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On New Closed Sets in Grill Topological Spaces

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I. Introduction

N. Levine[1] introduced the concept of generalized closed sets in topological spaces. Later many authors introduced new types closed sets in topological spaces and established their properties. They also studied the relationship with other types of closed sets in topological spaces. The concept of Grill was first introduced by Choquet [2] in the year 1947.Some authors introduced the concept of generalized closed set in Grill topological spaces in later years. In 2012, Dhananjoy Mandal and M. N. Mukherjee[3], introduced the concept of Gg-closed set in Grill topological spaces and studied their properties. In the year 2017, M. Kaleswari and others[4] introduced the concept of Gg*-closed sets in Grill topological spaces and established some of their properties. With their inspiration the concept of Gg**-closed set in a Grill topological space was introduced in the present work and studied some of their properties.

II. Preliminaries

Definition 2.1

A Grillon a topological space (X,τ) is a nonempty collection G of nonempty subsets of X such that

(i) $A \in G, A \subseteq B \subseteq X \Rightarrow B \in G$

(ii) $A \subseteq X, B \subseteq X, A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

If G is a Grill on a topological space (X,τ), then it is called a Grill topological space denoted with (X,τ,G).

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1. N. Levine, Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo 19 (1970), 89–96.

Definition 2.2

Let (X,τ,G) be a Grill topological space and A is any subset of X. The operator $\phi: P(A) \to P(A)$ is defined as $\phi(A) = \{x \in X \,/ U \cap A \in G, \forall U \in \tau(x)\}$ where $\tau(x)$ denotes the neighbourhood of x in the space X.

Definition 2.3:

A subset A of a Grill topological space (X,τ,G) is said to be Gg-closed if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The complement of a Gg-closed set is a Gg-open set.

Definition 2.4: [6]

A subset A of a Grill topological space (X,τ,G) is said to be Gg*-closed if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X. The complement of a Gg*-closed set is a Gg*-open set.

Throughout the paper, by a space X we always mean a topological space (X,τ) with no separation axioms assumed. For any subset A of the space X, the closure of A is denoted with $cl(A)$ and interior of the subset A is denoted with $\text{int}(A)$.

Theorem 2.5:[5]

Let (X,τ,G) be a Grill topological space. Then for any $A \subseteq X, B \subseteq X$ the following hold:

(a) $A \subseteq B \Rightarrow \phi(A) \subseteq \phi(B)$

(b) $\phi(A \cup B) = \phi(A) \cup \phi(B)$

(c) $\phi(\phi(A)) \subseteq \phi(A) = cl(\phi(A)) \subseteq cl(A)$

III. Gg**-Closed Sets in Grill Topological Spaces

In this section a new type of closed set was defined in a Grill topological space with an example.

Definition 3.1:

A subset A of a Grill topological space (X,τ,G) is said to be Gg**-closed set if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is g*-open in X. The complement of a Gg**-closed set is a Gg**-open set.

Example 3.2:

Consider $X = \{a,b,c\}$, $\tau = \{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}, \{b\}\}$ and $G = \{X, \{a\}, \{a,c\}$. Then, (X,τ,G) is a Grill topological space. In this space, g*-closed sets are $\{\varnothing, X, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$. Take the set $A = \{a,c\}$ in the space. Then, $\tau(a) = \{X, \{a\}, \{a,b\}, \{a,c\}$, $\tau(b) = \{X, \{b\}, \{a,b\}\}$ and $\tau(c) = \{X, \{a,c\}\}$.

Now, $\{a\} \cap A = \{a\} \in G, \{a,b\} \cap A = \{a\} \in G, \{a,c\} \cap A = \{a,c\} \in G, X \cap A = A \in G$. This shows that $a \in \phi(A)$. In a similar way we can check $b \notin \phi(A), c \in \phi(A)$. So, $\phi(A) = \{\text{a,c}\}$.

Also, the g*-open sets containing A are $\{X, \{a,c\}\}$ and each of the sets contain $\phi(A)$. Hence, the set $A = \{a,c\}$ is a Gg**-closed set in the Grill topological space (X,τ,G).

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IV. Properties of Gg**-Closed Sets

This section is dedicated to study some simple properties of Gg**-closed sets.

Theorem 4.1:

In a Grill topological space (X,τ,G), every non-member of G is Gg**-closed.

Proof:

Let A be any non-member of G and U be a g*-open set containing A. Then, $A \cap U = A \notin G$. This shows that $\phi(A) = \{\ \} \subseteq U$ and hence A is Gg**-closed set.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.2:

Consider the Grill topological space (X,τ,G) defined by the sets $X = \{a,b,c\}$, $\tau = \{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}, \{b\}\}$, $G = \{X, \{a\}, \{a,c\}\}$. In this space $A = \{a,c\}$ is a Gg**-closed set but it is a member of the grill G.

Theorem 4.3:

In a Grill topological space (X,τ,G), every closed set is a Gg**-closed set.

Proof:

Let A be any closed set in the Grill topological space (X,τ,G). Then, $A = cl(A)$. Let U be any g*-open set containing A. Then, U is a g*-open set containing $cl(A)$. We claim that $\phi(A) \subseteq U$. Suppose $x \in \phi(A)$. Then, $A \cap U \in G, \forall U \in \tau(x)$. This implies that $x \in cl(A)$ and so $x \in U$. Hence, $\phi(A) \subseteq U$ as we claimed.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.4:

Let $X = \{a,b,c\}, \tau = \{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}, \{b\}\}$ and $G = \{X, \{a\}, \{a,c\}$ be a grill. Then (X,τ,G) is a Grill topological space. In the space, $A = \{b,c\}$ is a Gg**-closed set but it is not a closed set.

Theorem 4.5:

In a grill topological space (X,τ,G), every $g*-$ closed set is a Gg**-closed set.

Proof:

Let A be ag*-closed set in the Grill topological space (X,τ,G) and U be any g*-open set containing A. Then, $\phi(A) \subseteq U$. Hence, A is a Gg**- closed set.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.6:

Let $X=\{a,b,c\},\tau=\{\varnothing,X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}\}$ be a grill. Then, (X,τ,G) is a Grill topological space. In the space, $A=\{a,b\}$ is a Gg**-closed set, but it is not a g*-closed set.

Theorem 4.7:

In a Grill topological space (X,τ,G), every Gg*-closed set is a Gg**-closed set.

Proof:

Let A be any Gg*-closed set in the Grill topological space (X,τ,G) and U be any g*-open set containing A. Then, $\phi(A)\subseteq U$. Hence, A is a Gg**-closed set.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.8:

Let $X=\{a,b,c\},\tau=\{\varnothing,X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}\}$ be a grill. Then, (X,τ,G) is a grill topological space. In the space, $A=\{a,b\}$ is a Gg**-closed setbut it is not a Gg*-closed set.

Theorem 4.9:

In a Grill topological space (X,τ,G) union of any two Gg**-closed setsis aGg**-closed set.

Proof:

Let A,B be any two Gg**-closed sets in a Grill topological space (X,τ,G). Let U be any g*-open set containing $A\cup B$. Since $A\subseteq A\cup B, B\subseteq A\cup B$, U is a g*-open set containing A and B also. Since, both the sets are Gg**-closed, $\phi(A)\subseteq U,\phi(B)\subseteq U$. But $\phi(A\cup B)=\phi(A)\cup\phi(B)\subseteq U$. This shows that the set $A\cup B$ is a Gg**-closed set.

Remark:

In a Grill topological space (X,τ,G), intersection of two Gg**-closed sets need not be a Gg**-closed set. This can be seen in the following example.

Example 4.10:

Let $X=\{a,b,c\}$, $\tau=\{\varnothing,X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}\}$ be a grill. Then, (X,τ,G) is a Grill topological space. In the space, the subsets $A=\{a,b\}$ and $B=\{a,c\}$ are Gg**-closed sets, but the intersection $\{a\}$ is not a Gg**-closed set.

Theorem 4.11:

If A,B are two subsets of a grill topological space (X,τ,G) such that A is a Gg**-closed setand $A\subseteq B\subseteq\phi(A)$, then B is a Gg**-closed set.

Proof:

Let U be any g*-open set containing B in the Grill topological space (X,τ,G). This implies, U is ag*-open set containing A also. Since A is a Gg**-closed, $\phi(A)\subseteq U$.

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But, $B \subseteq \phi(A) \Rightarrow \phi(B) \subseteq \phi(\phi(A)) = \phi(A)$. This shows that $\phi(B) \subseteq U$ and hence B is a Gg**-closed set.

V. Conclusion

In this paper an attempt was made to introduce the concept of Gg**-closed sets in a Grill topological space. Some basic properties of these sets were discussed. In continuation to this, continuity using these closed sets can be studied in future work.

Notes

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