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Color of Flavor of Leptons

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Abstract- This paper suggests the possible existence of lepton color $(l_R, l_G, l_B), l_{RGB}$ for charged leptons and neutral leptons as quarks possess $(q_R, q_G, q_B), q_{RGB}$. Further lepton number conservation could be explained by lepton color scalar products. More details are given to search for the relationship between the broken lepton number and PMNS. And a speculation between broken lepton number and dark matter-energy.

Keywords: flavor similarity, color similarity, lepton number conservation, Pontecorvo-Maki-Nakagawa-Sakata Matrix PMNS, lepton color scalar product, lepton color brokenon $\vec{\pi}_\beta$, neutrino lepton number $L(v^\dagger v)$, broken neutrino lepton number $L_{\text{Broken}}(v)$, neutrino mass lepton number $L_{\text{Mass}}(v)$, neutrino lepton coupling constant $G_L(v)$, neutrino background lepton number $L_{\text{Background}}(v)$, dark matter-energy $E_{\text{DME}}(v)$.

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This paper suggests the possible existence of lepton color $(l_R, l_G, l_B), l_{RGB}$ for charged leptons and neutral leptons as quarks possess $(q_R, q_G, q_B), q_{RGB}$. Further lepton number conservation could be explained by *leptoncolor scalar products*. More details are given to search for the relationship between the broken lepton number and PMNS. And a speculation between broken lepton number and dark matter-energy.

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0. INTRODUCTION

• In particle physics, six leptons and six antileptons are labelled by a group of unified three symbols L_e, L_μ, L_τ (00.1), which are their identity quantum numbers in particle community.

$$L_e, L_\mu, L_\tau \quad (00.1)$$

The values of L_e, L_μ , and L_τ are illustrated in Table1 below, in which there are all together 36 positions filled with integers +1, -1 and zero.

Experimental scientists discovery: L_e, L_μ , and L_τ are conserved separately in weak interactions for leptons and their antileptons. especialty the sum $L = L_e + L_\mu + L_\tau$ are conserved too, which are along with the horizontal direction in Table1 [1]

Theoretical scientists wonder: Is there an unified math approach which could explain *Where The values, integers +1, -1 and zero, of L_e, L_μ, L_τ of leptons and of antileptons, that appear in Table1, come from ?*

• The experimental searchers for neutrino oscilations, they obtain Pontecorvo-Maki-Nakagawa-Sakata Matrix, PMNS $(V_L^e)^\dagger V_L^\nu$ [2], which is the extremely pectfect achievement from their spirits of awe and adventure. long time persistent effor in analysing the data collected from weak interaction. There are many profound digits appear in the nine elements of PMNS that given in Matrix(1) and Matrix(2) below.

Theoretical searchers wonder: the origin of these mysterious digits, that occupy the elements of these matrices, *where the odd digit arrangements come from ?*

Table 1: The values of lepton numbers

Particle	Lepton number L		Electron Lepton number L_e	Muon Lepton number L_μ	Tau Lepton number L_τ
ν_e	+1		+1	0	0
e^-	+1		+1	0	0
ν_μ	+1		0	+1	0
μ^-	+1		0	+1	0
ν_τ	+1		0	0	+1
τ^-	+1		0	0	+1
$\bar{\nu}_e$	-1		-1	0	0
e^+	-1		-1	0	0
$\bar{\nu}_\mu$	-1		0	-1	0
μ^+	-1		0	-1	0
$\bar{\nu}_\tau$	-1		0	0	-1
τ^+	-1		0	0	-1
Anti Particle	Lepton number L		Electron Lepton number L_e	Muon Lepton number L_μ	Tau Lepton number L_τ



- $$|U^{PMNS}| = \begin{pmatrix} 0.797 \rightarrow 0.842 & 0.518 \rightarrow 0.585 & 0.143 \rightarrow 0.156 \\ 0.243 \rightarrow 0.490 & 0.473 \rightarrow 0.674 & 0.651 \rightarrow 0.772 \\ 0.295 \rightarrow 0.525 & 0.493 \rightarrow 0.688 & 0.618 \rightarrow 0.744 \end{pmatrix}$$

Matrix(1)

" The U^{PMNS} looks very different from that of CKM, which is very diagonal. The leptonic sector is characterized by a very large degree of mixing. As of today, the origin of this pattern remains an unsolved and profound puzzle! ". André Rubbia [3]

And

- $$|V_{PMES}| = \begin{pmatrix} 0.801 - 0.845 & 0.513 - 0.579 & 0.143 - 0.155 \\ 0.234 - 0.500 & 0.471 - 0.689 & 0.637 - 0.776 \\ 0.271 - 0.525 & 0.477 - 0.694 & 0.613 - 0.756 \end{pmatrix}$$

Matrix(2)

" As of today, assuming the unitary of the matrix and three active neutrinos, the absolute values of the elements of the matrix are estimated from a global fit of all measurements, which gives (Esteban et al., 2020) " PASCAL PAGANINI [4]

And

- $$U_{PMNS} = \begin{pmatrix} \bullet & \bullet & \cdot \\ \cdot & \bullet & \bullet \\ \cdot & \bullet & \bullet \end{pmatrix} = (V_{CKM}) \cdot P, \quad P = \text{diag}\left(e^{\frac{i\alpha_1}{2}}, e^{\frac{i\alpha_2}{2}}, 1\right); \quad V_{CKM} = \begin{pmatrix} \bullet & \cdot & \cdot \\ \cdot & \bullet & \cdot \\ \cdot & \cdot & \bullet \end{pmatrix}$$

Matrix(3)
Matrix(0)

The magnitudes of the element digits of PMNS in Matrix(1) and Matrix(2) could be graphical approached by the circle areas in Matrix(3) qualitatively above [5]. Andrzej J. Buras:

" The explanation of these patterns and in particular why they differ from each other remains an important goal of the theorists. " and " Why the mixing angles of CKM matrix so small and those of PMNS matrix rather large? "

1. COLOR OF FLAVOR OF LEPTONS

This paper attempts to explore and answer the above two " *come from ?* " questions. So two concepts of • *Color Similarity* and • *lepton color* are introduced following

Current particle physics shows an amazing *Flovor Similarity* between three generations of quark and those of leptons below

$$\text{six quarks } \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \Leftrightarrow \text{six leptons } \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \quad (0.00)$$

$$\text{as well antiquarks } \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}, \begin{pmatrix} \bar{c} \\ \bar{s} \end{pmatrix}, \begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix} \Leftrightarrow \text{antileptons } \begin{pmatrix} \bar{\nu}_e \\ e^+ \end{pmatrix}, \begin{pmatrix} \bar{\nu}_\mu \\ \mu^+ \end{pmatrix}, \begin{pmatrix} \bar{\nu}_\tau \\ \tau^+ \end{pmatrix} \quad (0.00)$$

Since six quarks and six antiquarks have been unified by means of " fundamental color representation of six flavor t, c, u, d, s, b of quarks and their six antiquarks " [6,7,8,9,10], And in this idea, why not continues to have a try to do the six flavor $\nu_\tau, \nu_\mu, \nu_e, e^-, \mu^-, \tau^-$ of leptons and the six antileptons ? That is, besides *Flovor Similarity* (0.00), there may be

$$\bullet \quad \text{Color Similarity} \quad \text{quark } (q_R, q_G, q_B), q_{\text{RGB}} \Leftrightarrow \text{lepton } (l_R, l_G, l_B), l_{\text{RGB}} \quad (\widetilde{0.00})$$

Base on $(\widetilde{0.00})$, This paper postulates an **ansatz**, If *the six fermion leptons & their antileptons* (0.00) *also are attributed to color degree of freedom* as six fermion quarks & their antiquarks were done in Stand Model historically, and then the third component $I_3(l)$ isospin of a lepton l could be written by • *lepton color* $l_{\text{RGB}}, (l_R, l_G, l_B)$ as below

$$I_3(l) = \frac{1}{3} (l_R + l_G + l_B) \equiv I_3(l_{\text{RGB}}) \quad (0.0)$$

Or details

$$\vec{u} = (u_R, u_G, u_B) \Rightarrow \vec{v}_e = (v_{eR}, v_{eG}, v_{eB}) = \left(\frac{-1}{2}, \frac{+1}{2}, \frac{+3}{2} \right), \quad I_3(v_e) = \frac{1}{3} \left(\frac{-1}{2} + \frac{+1}{2} + \frac{+3}{2} \right) = \frac{+1}{2} \quad (0.1)$$

$$\vec{d} = (d_R, d_G, d_B) \Rightarrow \vec{e}^- = (e_{eR}^-, e_{eG}^-, e_{eB}^-) = \left(\frac{-3}{2}, \frac{-1}{2}, \frac{+1}{2} \right), \quad I_3(e^-) = \frac{1}{3} \left(\frac{-3}{2} + \frac{-1}{2} + \frac{+1}{2} \right) = \frac{-1}{2} \quad (0.2)$$

$$\vec{c} = (c_R, c_G, c_B) \Rightarrow \vec{v}_\mu = (v_{\mu R}, v_{\mu G}, v_{\mu B}) = \left(\frac{+1}{2}, \frac{+3}{2}, \frac{+5}{2} \right), \quad I_3(v_\mu) = \frac{1}{3} \left(\frac{+1}{2} + \frac{+3}{2} + \frac{+5}{2} \right) = \frac{+3}{2} \quad (0.3)$$

$$\vec{s} = (s_R, s_G, s_B) \Rightarrow \vec{\mu}^- = (\mu_{\mu R}^-, \mu_{\mu G}^-, \mu_{\mu B}^-) = \left(\frac{-5}{2}, \frac{-3}{2}, \frac{-1}{2} \right), \quad I_3(\mu^-) = \frac{1}{3} \left(\frac{-5}{2} + \frac{-3}{2} + \frac{-1}{2} \right) = \frac{-3}{2} \quad (0.4)$$

$$\vec{t} = (t_R, t_G, t_B) \Rightarrow \vec{v}_\tau = (v_{\tau R}, v_{\tau G}, v_{\tau B}) = \left(\frac{+3}{2}, \frac{+5}{2}, \frac{+7}{2} \right), \quad I_3(v_\tau) = \frac{1}{3} \left(\frac{+3}{2} + \frac{+5}{2} + \frac{+7}{2} \right) = \frac{+5}{2} \quad (0.5)$$

$$\vec{b} = (b_R, b_G, b_B) \Rightarrow \vec{\tau}^- = (\tau_{\tau R}^-, \tau_{\tau G}^-, \tau_{\tau B}^-) = \left(\frac{-7}{2}, \frac{-5}{2}, \frac{-3}{2} \right), \quad I_3(\tau^-) = \frac{1}{3} \left(\frac{-7}{2} + \frac{-5}{2} + \frac{-3}{2} \right) = \frac{-5}{2} \quad (0.6)$$

Further we, very naturally, could obtain a universal color representation of leptons and antileptons below, which is the analogous to that of quarks [7]. And isospin $I_3(l_{\text{RGB}})$ (0.0) is an underlying classifications of three generation of leptons.

Table 2: Fundamental Color Representation of flavor $\nu_\tau, \nu_\mu, \nu_e, e^-, \mu^-, \tau^-$ of leptonss and their antileptons

lepton flavor r	ν_τ		ν_μ		ν_e		e^-		μ^-		τ^-
$I_3(l_\alpha)$	$I_3(\nu_\tau) \frac{+5}{2}$		$I_3(\nu_\mu) \frac{+3}{2}$		$I_3(\nu_e) \frac{+1}{2}$		$I_3(e^-) \frac{-1}{2}$		$I_3(\mu^-) \frac{-3}{2}$		$I_3(\tau^-) \frac{-5}{2}$
l_α	$\nu_{\tau R} \quad \nu_{\tau G} \quad \nu_{\tau B}$		$\nu_{\mu R} \quad \nu_{\mu G} \quad \nu_{\mu B}$		$\nu_{e R} \quad \nu_{e G} \quad \nu_{e B}$		$e_R^- \quad e_G^- \quad e_B^-$		$\mu_R^- \quad \mu_G^- \quad \mu_B^-$		$\tau_R^- \quad \tau_G^- \quad \tau_B^-$
l_{RGB}	$\frac{+3}{2} \quad \frac{+5}{2} \quad \frac{+7}{2}$		$\frac{+1}{2} \quad \frac{+3}{2} \quad \frac{+5}{2}$		$\frac{-1}{2} \quad \frac{+1}{2} \quad \frac{+3}{2}$		$\frac{-3}{2} \quad \frac{-1}{2} \quad \frac{+1}{2}$		$\frac{-5}{2} \quad \frac{-3}{2} \quad \frac{-1}{2}$		$\frac{-7}{2} \quad \frac{-5}{2} \quad \frac{-3}{2}$
	$\nu_{\tau R} + \nu_{\tau G} + \nu_{\tau B}$		$\nu_{\mu R} + \nu_{\mu G} + \nu_{\mu B}$		$\nu_{e R} + \nu_{e G} + \nu_{e B}$		$e_R^- + e_G^- + e_B^-$		$\mu_R^- + \mu_G^- + \mu_B^-$		$\tau_R^- + \tau_G^- + \tau_B^-$
	$\frac{+15}{2}$		$\frac{+9}{2}$		$\frac{+3}{2}$		$\frac{-3}{2}$		$\frac{-9}{2}$		$\frac{-15}{2}$
•	—————	•	—————	•	—————	•	—————	•	—————	•	—————
anti- lepton											
flavor $\bar{r}$	$\bar{\nu}_\tau$		$\bar{\nu}_\mu$		$\bar{\nu}_e$		e^+		μ^+		τ^+
$I_3(\bar{l}_\alpha)$	$I_3(\bar{\nu}_\tau) \frac{-5}{2}$		$I_3(\bar{\nu}_\mu) \frac{-3}{2}$		$I_3(\bar{\nu}_e) \frac{-1}{2}$		$I_3(e^+) \frac{+1}{2}$		$I_3(\mu^+) \frac{+3}{2}$		$I_3(\tau^+) \frac{+5}{2}$
$\bar{l}_\alpha$	$\bar{\nu}_{\tau R} \quad \bar{\nu}_{\tau G} \quad \bar{\nu}_{\tau B}$		$\bar{\nu}_{\mu R} \quad \bar{\nu}_{\mu G} \quad \bar{\nu}_{\mu B}$		$\bar{\nu}_{e R} \quad \bar{\nu}_{e G} \quad \bar{\nu}_{e B}$		$e_R^+ \quad e_G^+ \quad e_B^+$		$\mu_R^+ \quad \mu_G^+ \quad \mu_B^+$		$\tau_R^+ \quad \tau_G^+ \quad \tau_B^+$
$\bar{l}_{RGB}$	$\frac{-3}{2} \quad \frac{-5}{2} \quad \frac{-7}{2}$		$\frac{-1}{2} \quad \frac{-3}{2} \quad \frac{-5}{2}$		$\frac{+1}{2} \quad \frac{-1}{2} \quad \frac{-3}{2}$		$\frac{+3}{2} \quad \frac{+1}{2} \quad \frac{-1}{2}$		$\frac{+5}{2} \quad \frac{+3}{2} \quad \frac{+1}{2}$		$\frac{+7}{2} \quad \frac{+5}{2} \quad \frac{+3}{2}$
	$\bar{\nu}_{\tau R} + \bar{\nu}_{\tau G} + \bar{\nu}_{\tau B}$		$\bar{\nu}_{\mu R} + \bar{\nu}_{\mu G} + \bar{\nu}_{\mu B}$		$\bar{\nu}_{e R} + \bar{\nu}_{e G} + \bar{\nu}_{e B}$		$e_R^+ + e_G^+ + e_B^+$		$\mu_R^+ + \mu_G^+ + \mu_B^+$		$\tau_R^+ + \tau_G^+ + \tau_B^+$
	$\frac{-15}{2}$		$\frac{-9}{2}$		$\frac{-3}{2}$		$\frac{+9}{6} = \frac{+3}{2}$		$\frac{+27}{6} = \frac{+9}{2}$		$\frac{+45}{6} = \frac{+15}{2}$

• Remark: we see the color values l_R, l_G, l_B of each lepton in Table2 are selected from the third components $\pi_3(l)$ of one-half spin $\vec{\pi}(l)$ below

$$\pi_3(l) = \dots, \frac{-9}{2}, \frac{-7}{2}, \frac{-5}{2}, \frac{-3}{2}, \frac{-1}{2}, \frac{+1}{2}, \frac{+3}{2}, \frac{+5}{2}, \frac{+7}{2}, \frac{+9}{2}, \dots \subseteq l_{\text{RGB}} \equiv l_R, l_G, l_B \quad (0.7)$$

$$\vec{\pi}(l) \times \vec{\pi}(l) = i\vec{\pi}(l) \quad (0.8)$$

Comparability between color freedom of quarks [1] and that of leptons above:

Quark are closed-color, color confinement, quark color $q_\alpha = q_{\text{RGB}} \subseteq$ one-sixth series [7]

Lepton are opened-color, color freer, lepton color $l_\alpha = l_{\text{RGB}} \subseteq$ one-half series (0.7).

Both quark $\pi_3(q)$ and lepton $\pi_3(l)$ are the third components of Non-Hermitian spin angular momentum $\vec{\pi}$ in Spin Topological Space STS.[11]. $\pi_3(q)$ and $\pi_3(l)$ are Hermitian operators in STS.

This paper are particular about neutrino lepton numbers, further Table1 is down to Table3 below

Table 3: Lepton Numbers of Neutrinos ν_e, ν_μ, ν_τ

Particle	Lepton number L		Electron Lepton number L_e	Muon Lepton number L_μ	Tau Lepton number L_τ
ν_e	+1		+1	0	0
ν_μ	+1		0	+1	0
ν_τ	+1		0	0	+1

2. COLOR SCALAR PRODUCTS $L(v^\dagger v')$ OF NEUTRINO LEPTON NUMBER

Inspire with the idea of quarkcolor scalar product [10] and base on **ansatz**, *Color Similarity* mentioned previously, further an amusing *leptoncolor scalar product* is introduced to essential of lepton numbers L_e, L_μ, L_τ of six leptons and six antileptons in Table1. This paper lays emphases on neutral leptons, neutrinos, then the *leptoncolor scalar product* representations of Table 3 are given on details following (0)

$$L(v^\dagger v') = \overrightarrow{v^\dagger} \cdot \overrightarrow{v'} = \begin{pmatrix} \overrightarrow{v_e^\dagger} \cdot \overrightarrow{v_e} & \overrightarrow{v_e^\dagger} \cdot \overrightarrow{v_\mu} & \overrightarrow{v_e^\dagger} \cdot \overrightarrow{v_\tau} \\ \overrightarrow{v_\mu^\dagger} \cdot \overrightarrow{v_e} & \overrightarrow{v_\mu^\dagger} \cdot \overrightarrow{v_\mu} & \overrightarrow{v_\mu^\dagger} \cdot \overrightarrow{v_\tau} \\ \overrightarrow{v_\tau^\dagger} \cdot \overrightarrow{v_e} & \overrightarrow{v_\tau^\dagger} \cdot \overrightarrow{v_\mu} & \overrightarrow{v_\tau^\dagger} \cdot \overrightarrow{v_\tau} \end{pmatrix} = \begin{pmatrix} +1.000 & 0.000 & 0.000 \\ 0.000 & +1.000 & 0.000 \\ 0.000 & 0.000 & +1.000 \end{pmatrix} = \begin{pmatrix} \mathbf{[A]} & \mathbf{[D]} & \mathbf{[F]} \\ \mathbf{[E]} & \mathbf{[B]} & \mathbf{[H]} \\ \mathbf{[G]} & \mathbf{[I]} & \mathbf{[C]} \end{pmatrix} \quad (0)$$

Matrix(4) Matrix(5)

Matrix(5) is the digit graph of lepton number conservation (Table1).

From now on, we discuss the case of the left handed neutrino family $v = v_L$.

Example V_{11} of $\overrightarrow{v_e^\dagger} \cdot \overrightarrow{v_e} = +1.000$ in expression (0) with Table3 below (left handed neutrino v_L) is given explicitly following

$$\overrightarrow{v_e^\dagger} = \overrightarrow{v_e} + \overrightarrow{\Phi_{v_e^\dagger}} \quad (1)$$

$$\overrightarrow{v_e} = \overrightarrow{v_e} + \overrightarrow{\Phi_{v_e}} \quad (2)$$

and *lepton weak interaction paring* $\overrightarrow{\Phi}$

$$\begin{aligned} \overrightarrow{\Phi_{v_e^\dagger}} &= \left(\frac{+4}{6}, \frac{+1}{6}, \frac{-5}{6} \right), & I_3(\Phi_{v_e^\dagger}) &= \frac{1}{3} \left(\frac{+4}{6} + \frac{+1}{6} + \frac{-5}{6} \right) = 0 \\ \overrightarrow{\Phi_{v_e}} &= \left(\frac{+1}{2}, \frac{-1}{2}, \frac{0}{2} \right), & I_3(\Phi_{v_e}) &= \frac{1}{3} \left(\frac{+1}{2} + \frac{-1}{2} + \frac{0}{2} \right) = 0 \\ \overrightarrow{\Phi_{v_e^\dagger}} \cdot \overrightarrow{\Phi_{v_e}} &= \left(\frac{+4}{6}, \frac{+1}{6}, \frac{-5}{6} \right) \left(\frac{+1}{2}, \frac{-1}{2}, \frac{0}{2} \right) = \frac{+1}{4} \end{aligned} \quad (3)$$

Further

$$\begin{aligned}\vec{v}_e^{\dagger'} &= \vec{v}_e + \vec{\Phi}_{v_e^{\dagger}} = \left(\frac{-1}{2}, \frac{+1}{2}, \frac{+3}{2}\right) + \left(\frac{+4}{6}, \frac{+1}{6}, \frac{-5}{6}\right) = \left(\frac{+1}{6}, \frac{+4}{6}, \frac{+4}{6}\right) \\ I_3(v_e^{\dagger'}) &= \frac{1}{3} \left(\frac{+1}{6} + \frac{+4}{6} + \frac{+4}{6}\right) = \frac{1}{3} \left(\frac{+9}{6}\right) = \frac{+1}{2}\end{aligned}\quad (4)$$

$$\begin{aligned}\vec{v}_e^{\dagger} &= \vec{v}_e + \vec{\Phi}_{v_e} = \left(\frac{-1}{2}, \frac{+1}{2}, \frac{+3}{2}\right) + \left(\frac{+1}{2}, \frac{-1}{2}, \frac{0}{2}\right) = \left(\frac{0}{6}, \frac{0}{6}, \frac{+9}{6}\right) \\ I_3(v_e^{\dagger}) &= \frac{1}{3} \left(\frac{0}{6} + \frac{0}{6} + \frac{+9}{6}\right) = \frac{1}{3} \left(\frac{+9}{6}\right) = \frac{+1}{2}\end{aligned}\quad (5)$$

Finally

$$\begin{aligned}\vec{v}_e^{\dagger'} \cdot \vec{v}_e^{\dagger} &= (\vec{v}_e + \vec{\Phi}_{v_e^{\dagger}}) \cdot (\vec{v}_e + \vec{\Phi}_{v_e}) = \left(\frac{+1}{6}, \frac{+4}{6}, \frac{+4}{6}\right) \cdot \left(\frac{0}{6}, \frac{0}{6}, \frac{+9}{6}\right) \\ &= \frac{1}{36} \{ 0 + 0 + 36 \} = \frac{1}{36} \{ +36 \} = +1.000\end{aligned}\quad (6)$$

The processes of the eight other elements in expression (0) could be obtained as the same way (as notation "♦") following:

$$\bullet \bullet \quad \vec{v}_\alpha^{\dagger'} \cdot \vec{v}_e^{\dagger} = \begin{pmatrix} \vec{v}_e^{\dagger'} \cdot \vec{v}_e^{\dagger} & \text{[A]} \\ \vec{v}_\mu^{\dagger'} \cdot \vec{v}_e^{\dagger} & \text{[E]} \\ \vec{v}_\tau^{\dagger'} \cdot \vec{v}_e^{\dagger} & \text{[G]} \end{pmatrix} = \begin{pmatrix} \frac{+1}{6} & \frac{+4}{6} & \frac{+4}{6} \\ \frac{+12}{6} & \frac{+15}{6} & \frac{0}{6} \\ \frac{+21}{6} & \frac{+24}{6} & \frac{0}{6} \end{pmatrix} \begin{pmatrix} \frac{0}{6} \\ \frac{0}{6} \\ \frac{+9}{6} \end{pmatrix} = \begin{pmatrix} \frac{+36}{36} = 1.000 \\ 0.000 \\ 0.000 \end{pmatrix} \quad (7)$$

$$\bullet \bullet \quad \vec{v}_\alpha^{\dagger'} \cdot \vec{v}_\mu^{\dagger} = \begin{pmatrix} \vec{v}_e^{\dagger'} \cdot \vec{v}_\mu^{\dagger} & \text{[D]} \\ \vec{v}_\mu^{\dagger'} \cdot \vec{v}_\mu^{\dagger} & \text{[B]} \\ \vec{v}_\tau^{\dagger'} \cdot \vec{v}_\mu^{\dagger} & \text{[I]} \end{pmatrix} = \begin{pmatrix} \frac{+6}{6} & \frac{+9}{6} & \frac{-6}{6} \\ \frac{+19}{6} & \frac{+22}{6} & \frac{-14}{6} \\ \frac{+36}{6} & \frac{+39}{6} & \frac{-30}{6} \end{pmatrix} \begin{pmatrix} \frac{+6}{6} \\ \frac{+6}{6} \\ \frac{+15}{6} \end{pmatrix} = \begin{pmatrix} 0.000 \\ \frac{+36}{36} = 1.000 \\ 0.000 \end{pmatrix} \quad (8)$$

$$\bullet \bullet \quad \vec{v}_\alpha^{\dagger'} \cdot \vec{v}_\tau^{\dagger} = \begin{pmatrix} \vec{v}_e^{\dagger'} \cdot \vec{v}_\tau^{\dagger} & \text{[F]} \\ \vec{v}_\mu^{\dagger'} \cdot \vec{v}_\tau^{\dagger} & \text{[H]} \\ \vec{v}_\tau^{\dagger'} \cdot \vec{v}_\tau^{\dagger} & \text{[C]} \end{pmatrix} = \begin{pmatrix} \frac{+9}{6} & \frac{+12}{6} & \frac{-12}{6} \\ \frac{+30}{6} & \frac{+33}{6} & \frac{-36}{6} \\ \frac{+49}{6} & \frac{+52}{6} & \frac{-56}{6} \end{pmatrix} \begin{pmatrix} \frac{+12}{6} \\ \frac{+12}{6} \\ \frac{+21}{6} \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.000 \\ \frac{+36}{36} = 1.000 \end{pmatrix} \quad (9)$$

3. COLOR SCALAR PRODUCTS $\vec{v}^{\dagger} \cdot \vec{n}$ OF BROKEN NEUTRINO LEPTON NUMBER $L_{\text{BROKEN}}(v)$

- Definition of Broken Neutrino Lepton Number

$$L_{\text{Broken}}(v) = \vec{v}_\alpha^{\dagger} \cdot \vec{n}_\beta = \begin{pmatrix} \vec{v}_e^{\dagger} \cdot \vec{n}_e & \vec{v}_e^{\dagger} \cdot \vec{n}_\mu & \vec{v}_e^{\dagger} \cdot \vec{n}_\tau \\ \vec{v}_\mu^{\dagger} \cdot \vec{n}_e & \vec{v}_\mu^{\dagger} \cdot \vec{n}_\mu & \vec{v}_\mu^{\dagger} \cdot \vec{n}_\tau \\ \vec{v}_\tau^{\dagger} \cdot \vec{n}_e & \vec{v}_\tau^{\dagger} \cdot \vec{n}_\mu & \vec{v}_\tau^{\dagger} \cdot \vec{n}_\tau \end{pmatrix}, \quad \alpha, \beta = e, \mu, \tau \quad (10)$$

Where below

- *Leptoncolor brokenon* $\vec{n}_\beta$ of lepton weak interaction paring $\vec{\Phi}$ in neutrino flavor oscillations are given following

$$\vec{n}_\beta = (n_1, n_2, n_3)_\beta, \quad \beta = e, \mu, \tau \quad (11)$$

OR

$$\vec{n}_e = (n_1, n_2, n_3)_e = \left(\frac{-12}{26}, \frac{+12}{26}, \frac{-27}{26} \right) \quad (11.1)$$

$$\vec{n}_\mu = (n_1, n_2, n_3)_\mu = \left(\frac{-5530}{3120}, \frac{-886}{3120}, \frac{-7987}{3120} \right) \quad (11.2)$$

$$\vec{n}_\tau = (n_1, n_2, n_3)_\tau = \left(\frac{-292}{120}, \frac{-328}{120}, \frac{-553}{120} \right) \quad (11.3)$$

Using Table2 and (11.1),(11.2),(11.3), the nine elements in expression (10) could be obtained following:

An example of color scalar products with $\vec{v}_\alpha^{\dagger} \cdot \vec{n}_e$, $\vec{v}_\mu^{\dagger} \cdot \vec{n}_e$, $\vec{v}_\tau^{\dagger} \cdot \vec{n}_e$ (12.1)♦ are given below

$$V_{11} \vec{v}_e^{\dagger} \cdot \vec{n}_e = \left(\frac{+1}{6}, \frac{+4}{6}, \frac{+4}{6} \right) \left(\frac{-12}{26}, \frac{+12}{26}, \frac{-27}{26} \right) = \frac{1}{156} (-12 + 48 - 108) = \frac{-72}{156} = \frac{-12}{26} = \frac{-120}{260}$$

$$V_{21} \vec{v}_\mu^{\dagger} \cdot \vec{n}_e = \left(\frac{+12}{6}, \frac{+15}{6}, \frac{0}{6} \right) \left(\frac{-12}{26}, \frac{+12}{26}, \frac{-27}{26} \right) = \frac{1}{156} (-144 + 180 + 0) = \frac{+36}{156} = \frac{+6}{26} = \frac{+60}{260}$$

$$V_{31} \vec{v}_\tau^{\dagger} \cdot \vec{n}_e = \left(\frac{+21}{6}, \frac{+24}{6}, \frac{0}{6} \right) \left(\frac{-12}{26}, \frac{+12}{26}, \frac{-27}{26} \right) = \frac{1}{156} (-252 + 288 + 0) = \frac{+36}{156} = \frac{+6}{26} = \frac{+60}{260}$$

As the same way (as notation "♦")

$$\bullet \bullet \quad \vec{v}_\alpha^\dagger \cdot \vec{n}_e = \begin{pmatrix} \vec{v}_e^\dagger \cdot \vec{n}_e & \text{【A】} \\ \vec{v}_\mu^\dagger \cdot \vec{n}_e & \text{【E】} \\ \vec{v}_\tau^\dagger \cdot \vec{n}_e & \text{【G】} \end{pmatrix} = \begin{pmatrix} \frac{+1}{6} & \frac{+4}{6} & \frac{+4}{6} \\ \frac{+12}{6} & \frac{+15}{6} & \frac{0}{6} \\ \frac{+21}{6} & \frac{+24}{6} & \frac{0}{6} \end{pmatrix} \begin{pmatrix} \frac{-12}{26} \\ \frac{+12}{26} \\ \frac{-27}{26} \end{pmatrix} = \begin{pmatrix} \frac{-120}{260} \\ \frac{+60}{260} \\ \frac{+60}{260} \end{pmatrix} \quad (12.1) \spadesuit$$

$$\bullet \bullet \quad \vec{v}_\alpha^\dagger \cdot \vec{n}_\mu = \begin{pmatrix} \vec{v}_e^\dagger \cdot \vec{n}_\mu & \text{【D】} \\ \vec{v}_\mu^\dagger \cdot \vec{n}_\mu & \text{【B】} \\ \vec{v}_\tau^\dagger \cdot \vec{n}_\mu & \text{【I】} \end{pmatrix} = \begin{pmatrix} \frac{+6}{6} & \frac{+9}{6} & \frac{-6}{6} \\ \frac{+19}{6} & \frac{+22}{6} & \frac{-14}{6} \\ \frac{+36}{6} & \frac{+39}{6} & \frac{-30}{6} \end{pmatrix} \begin{pmatrix} \frac{-5530}{3120} \\ \frac{-886}{3120} \\ \frac{-7987}{3120} \end{pmatrix} = \begin{pmatrix} \frac{-94}{260} \\ \frac{-177}{260} \\ \frac{+83}{260} \end{pmatrix} \quad (12.2)$$

$$\bullet \bullet \quad \vec{v}_\alpha^\dagger \cdot \vec{n}_\tau = \begin{pmatrix} \vec{v}_e^\dagger \cdot \vec{n}_\tau & \text{【F】} \\ \vec{v}_\mu^\dagger \cdot \vec{n}_\tau & \text{【H】} \\ \vec{v}_\tau^\dagger \cdot \vec{n}_\tau & \text{【C】} \end{pmatrix} = \begin{pmatrix} \frac{+9}{6} & \frac{+12}{6} & \frac{-12}{6} \\ \frac{+30}{6} & \frac{+33}{6} & \frac{-36}{6} \\ \frac{+49}{6} & \frac{+52}{6} & \frac{-56}{6} \end{pmatrix} \begin{pmatrix} \frac{-292}{120} \\ \frac{-328}{120} \\ \frac{-553}{120} \end{pmatrix} = \begin{pmatrix} \frac{+26}{260} \\ \frac{+117}{260} \\ \frac{-143}{260} \end{pmatrix} \quad (12.3)$$

The above three results are put into (10), obtain below

$$L_{\text{Broken}}(v) = \begin{pmatrix} \vec{v}_e^\dagger \cdot \vec{n}_e & \vec{v}_e^\dagger \cdot \vec{n}_\mu & \vec{v}_e^\dagger \cdot \vec{n}_\tau \\ \vec{v}_\mu^\dagger \cdot \vec{n}_e & \vec{v}_\mu^\dagger \cdot \vec{n}_\mu & \vec{v}_\mu^\dagger \cdot \vec{n}_\tau \\ \vec{v}_\tau^\dagger \cdot \vec{n}_e & \vec{v}_\tau^\dagger \cdot \vec{n}_\mu & \vec{v}_\tau^\dagger \cdot \vec{n}_\tau \end{pmatrix} = \begin{pmatrix} \frac{-120}{260} & \frac{+94}{260} & \frac{+26}{260} & 0.000 \\ \frac{+60}{260} & \frac{-177}{260} & \frac{+117}{260} & 0.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{-143}{260} & 0.000 \\ 0.000 & 0.000 & 0.000 & \frac{-440}{260} = \frac{-22}{13} \end{pmatrix} = \begin{pmatrix} \text{【A】} & \text{【D】} & \text{【F】} \\ \text{【E】} & \text{【B】} & \text{【H】} \\ \text{【G】} & \text{【I】} & \text{【C】} \end{pmatrix} \quad (13)$$

Matrix(6) Matrix(7)

Matrix(7) is the digit graph of broken neutrino lepton number $L_{\text{Broken}}(v)$.

4. NEUTRINO MASS LEPTON NUMBER $L_{\text{MASS}}(v)$, NEUTRINO LEPTON COUPLING CONSTANT $G_L(v)$ AND PMNS U_{PMNS}

Plus $L(v^{\dagger}v')$ (0) and $L_{\text{Broken}}(v)$ (13), Obtain (14) below

$$L(v^{\dagger}v') + L_{\text{Broken}}(v) = \begin{pmatrix} \vec{v}_e^{\dagger} \cdot \vec{v}_e & \vec{v}_e^{\dagger} \cdot \vec{v}_{\mu} & \vec{v}_e^{\dagger} \cdot \vec{v}_{\tau} \\ \vec{v}_{\mu}^{\dagger} \cdot \vec{v}_e & \vec{v}_{\mu}^{\dagger} \cdot \vec{v}_{\mu} & \vec{v}_{\mu}^{\dagger} \cdot \vec{v}_{\tau} \\ \vec{v}_{\tau}^{\dagger} \cdot \vec{v}_e & \vec{v}_{\tau}^{\dagger} \cdot \vec{v}_{\mu} & \vec{v}_{\tau}^{\dagger} \cdot \vec{v}_{\tau} \end{pmatrix} + \begin{pmatrix} \vec{v}_e^{\dagger} \cdot \vec{n}_e & \vec{v}_e^{\dagger} \cdot \vec{n}_{\mu} & \vec{v}_e^{\dagger} \cdot \vec{n}_{\tau} \\ \vec{v}_{\mu}^{\dagger} \cdot \vec{n}_e & \vec{v}_{\mu}^{\dagger} \cdot \vec{n}_{\mu} & \vec{v}_{\mu}^{\dagger} \cdot \vec{n}_{\tau} \\ \vec{v}_{\tau}^{\dagger} \cdot \vec{n}_e & \vec{v}_{\tau}^{\dagger} \cdot \vec{n}_{\mu} & \vec{v}_{\tau}^{\dagger} \cdot \vec{n}_{\tau} \end{pmatrix}$$

$$\begin{pmatrix} +1.000 & 0.000 & 0.000 & +1.000 \\ 0.000 & +1.000 & 0.000 & +1.000 \\ 0.000 & 0.000 & +1.000 & +1.000 \\ +1.000 & +1.000 & +1.000 & +3.000 \end{pmatrix} + \begin{pmatrix} \frac{-120}{260} & \frac{+94}{260} & \frac{+26}{260} & 0.000 \\ \frac{+60}{260} & \frac{-177}{260} & \frac{+117}{260} & 0.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{-143}{260} & 0.000 \\ 0.000 & 0.000 & 0.000 & \frac{-440}{260} = \frac{-22}{13} \end{pmatrix} = \begin{pmatrix} \frac{+140}{260} & \frac{+94}{260} & \frac{+26}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ 1.000 & 1.000 & 1.000 & \frac{+340}{260} = \frac{+17}{13} \end{pmatrix} \quad (14)$$

We call the right term in expression (14), *Neutrino Mass Lepton Number* $L_{\text{MASS}}(v)$ below

•

$$L_{\text{MASS}}(v) = \begin{pmatrix} \frac{+140}{260} & \frac{+94}{260} & \frac{+26}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ 1.000 & 1.000 & 1.000 & \frac{+340}{260} = \frac{+17}{13} \end{pmatrix} \quad (15)$$

- Introduce a marvellous constant, $G_L(v) = \left(\frac{13}{8}\right)$ (16)

Multiply $G_L(v)$ (16) and $L_{\text{Mass}}(v)$ (15), (or Matrix(10) below)

$$\begin{aligned}
 & G_L(v) \cdot L_{\text{Mass}}(v) \\
 &= G_L(v) \cdot \begin{pmatrix} \frac{+140}{260} & \frac{+94}{260} & \frac{+26}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ 1.000 & 1.000 & 1.000 & \frac{+17}{13} \end{pmatrix} = \left(\frac{13}{8}\right) \begin{pmatrix} \frac{+140}{260} & \frac{+94}{260} & \frac{+26}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ \frac{+60}{260} & \frac{+83}{260} & \frac{+117}{260} & 1.000 \\ 1.000 & 1.000 & 1.000 & \frac{+17}{13} \end{pmatrix} \\
 &= \begin{pmatrix} \frac{+140}{160} & \frac{+94}{160} & \frac{+26}{160} & \frac{+260}{160} \\ \frac{+60}{160} & \frac{+83}{160} & \frac{+117}{160} & \frac{+260}{160} \\ \frac{+60}{160} & \frac{+83}{160} & \frac{+117}{160} & \frac{+260}{160} \\ \frac{+260}{160} & \frac{+260}{160} & \frac{+260}{160} & \frac{+17}{8} \end{pmatrix} = \begin{pmatrix} 0.875 & 0.5875 & 0.1625 & 1.625 \\ 0.375 & 0.51875 & 0.73125 & 1.625 \\ 0.375 & 0.51875 & 0.73125 & 1.625 \\ 1.625 & 1.625 & 1.625 & 2.125 \end{pmatrix} = |U_{\text{PMNS}}| \quad (17)
 \end{aligned}$$

Comparing (17) with Matrix(1), Matrix(2) and Matrix(3) mentioned previously, we obtain an interesting result below:

$$\blacksquare \blacksquare \blacksquare \blacksquare \blacksquare \quad |U_{\text{PMNS}}| = \text{Neutrino Lepton Coupling Constant } G_L(v) \cdot \text{Neutrino Mass Lepton Number } L_{\text{Mass}}(v) \quad \blacksquare \blacksquare \blacksquare \blacksquare \blacksquare$$

5. NEUTRINO BACKGROUND LEPTON NUMBER $L_{\text{Background}}(\nu)$ AND DARK MATTER-ENERGY $E_{\text{DME}}(\nu)$

For a more understanding of formula (14) and formula (17), we transform the left second term, $L_{\text{Broken}}(\nu)$ to the right side, further formula (14) turns to (18) below

$$\begin{pmatrix} +1.000_{11} & 0.000_{12} & 0.000_{13} & +1.000 \\ 0.000_{21} & +1.000_{22} & 0.000_{23} & +1.000 \\ 0.000_{31} & 0.000_{32} & +1.000_{33} & +1.000 \\ +1.000 & +1.000 & +1.000 & +3.000 \end{pmatrix} = \begin{pmatrix} \frac{+120}{260}_{11} & \frac{-94}{260}_{12} & \frac{-26}{260}_{13} & 0.000 \\ \frac{-60}{260}_{21} & \frac{+177}{260}_{22} & \frac{-117}{260}_{23} & 0.000 \\ \frac{-60}{260}_{31} & \frac{-83}{260}_{32} & \frac{+143}{260}_{33} & 0.000 \\ 0.000 & 0.000 & 0.000 & \frac{+440}{260} = \frac{+22}{13} \end{pmatrix} + \begin{pmatrix} \frac{+140}{260}_{11} & \frac{+94}{260}_{12} & \frac{+26}{260}_{13} & 1.000 \\ \frac{+60}{260}_{21} & \frac{+83}{260}_{22} & \frac{+117}{260}_{23} & 1.000 \\ \frac{+60}{260}_{31} & \frac{+83}{260}_{32} & \frac{+117}{260}_{33} & 1.000 \\ 1.000 & 1.000 & 1.000 & \frac{+340}{260} = \frac{+17}{13} \end{pmatrix} \quad (18)$$

Matrix(8)

=

Matrix(9)

+

Matrix(10)

$\uparrow \uparrow \uparrow$

$\uparrow \uparrow \uparrow$

$\uparrow \uparrow \uparrow$

$L(\nu^+ \nu^-)$

$L_{\text{Background}}(\nu)$

$L_{\text{Mass}}(\nu)$

Lepton Number Conservation

=

Neutrino Background Lepton Number

+

Neutrino Mass Lepton Number

$$+3.000 = \frac{39}{13}$$

$$\frac{22}{13} / \frac{39}{13} = \frac{22}{39} \approx 56\%$$

$$\frac{+17}{13} / \frac{39}{13} = \frac{17}{39} \approx 44\%$$

Here

$$\text{Neutrino Background Lepton Number } L_{\text{Background}}(\nu) = - (\text{Broken Neutrino Lepton Number } L_{\text{Broken}}(\nu)) \quad (19)$$

Next analogous to the process of (16) and (17), Matrix(10), we multiply $G_L(\nu)$ (16) and Matrix(9) in (18) above. Further obtain (20) following

$$G_L(v) \cdot L_{\text{Background}}(v)$$

$$= G_L(v) \cdot \begin{pmatrix} \frac{+120}{260} & \frac{-94}{260} & \frac{-26}{260} & 0.000 \\ \frac{-60}{260} & \frac{+177}{260} & \frac{-117}{260} & 0.000 \\ \frac{-60}{260} & \frac{-83}{260} & \frac{+143}{260} & 0.000 \\ 0.000 & 0.000 & 0.000 & \frac{+22}{13} \end{pmatrix} = \left(\frac{13}{8} \right) \begin{pmatrix} \frac{+120}{260} & \frac{-94}{260} & \frac{-26}{260} & 0.000 \\ \frac{-60}{260} & \frac{+177}{260} & \frac{-117}{260} & 0.000 \\ \frac{-60}{260} & \frac{-83}{260} & \frac{+143}{260} & 0.000 \\ 0.000 & 0.000 & 0.000 & \frac{+22}{13} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{+120}{160} & \frac{-94}{160} & \frac{-26}{160} & 0.000 \\ \frac{-60}{160} & \frac{+177}{160} & \frac{-117}{160} & 0.000 \\ \frac{-60}{160} & \frac{-83}{160} & \frac{-143}{160} & 0.000 \\ 0.000 & 0.000 & 0.000 & \frac{+22}{8} \end{pmatrix} = \begin{pmatrix} +0.750 & -0.5875 & -0.1625 & 0.000 \\ -0.375 & +1.10625 & -0.73125 & 0.000 \\ -0.375 & -0.51875 & +0.89375 & 0.000 \\ 0.000 & 0.000 & 0.000 & 2.75 \end{pmatrix} = U_{\text{Background}} \quad (20)$$

We call formular (20) $U_{\text{Background}} = G_L(v) \cdot L_{\text{Background}}(v)$, **Neutrino Dark Matter-Energy** $E_{\text{DME}}(v)$, resulted from neutrino background lepton number, which accounts to 56% of that total lepton number conservation, a slight large than $|U_{\text{PMNS}}|$ 44% resulted from Neutrino Mass Lepton Number

- To obtain experimental observations, after the diagonalization of neutrino lepton number (18), obtain (21) below:

$$L(v^{\dagger} v')_{\text{diagonal}} = \begin{pmatrix} \frac{+13}{13} & 0 & 0 & \frac{+13}{13} \\ 0 & \frac{+13}{13} & 0 & \frac{+13}{13} \\ 0 & 0 & \frac{+13}{13} & \frac{+13}{13} \\ \frac{+13}{13} & \frac{+13}{13} & \frac{+13}{13} & \frac{+39}{13} \end{pmatrix} = \begin{pmatrix} \frac{0}{13} & 0 & 0 & \frac{0}{13} \\ 0 & \frac{+9}{13} & 0 & \frac{+9}{13} \\ 0 & 0 & \frac{+13}{13} & \frac{+13}{13} \\ \frac{0}{13} & \frac{+9}{13} & \frac{+13}{13} & \frac{+22}{13} \end{pmatrix} + \begin{pmatrix} \frac{+13}{13} & 0 & 0 & \frac{+13}{13} \\ 0 & \frac{+4}{13} & 0 & \frac{+4}{13} \\ 0 & 0 & \frac{0}{13} & \frac{0}{13} \\ \frac{+13}{13} & \frac{+4}{13} & \frac{0}{13} & \frac{+17}{13} \end{pmatrix} \quad (21)$$

$$\text{lepton number } L(v^{\dagger} v') = L_{\text{Background}}(v), E_{\text{DME}}(v) + L_{\text{Mass}}(v)$$

■a Then multiply $G_L(v) = (\frac{13}{8})$ and (21), gain (22) below

$$G_L(v) = \begin{pmatrix} \frac{+13}{8} & 0 & 0 & \frac{+13}{8} \\ 0 & \frac{+13}{8} & 0 & \frac{+13}{8} \\ 0 & 0 & \frac{+13}{8} & \frac{+13}{8} \\ \frac{+13}{8} & \frac{+13}{8} & \frac{+13}{8} & \frac{+39}{8} \end{pmatrix} = \begin{pmatrix} \frac{0}{8} & 0 & 0 & \frac{0}{8} \\ 0 & \frac{+9}{8} & 0 & \frac{+9}{8} \\ 0 & 0 & \frac{+13}{8} & \frac{+13}{8} \\ \frac{0}{8} & \frac{+9}{8} & \frac{+13}{8} & \frac{+22}{8} \end{pmatrix} + \begin{pmatrix} \frac{+13}{8} & 0 & 0 & \frac{+13}{8} \\ 0 & \frac{+4}{8} & 0 & \frac{+4}{8} \\ 0 & 0 & \frac{0}{8} & \frac{0}{8} \\ \frac{+13}{8} & \frac{+4}{8} & \frac{0}{8} & \frac{+17}{8} \end{pmatrix} \quad (22)$$

$$\begin{array}{ccccc} \text{Matrix(11)} & = & \text{Matrix(12)} & + & \text{Matrix(13)} \\ G_L(v) & = & G_L(v)_{\text{Background}} & + & G_L(v)_{\text{Mass}} \end{array}$$

$$\begin{array}{lll} \text{Trace of Matrix: } \frac{+13}{8} + \frac{+13}{8} + \frac{+13}{8} = \frac{+39}{8} & \frac{+0}{8} + \frac{+9}{8} + \frac{+13}{8} = \frac{+22}{8} & \frac{+13}{8} + \frac{+4}{8} + \frac{0}{8} = \frac{+17}{8} \\ \frac{+22}{8} / \frac{+39}{8} = \approx 56\% & & \frac{+17}{8} / \frac{+39}{8} = \approx 44\% \end{array}$$

■b Last, again turn to (18), multiply $G_L(v) = (\frac{13}{8})$ and (18). gain (23) below

$$\begin{pmatrix} \frac{+13}{8} & 0 & 0 & \frac{+13}{8} \\ 0 & \frac{+13}{8} & 0 & \frac{+13}{8} \\ 0 & 0 & \frac{+13}{8} & \frac{+13}{8} \\ \frac{+13}{8} & \frac{+13}{8} & \frac{+13}{8} & \frac{+39}{8} \end{pmatrix} = \begin{pmatrix} \frac{+120}{160} & \frac{-94}{160} & \frac{-26}{160} & \frac{0}{160} \\ \frac{-60}{160} & \frac{+177}{160} & \frac{-117}{160} & \frac{0}{160} \\ \frac{-60}{160} & \frac{-83}{160} & \frac{+143}{160} & \frac{0}{160} \\ \frac{0}{160} & \frac{0}{160} & \frac{0}{160} & \frac{+440}{160} = \frac{+22}{8} \end{pmatrix} + \begin{pmatrix} \frac{+140}{160} & \frac{+94}{160} & \frac{+26}{160} & \frac{+260}{160} \\ \frac{+60}{160} & \frac{+83}{160} & \frac{+117}{160} & \frac{+260}{160} \\ \frac{+60}{160} & \frac{+83}{160} & \frac{+117}{160} & \frac{+260}{160} \\ \frac{+260}{160} & \frac{+260}{160} & \frac{+260}{160} & \frac{+340}{160} = \frac{+17}{8} \end{pmatrix} \quad (23)$$

If the diagonalization of (23), we could obtain (22) again.

CONCLUSIONS

• The conservation of Lepton number $L(v^\dagger v')$ (18) is characterized by Matrix(8): in which there are three classifications of matrix element digit-sum below

$$1) \text{ Horizontal sum: } 11 + 12 + 13 = +1.000 + 0.000 + 0.000 = +1.000$$

$$21 + 22 + 23 = 0.000 + 1.000 + 0.000 = +1.000$$

$$31 + 32 + 33 = 0.000 + 0.000 + 1.000 = +1.000$$

$$2) \text{ Vertical sum: } 11 + 21 + 31 = +1.000 + 0.000 + 0.000 = +1.000$$

$$12 + 22 + 32 = 0.000 + 1.000 + 0.000 = +1.000$$

$$13 + 23 + 33 = 0.000 + 0.000 + 1.000 = +1.000$$

$$3) \text{ Diagonal sum: } 11 + 22 + 33 = +1.000 + 1.000 + 1.000 = +3.000$$

(24)

• Neutrino background lepton number $L_{\text{Background}}(v)$ is characterized by Matrix(9) below :

$$4) \text{ Horizontal sum: } 11 + 12 + 13 = \frac{+120}{260} + \frac{-94}{260} + \frac{-26}{260} = 0.000$$

$$21 + 22 + 23 = \frac{-60}{260} + \frac{+177}{260} + \frac{-117}{260} = 0.000$$

$$31 + 32 + 33 = \frac{-60}{260} + \frac{-83}{260} + \frac{+143}{260} = 0.000$$

$$5) \text{ Vertical sum: } 11 + 21 + 31 = \frac{+120}{260} + \frac{-60}{260} + \frac{-60}{260} = 0.000$$

$$12 + 22 + 32 = \frac{-94}{260} + \frac{+177}{260} + \frac{-83}{260} = 0.000$$

$$13 + 23 + 33 = \frac{-26}{260} + \frac{-117}{260} + \frac{+143}{260} = 0.000$$

$$6) \text{ Diagonal sum: } 11 + 22 + 33 = \frac{+120}{260} + \frac{+177}{260} + \frac{+143}{260} = \frac{+440}{260} = \frac{+22}{13} \neq 0.000$$

(25)

- Neutrino mass lepton number $L_{\text{Mass}}(v)$ is characterized by Matrix(10) below:

$$7) \text{ Horizontal sum: } 11 + 12 + 13 = \frac{+140}{260} + \frac{+94}{260} + \frac{+26}{260} = +1.000$$

$$21 + 22 + 23 = \frac{+60}{260} + \frac{+83}{260} + \frac{+117}{260} = +1.000$$

$$31 + 32 + 33 = \frac{+60}{260} + \frac{+83}{260} + \frac{+117}{260} = +1.000$$

$$8) \text{ Vertical sum: } 11 + 21 + 31 = \frac{+140}{260} + \frac{+60}{260} + \frac{+60}{260} = +1.000$$

$$12 + 22 + 32 = \frac{+94}{260} + \frac{+83}{260} + \frac{+83}{260} = +1.000$$

$$13 + 23 + 33 = \frac{+26}{260} + \frac{+117}{260} + \frac{+117}{260} = +1.000$$

$$9) \text{ Diagonal sum: } 11 + 22 + 33 = \frac{+140}{260} + \frac{+83}{260} + \frac{+117}{260} = \frac{+340}{260} = \frac{+17}{13} \neq +3.000 \quad (26)$$

We see:

Lepton number $L(v^\dagger v')$ Matrix(8) could be decomposed into two parts, Matrix(9) $L_{\text{Background}}(v)$ and Matrix(10) $L_{\text{Mass}}(v)$.

1), 2) and 3) show: the conservation properties of Matrix(8) $L(v^\dagger v')$ in three different directions.

4), 5) show: the conservation properties of Matrix(9) $L_{\text{Background}}(v)$; but (6) shows the value of neutrino background lepton number is not zero, no a " really pure vacuum " ! its diagonal sum 6), trace is detectable.

7), 8) show: the conservation properties of Matrix(10) $L_{\text{Mass}}(v)$; but (9) shows the value of neutrino mass lepton number is Only Partly BE Broken in diagonal direction 9). BUT those along with horizontal and vertical directions are still conserved.

All that happened, are due to the *Leptoncolor brokenon* $\vec{n}_\beta$ of lepton weak interaction paring $\vec{\Phi}$ in neutrino flavor oscillations

$$\vec{n}_\beta = (n_1, n_2, n_3)_\beta, \quad \beta = e, \mu, \tau \quad (11)$$

The nine digits, $\vec{n}_\beta$ which are restricted to the seven digit-sums conditions previously mentioned, three horizontal sums, three vertical sums and one diagonal sum. Remain two dependent parametes. that may be related to $\vec{n}_\beta \approx$ Higgs particle ?

Physical observable matrix $|U_{\text{PMNS}}| = G_L(v) \cdot (\text{broken quantum operator, neutrino mass lepton number } L_{\text{Mass}}(v))$

Neutrino dark matter-energy $E_{\text{DME}}(v) = U_{\text{Background}} = G_L(v) \cdot (\text{broken quantum operator, neutrino backgroud lepton number } L_{\text{Background}}(v))$

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