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Abstract

This paper calculates and obtains the theoretical value of EoS parameter as w = – 1 via the Feynman path integral formalism for the evolution of the quantum gravitational field without any a posteriori assumptions, and provides a plausible explanation for the potential deviations of dark energy density and EoS parameter.

Keywords: *accelerated expansion, Dark energy, dark energy density, equation of state parameter, Feynman path integral, graviton, Noncommutative quantum gravity, quantum gravity*

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1. Introduction

In the paper [1], [2], [3], [4], [5], [6], [7] and [8], we introduce the noncommutative quantum gravity and the Feynman path integral of the quantum gravitational field. In this paper, we will calculate the propagator of the gravitational field in the quantum superposition state of a two-source system via the Feynman path integral. Let the initial state gravitational field of the two-source system be prepared in a quantum superposition configuration. In the evolution of the gravitational field of the two-source system, we will obtain a virtual gravitational field. If the two gravitational sources in the final state are separated by a sufficiently large distance such that the quantum superposition effect vanishes, an auxiliary field needs to be introduced to decohere the final state from the quantum superposition state.

2. Quantum Superposition and Energy

In the paper [1] and [2] we introduce a wave packet approximate to the Dirac δ-function which can be explained as a semiclassical graviton. It can be written as follows:

$$\xi^i(x,r) = \begin{cases} \xi^r = r + C^r(x)\exp\left(-\dfrac{r}{l_P}\right) \\ \xi^\theta = \theta(x) \\ \xi^\phi = \phi(x) \\ \xi^t = t + C^t(x)\exp\left(-\dfrac{|t|}{t_P}\right) \end{cases} \tag{2.1}$$

The dynamic variables of ξ^i are $C^i(x) = \left(C^r(x), \theta(x), \phi(x), C^t(x)\right)$. Quantization only quantizes the dynamic variable $C^i(x)$. Therefore, for the sake of conciseness, in this paper, we directly identify $C^i(x)$ as the fundamental state function characterizing the gravitational field.

From the paper [1] we have the Lagrangian density of graviton is:

$$\mathcal{L} = -\frac{\eta^{\mu\nu}}{2}\frac{\partial\xi^i(x,r)}{\partial x^\mu}\frac{\partial\xi^j(x,r)}{\partial x^\nu}\eta_{ij} \tag{2.2}$$

For simplicity, assume that the initial state has two gravitational fields $C^i_{(1)}$ and $C^i_{(2)}$ and $C^i_{(2)} = k^i \cdot C^i_{(1)}$. Provided that the two gravitational sources are separated by a sufficiently large spatial interval, no quantum superposition effect can arise between $C^i_{(1)}$ and $C^i_{(2)}$, the initial state can be written as $C^i_{(1+2)} = C^i_{(1)} + C^i_{(2)}$, and the action can be written as $S_{(1+2)} = S[C^i_{(1)}] + S[C^i_{(2)}]$. Denote the final state as $\tilde{C}^i_{(1+2)}$. For the Lagrangian density [2.2], the propagator $K_{(1+2)}$ of $C^i_{(1+2)}$ can be written as follows:

$$\begin{aligned}K_{(1+2)} &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{iS_{(1+2)}/\hbar}\right)\\ &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{i\left[S[C^i_{(1)}]+S[C^i_{(2)}]\right]/\hbar}\right)\\ &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{iS[C^i_{(1)}]/\hbar}\cdot e^{iS[C^i_{(2)}]/\hbar}\right)\end{aligned} \tag{2.3}$$

If two gravitational sources are sufficiently close to each other or merge into one, quantum superposition effects will arise between $C^i_{(1)}$ and $C^i_{(2)}$. Denote the initial state of the two-source system with quantum superposition as $C^i_{(1\oplus2)}$, where $\oplus$ denotes the quantum superposition of states, the action can be written as $S_{(1\oplus2)} = S[C^i_{(1)} + C^i_{(2)}]$. Denote the final state as $\tilde{C}^i_{(1\oplus2)}$. For the Lagrangian density [2.2], the propagator $K_{(1\oplus2)}$ of $C^i_{(1\oplus2)}$ can be written as follows:

$$\begin{aligned}K_{(1\oplus2)} &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{iS_{(1\oplus2)}/\hbar}\right)\\ &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{iS[C^i_{(1)}+C^i_{(2)}]/\hbar}\right)\\ &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{iS[C^i_{(1)}]/\hbar}\right)^{(1+k^i)^2}\\ &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)}]\left(e^{iS[C^i_{(1)}]/\hbar}\cdot e^{iS[C^i_{(2)}]/\hbar}\cdot e^{iS[C^i_{(3)}]/\hbar}\right)\\ &= \frac{1+k^i}{1+k^i+\sqrt{2k^i}}\int \mathcal{D}[C^i_{(1)} + C^i_{(2)} + C^i_{(3)}]\left(e^{iS[C^i_{(1)}]/\hbar}\cdot e^{iS[C^i_{(2)}]/\hbar}\cdot e^{iS[C^i_{(3)}]/\hbar}\right)\end{aligned} \tag{2.4}$$

where the fictitious field $C^i_{(3)} = \sqrt{2k^i}\cdot C^i_{(1)}$.

The factor $\frac{1+k}{1+k+\sqrt{2k}}$, as an overall constant factor, is independent of field configurations and external sources, so it can be eliminated via functional integration when calculating all physical observables, such as correlation functions, scattering cross sections. This factor has no observable physical effects. While it may perturb vacuum fluctuations and related effects, it is not considered in this paper. Therefore, the propagator $K_{(1\oplus2)}$ associated with the two-source system exhibiting quantum superposition is equivalent to the propagator $K_{(1+2+3)}$ corresponding to the three-source system in the absence of quantum superposition:

$$\begin{aligned}K_{(1\oplus2)} &= K_{(1+2+3)}\\ &= \int \mathcal{D}[C^i_{(1)} + C^i_{(2)} + C^i_{(3)}]\left(e^{iS[C^i_{(1)}]/\hbar}\cdot e^{iS[C^i_{(2)}]/\hbar}\cdot e^{iS[C^i_{(3)}]/\hbar}\right)\end{aligned} \tag{2.5}$$

By comparing Eq. [2.3] and Eq. [2.4], we can see that $K_{(1+2)} \neq K_{(1\oplus2)}$. This indicates the emergence of a new physical degree of freedom.

Eq. [2.5] implies that the dynamical evolution of a two-source system $C^i_{(1\oplus2)}$ prepared in a quantum superposition state is formally equivalent to that of a three-source system $C^i_{(1+2+3)}$ without quantum superposition.

In the final state $\tilde{C}^i_{(1+2)}$, provided that the two gravitational sources are separated by a sufficiently large spatial distance, the quantum superposition between the final state $\tilde{C}^i_{(1)}$ and the final state $\tilde{C}^i_{(2)}$ is completely decohered. So it is necessary to introduce the field $\mathcal{C}^i$ to decohere the final state $\tilde{C}^i_{(1\oplus2)}$ out of quantum superposition into the final state $\tilde{C}^i_{(1+2)}$:

$$\begin{aligned}\tilde{C}^i_{(1+2)} &= \tilde{C}^i_{(1\oplus2)} + \mathcal{C}^i\\ &= \tilde{C}^i_{(1\oplus2)} - C^i_{(3)}\\ &= \tilde{C}^i_{(1+2+3)} - \sqrt{2k^i}C^i_{(1)}\end{aligned} \tag{2.6}$$

where $\mathcal{C}^i = -C^i_{(3)} = -\sqrt{2k^i}C^i_{(1)}$.

Apparently, the energy-momentum tensor of the field $\mathcal{C}^i$ is equal in magnitude to that of the fictitious field $C^i_{(3)}$ but opposite in algebraic sign, so the pressure P and the energy density ρ of the field $\mathcal{C}^i$ is negative: $P < 0,\ \rho < 0$. Assuming that $\mathcal{C}^i$ is a homogeneous and isotropic perfect fluid, the energy-momentum tensor of $\mathcal{C}^i$ can be written as follows:

$$T_{\mu\nu} = \begin{pmatrix} P & & & \\ & P & & \\ & & P & \\ & & & -\rho \end{pmatrix} \tag{2.7}$$

The trace of this energy-momentum tensor is equal to 0:

$$T = \mathrm{tr}\, T_{\mu\nu} = 0 \tag{2.8}$$

Using natural units, the second Friedmann equation can be written as:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(3P + \rho) \tag{2.9}$$

Substitute the pressure and the energy density of the field $\mathcal{C}^i$ into Eq. [2.9], we have:

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(3P + \rho) \\ &= -\frac{4\pi G}{3}(T + 2\rho) \\ &= -\frac{4\pi G}{3}(2\rho) \end{aligned} \tag{2.10}$$

Since the trace T vanishes and the energy density ρ is negative, the right-hand side of the second Friedmann equation is greater than 0. Therefore, the field $\mathcal{C}^i$ supports the accelerated expansion.

Let us now examine the accelerated expansion from the perspective of contemporary dark energy theories. In the Λ-CDM standard model, the energy-momentum tensor of dark energy can be written as:

$$T_{\mu\nu} = \begin{pmatrix} P_{(DE)} & & & \\ & P_{(DE)} & & \\ & & P_{(DE)} & \\ & & & -\rho_{(DE)} \end{pmatrix} \tag{2.11}$$

where $P_{(DE)}$ and $\rho_{(DE)}$ denote the pressure and energy density of dark energy, respectively. The second Friedmann equation in the Λ-CDM standard model can be written as:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(3P_{(DE)} + \rho_{(DE)}) \tag{2.12}$$

In the Λ-CDM standard model, we preset a positive dark energy density: $\rho_{(DE)} > 0$. Therefore, to ensure that the right-hand side of Eq. [2.12] is equal to the right-hand side of Eq. [2.10], the pressure $P_{(DE)}$ is required to be negative and $P_{(DE)} = -\rho_{(DE)}$. The second Friedmann equation can be written as:

$$\begin{aligned} \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}\big(3P_{(DE)} + \rho_{(DE)}\big) \\ &= -\frac{4\pi G}{3}\big(2P_{(DE)}\big) \\ &= -\frac{4\pi G}{3}\big(-2\rho_{(DE)}\big) \end{aligned} \tag{2.13}$$

Then the EoS parameter in the Λ-CDM standard model is

$$w = \frac{P_{(DE)}}{\rho_{(DE)}} = -1 \tag{2.14}$$

Apparently, within the framework of noncommutative quantum gravity theory, there is no need to introduce any a posteriori assumptions, and the EoS parameter $w = -1$ that previously required artificial predefinition in dark energy theories can be directly derived from first principles.

In the model of the two-source system $C^i_{(1\oplus 2)}$, if the decoherence process acting on the quantum superposition in the final state $\tilde{C}^i_{(1\oplus 2)}$ is incomplete, the energy density associated with the field $\mathcal{C}^i$ will manifest a deviation from its theoretically predicted baseline value. In regions where the quantum effects of the gravitational field are strong, the trace anomaly $T \neq 0$ will modify the value on the right-hand side of Eq.[2.10] . These also offers a theoretically consistent interpretation for the potential deviations of energy density and EoS parameter in dark energy observational studies.

3. Conclusion

Given that the Lagrangian density for the graviton is explicitly formulated within the framework of non-commutative quantum gravity, the Feynman path integral formalism can be employed to evaluate the dynamical evolution of the gravitational field. If there exists quantum superposition between the gravitational fields in the initial state, while no quantum superposition is present in the gravitational fields of the final state, then we need to introduce an auxiliary field $\mathcal{C}^i$ to decohere the final state from its quantum superposition. This field $\mathcal{C}^i$ supports the accelerated expansion. The EoS parameter of dark energy can be derived without invoking any a posteriori assumptions.

REFERENCES

[1] A New Approach to Quantum Gravity. DOI: https://doi.org/10.34257/GJSFRAVOL20IS2PG1

[2] Noncommutative Quantum Gravity and Symmetry of Klein-Gordon Equation. DOI: https://doi.org/10.34257/GJSFRAVOL22IS7PG19

[3] Noncommutative Quantum Gravity and Dark Matter. DOI: https://doi.org/10.34257/GJSFRAVOL23IS4PG7

[4] Macroscopic Effect of Quantum Gravity in General Static Isotropic Gravitational Field. DOI: https://doi.org/10.34257/GJSFRAVOL24IS2PG49

[5] Calculation of Macroscopic Effect of Quantum Gravity in General Static Isotropic Gravitational Field. DOI: https://doi.org/10.34257/GJSFRAVOL24IS4PG75

[6] The Role of Noncommutative Quantum Gravity in Galactic Dynamics and Dark Matter Phenomena. DOI: https://doi.org/10.34257/GJSFRAVOL25IS2PG13

[7] Quantum Superposition and the Emergence of Negative Energy in Gravitational Fields. DOI: https://doi.org/10.34257/GJSFRAVOL25IS5PG19

[8] Quantum Superposition Effect of Gravitational Field, Negative Pressure and Dark Energy. DOI: https://doi.org/10.34257/GJSFRA257434