\infty} d_k < \infty.$$

Then the trivial solution of (1) is uniformly strictly practical stable.

Proof :

Since

$$\sum_{k=1}^{\infty} b_k < \infty, \sum_{k=1}^{\infty} c_k < \infty.$$

It follows that

$$\prod_{k=1}^{\infty} \frac{1+c_k}{2} = M \text{ and } \prod_{k=1}^{\infty} \frac{1-b_k}{2} = N.$$

Obviously

$$1 \leq M \leq \infty, 0 < N \leq 1.$$

Let $0 \leq A_1 < \rho$ and $\sigma \geq t_0$ be given and $\sigma \in [\tau_{k-1}, \tau_k]$ for some $k \in Z^+$.

Choose $0 < \lambda_1 \leq A_1$ such that $Ma_1(\lambda_1) < b_1(A_1)$.

We claim that $\|\varphi\| \leq \lambda_1$ implies $\|x(t)\| < A_1$ for $t \geq \sigma$.

Obviously for any $t \in [\sigma - \tau, \sigma]$, there exist a $\theta \in [-\tau, 0]$, Such that

$$\begin{aligned} V_{\lambda_1}(t, x(t)) &= V_{\lambda_1}(\sigma + \theta, x(\sigma + \theta)) \\ &\leq a_1(\|x(\sigma + \theta)\|) = a_1(\|x_\sigma(\theta)\|) = a_1(\|\varphi(\theta)\|) \leq a_1(\lambda_1). \end{aligned}$$

Then we claim that

$$V_{\lambda_1}(t, x(t)) \leq a_1(\lambda_1) \text{ for } \sigma \leq t < \tau_k. \quad (3)$$

In fact, if inequality (3) does not hold, then there exist a $\hat{t} \in (\sigma, \tau_k)$ such that $V_{\lambda_1}(\hat{t}, x(\hat{t})) > a_1(\lambda_1) \geq V_{\lambda_1}(\sigma, x(\sigma))$ which implies that there is a $\bar{t} \in (\sigma, \hat{t})$ such that

$$D^+V_{\lambda_1}(\bar{t}, x(\bar{t})) > 0 \quad (4)$$

and

$$V_{\lambda_1}(\bar{t} + s, x(\bar{t} + s)) \leq V_{\lambda_1}(\bar{t}, x(\bar{t})) \text{ for } s \in [-\tau, 0].$$

By condition (ii), which implies $D^+V_{\lambda_1}(\bar{t}, x(\bar{t})) \leq 0$. This contradict inequality (4) so inequality (3) holds.

In view of inequality (3) and condition (iii), we have

$$\begin{aligned} V_{\lambda_1}(\tau_k, x(\tau_k)) &= V_{\lambda_1}(\tau_k, I_k(x(\tau_k^-)) + J_k(x(\tau_k^- - \tau))) \\ &\leq \frac{1+c_k}{2}[V_{\lambda_1}(\tau_k^-, x(\tau_k^-)) + V_{\lambda_1}(\tau_k^- - \tau, x(\tau_k^- - \tau))] \leq (1+c_k)a_1(\lambda_1) \end{aligned}$$

Next we prove that

$$V_{\lambda_1}(t, x(t)) \leq (1+c_k)a_1(\lambda_1) \text{ for } \tau_k \leq t \leq \tau_{k+1}. \quad (5)$$

If inequality (5) does not hold, then there is a $\hat{s} \in (\tau_k, \tau_{k+1})$ such that

$$V_{\lambda_1}(\hat{s}, x(\hat{s})) > (1+c_k)a_1(\lambda_1) \geq V_{\lambda_1}(\tau_k, x(\tau_k))$$

Which implies that there is a $\bar{s} \in (\tau_k, \hat{s})$ such that

$$D^+V_{\lambda_1}(\bar{s}, x(\bar{s})) > 0 \quad (6)$$

and

$$V_{\lambda_1}(\bar{s} + s, x(\bar{s} + s)) \leq V_{\lambda_1}(\bar{s}, x(\bar{s})) \text{ for } s \in [-\tau, 0].$$

By condition (ii) which implies that $D^+V_{\lambda_1}(\bar{s}, x(\bar{s})) \leq 0$. This contradicts inequality (6), so inequality (5) holds.

In view of inequality (6) and the condition (iii), we have

$$\begin{aligned} V_{\lambda_1}(\tau_{k+1}, x(\tau_{k+1})) &= V_{\lambda_1}(\tau_{k+1}, I_{k+1}(x(\tau_{k+1}^-)) + J_{k+1}(x(\tau_{k+1}^- - \tau))) \\ &\leq \frac{1+c_{k+1}}{2}[V_{\lambda_1}(\tau_{k+1}^-, x(\tau_{k+1}^-)) + V_{\lambda_1}(\tau_{k+1}^- - \tau, x(\tau_{k+1}^- - \tau))] \\ &\leq (1+c_{k+1})(1+c_k)a_1(\lambda_1). \end{aligned}$$

This together with inequality (3) yields

$$V_{\lambda_1}(t, x(t)) \leq Ma_1(\lambda_1).$$

From this and condition (i) we have

$$b_1(\|x(t)\|) \leq V_{\lambda_1}(t, x(t)) \leq Ma_1(\lambda_1) \leq b_1(A_1) \quad \text{for } t \geq \sigma.$$

Thus, we have $\|x(t)\| < A_1$ for $t \geq \sigma$.

Now, let $0 < \lambda_2 \leq \lambda_1$, and Choose $0 < A_2 \leq \lambda_2$ such that $a_2(A_2) < Nb_2(\lambda_2)$.

Next we claim that $\|\varphi\| > \lambda_2$ implies $\|x(t)\| > A_2$ for $t \geq \sigma$.

If this holds then $A_2 < \|x(t)\| < A_1$ for $t \geq \sigma$.

Obviously for any $t \in [\sigma - \tau, \sigma]$, there exist a $\theta \in [-\tau, 0]$, Such that

$$V_{\lambda_2}(t, x(t)) = V_{\lambda_2}(\sigma + \theta, x(\sigma + \theta)) \geq b_2(\|x(\sigma + \theta)\|) = b_2(\|x_\sigma(\theta)\|) = b_2(\|\varphi(\theta)\|) \geq b_2(\lambda_2).$$

Then we claim that

$$V_{\lambda_2}(t, x(t)) \geq b_2(\lambda_2) \quad \text{for } \sigma \leq t < \tau_k. \quad (7)$$

In fact, if inequality (7) does not hold, then there exist a $\bar{t} \in (\sigma, \tau_k)$ such that

$$V_{\lambda_2}(\bar{t}, x(\bar{t})) < b_2(\lambda_2) \leq V_{\lambda_2}(\sigma, x(\sigma)) \quad (8)$$

which implies that there is a $t_1 \in (\sigma, \bar{t})$ such that

$$D^+V_{\lambda_2}(t_1, x(t_1)) < 0 \quad (9)$$

and

$$V_{\lambda_2}(t_1 + s, x(t_1 + s)) \geq V_{\lambda_2}(t_1, x(t_1)) \quad \text{for } s \in [-\tau, 0].$$

By condition (v), which implies $D^+V_{\lambda_2}(t_1, x(t_1)) \geq 0$. This contradict inequality (9) so inequality (8) holds.

In view of inequality (7) and condition (vi), we have

$$V_{\lambda_2}(\tau_k, x(\tau_k)) = V_{\lambda_2}(\tau_k, I_k(x(\tau_k^-)) + J_k(x(\tau_k^- - \tau))) \geq \frac{1-d_k}{2}[V_{\lambda_2}(\tau_k^-, x(\tau_k^-)) + V_{\lambda_2}(\tau_k^- - \tau, x(\tau_k^- - \tau))] \geq (1-d_k)b_2(\lambda_2)$$

Next we prove that

$$V_{\lambda_2}(t, x(t)) \geq (1-d_k)b_2(\lambda_2) \quad \text{for } \tau_k \leq t \leq \tau_{k+1}. \quad (10)$$

If inequality (10) does not hold, then there is a $\hat{r} \in (\tau_k, \tau_{k+1})$ such that

$$V_{\lambda_2}(\hat{r}, x(\hat{r})) < (1-d_k)b_2(\lambda_2) \leq V_{\lambda_2}(\tau_k, x(\tau_k))$$

Which implies that there is a $\bar{r} \in (\tau_k, \hat{r})$ such that

$$D^+V_{\lambda_2}(\bar{r}, x(\bar{r})) < 0 \quad (11)$$

And

$$V_{\lambda_1}(\bar{r} + s, x(\bar{r} + s)) \leq V_{\lambda_1}(\bar{r}, x(\bar{r})) \quad \text{for } s \in [-\tau, 0].$$

By condition (v) which implies that $D^+V_{\lambda_2}(\bar{r}, x(\bar{r})) \geq 0$. This contradicts inequality (11), so inequality (10) holds.

In view of inequality (11) and the condition (vi), we have

Which implies that there is a $\bar{r} \in (\tau_k, \hat{r})$ such that

$$D^+V_{\lambda_2}(\bar{r}, x(\bar{r})) < 0 \quad (11)$$

And $V_{\lambda_1}(\bar{s} + s, x(\bar{s} + s)) \leq V_{\lambda_1}(\bar{s}, x(\bar{s}))$ for $s \in [-\tau, 0]$.

By condition (v) which implies that $D^+V_{\lambda_2}(\bar{r}, x(\bar{r})) \geq 0$. This contradicts inequality (11), so inequality (10) holds.

In view of inequality (11) and the condition (vi), we have

$$\begin{aligned} V_{\lambda_2}(\tau_{k+1}, x(\tau_{k+1})) &= V_{\lambda_2}(\tau_{k+1}, I_{k+1}(x(\tau_{k+1}^-)) + J_{k+1}(x(\tau_{k+1}^- - \tau))) \\ &\geq \frac{1-d_{k+1}}{2} [V_{\lambda_2}(\tau_{k+1}^-, x(\tau_{k+1}^-)) + V_{\lambda_2}(\tau_{k+1}^- - \tau, x(\tau_{k+1}^- - \tau))] \\ &\geq (1-d_{k+1})(1-d_k)b_2(\lambda_2). \end{aligned}$$

By simple induction, we can prove that in general that for $k=0,1,2,\dots$

$$V_{\lambda_2}(t, x(t)) \geq (1-d_{k+m})\dots(1-d_k)b_2(\lambda_2) \quad \text{for } \tau_{k+m} \leq t < \tau_{k+m+1}.$$

This together with inequality (7) yields $V_{\lambda_2}(t, x(t)) \geq Nb_2(\lambda_2)$.

From this and condition (iv) we have

$$a_2(\|x(t)\|) \geq V_{\lambda_2}(t, x(t)) \geq Nb_2(\lambda_2) > a_2(A_2) \quad \text{for } t \geq \sigma.$$

Thus, we have $\|x(t)\| > A_2$ for $t \geq \sigma$.

Thus, the trivial solution of (1) is uniformly strictly practical stable.

IV. CONCLUSION

In this paper, the strict practical stability of impulsive functional differential equations in which the state variables on the impulses are related to the time delay is considered. By using Lyapunov functions and Razumikhin technique, we have obtained some results for the strict practical stability. Strict practical stability theorem for impulsive functional differential equation have been extended to impulsive functional differential equations in which the state variables on the impulses are related to the time delay.

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