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Structure of Regular Semigroups

By P. Sreenivasulu Reddy & Mulugeta Dawud

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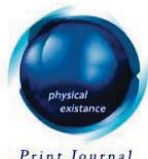
Abstract- This paper concerned with basic concepts and some results on (idempotent) semigroup satisfying the identities of three variables. The motivation of taking three for the number of variables has come from the fact that many important identities on idempotent semigroups are written by three or fewer independent variables. We consider the semigroup satisfying the property $abc = ac$ and prove that it is left semi-normal and right quasi-normal. Again an idempotent semigroup with an identity $aba = ab$ and $aba = ba$ ($ab = a$, $ab = b$) is always a semilattices and normal. An idempotent semigroup is normal if and only if it is both left quasi-normal and right quasi-normal. If a semigroup is rectangular then it is left and right semi-regular.

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Structure of Regular Semigroups

P. Sreenivasulu Reddy ^α & Mulugeta Dawud ^σ

Abstract- This paper concerned with basic concepts and some results on (idempotent) semigroup satisfying the identities of three variables. The motivation of taking three for the number of variables has come from the fact that many important identities on idempotent semigroups are written by three or fewer independent variables. We consider the semigroup satisfying the property $abc = ac$ and prove that it is left semi-normal and right quasi-normal. Again an idempotent semigroup with an identity $aba = ab$ and $aba = ba$ ($ab = a$, $ab = b$) is always a semilattices and normal. An idempotent semigroup is normal if and only if it is both left quasi-normal and right quasi-normal. If a semigroup is rectangular then it is left and right semi-regular.

I. PRELIMINARIES AND BASIC PROPERTIES OF REGULAR SEMIGROUPS

In this section we present some basic concepts of semigroups and other definitions needed for the study of this chapter and the subsequent chapters.

- **Definition:** A semigroup $(S, .)$ is said to be left(right) singular if it satisfies the identity $ab = a$ ($ab = b$) for all a, b in S
- **Definition:** A semigroup $(S, .)$ is rectangular if it satisfies the identity $aba = a$ for all a, b in S .
- **Definition:** A semigroup $(S, .)$ is called left(right) regular if it satisfies the identity $aba = ab$ ($aba = ba$) for all a, b in S .
- **Definition:** A semigroup $(S, .)$ is called regular if it satisfies the identity $abca = abaca$ for all a, b, c in S
- **Definition:** A semigroup $(S, .)$ is said to be total if every element of S can be written as the product of two elements of S . i.e, $S^2 = S$.
- **Definition:** A semigroup $(S, .)$ is said to be left(right) normal if $abc = acb$ ($abc = bac$) for all a, b, c in S .
- **Definition:** A semigroup $(S, .)$ is said to be normal if satisfies the identity $abca = acba$ for all a, b, c in S .
- **Definition:** A semigroup $(S, .)$ is said to be left(right) quasi-normal if it satisfies the identity $abc = acbc$ ($abc = abac$) for all a, b, c in S .
- **Definition:** A semigroup $(S, .)$ is said to be left (right) semi-normal if it satisfies the identity $abca = acbca$ ($abca = abcbca$) for all a, b, c in S .
- **Definition:** A semigroup $(S, .)$ is said to be left(right) semi-regular if it satisfies the identity $abca = abacabca$ ($abca = abcabaca$) if for all a, b, c in S .
- **Result:** [11,12] A left (right) singular semigroup is rectangular.
- **Theorem:** Every left(right) singular semigroup is total.

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Proof: Let $(S, .)$ be a left(right) singular semigroup.

Then $ab = a$ for any a, b in S

To prove that S is total we have to prove $S^2 = S$

We know that $S \subseteq S^2$

To prove $S^2 \subseteq S$

Let $x \in S^2 \Rightarrow x = a.b$ for any a, b in S

$x = a$

$x \in S$

$S^2 \subseteq S$

$\therefore S = S^2$

Hence $(S, .)$ is total.

- *Note:* From the result 2.1.11 every left(right) singular semigroup is rectangular and again from theorem 2.1.12 it is total. Hence every rectangular semigroup is total.
- *Theorem:* A semigroup satisfying the singular properties is always a semilattice.
- *Theorem:* Let $(S, .)$ be a semigroup. If S is left and right regular then S is a semilattice.

Proof: Let $(S, .)$ be a semigroup with left regular and right regular then

$aba = ab$ (left regular)

$aba = ba$ (right regular) for all a, b in S .

From the above we have $ab = ba$

$\therefore (S, .)$ is a commutative.

To prove that $(S, .)$ is band, let $aba = ab \Rightarrow a(ba) = ab \Rightarrow abab = ab$ ($ba = bab$)

$\Rightarrow (ab)(ab) = ab \Rightarrow (ab)^2 = ab$ put $a = b \Rightarrow (a.a)^2 = a.a \Rightarrow (a.a)^2 = a^2$ $a.a = a$

$\therefore (S, .)$ is a band

So, if S is left and right regular then $(S, .)$ is commutative band

Hence $(S, .)$ is a semilattice.

- *Lemma:* A left (right) regular semigroup is regular.
- *Note:* Every regular semigroup need not be a left (right) regular.
- *Lemma:* A right (left) singular semigroup is regular.
- *Lemma:* A left (right) normal semigroup is normal.
- *Theorem:* Let $(S, .)$ be a semigroup. If S is both left and right regular then S is normal.
- *Lemma:* A left (right) regular semigroup is right (left) quasi-normal.
- *Lemma:* A left (right) regular semigroup is right (left) semi-normal.
- *Theorem:* A left (right) regular semigroup is left (right) semi-regular
- *Theorem:* If an idempotent semigroup satisfies the right (left) quasi-normal then it is right(left) semi-regular.
- *Lemma:* A semigroup $(S, .)$ with left (right) quasi-normal is right (left) semi-normal.
- *Theorem:* If a semigroup $(S, .)$ is rectangular then $(S, .)$ is right (left) semi-regular.
- *Note:* Similarly, we prove that,

1. A semigroup $(S, .)$ is regular then $(S, .)$ is left (right) semi-regular.
2. A semigroup $(S, .)$ satisfies the left (right) semi-normal property is right (left) semi-regular.

- *Theorem:* An idempotent semigroup $(S, .)$ is normal if and only if it is both left and right quasi-normal.

Proof: Let $(S, .)$ be an idempotent semigroup.

Assume that $(S, .)$ is both left and right quasi-normal

$$\begin{aligned} \text{then} \quad abc &= acbc && (\text{left quasi-normal}) \\ abc &= abac && (\text{right quasi-normal}) \end{aligned}$$

To prove that $(S, .)$ is normal. Since S is right quasi-normal, we have

$$\begin{aligned} abc &= abac \Rightarrow abc = a(bac) \Rightarrow abc = abcac \quad (\text{left quasi-normal}) \\ \Rightarrow abc &= (abc)ac \Rightarrow abc = acbcac \quad (\text{left quasi normal}) \\ \Rightarrow abc &= ac(bcac) \Rightarrow abc = acbac \quad (\text{left quasi normal}) \\ \Rightarrow abca &= acbaca \Rightarrow abc = ac(baca) \Rightarrow abc = acbca \Rightarrow abc = a(cbca) \Rightarrow abc = acba \\ \Rightarrow abca &= acba. \end{aligned}$$

Hence $(S, .)$ is normal.

Conversely; let $(S, .)$ be normal then $abca = acba$

we show that $(S, .)$ is both left quasi-normal and right quasi-normal.

$$\begin{aligned} \text{Consider} \quad abc &= abc.c \Rightarrow abc = a(bc)c \Rightarrow abc = acbc \Rightarrow abc = acbc. \\ (S, .) &\text{ is left quasi-normal.} \end{aligned}$$

$$\text{Symilarly, } abc = a.abc \Rightarrow abc = a(ab)c \Rightarrow abc = abac \Rightarrow abc = abac.$$

Therefore $(S, .)$ is right quasi-normal.

II. SEMIGROUP SATISFYING THE IDENTITY $abc = ac$.

In this section we discuss if a semigroup S satisfying the identity $abc = ac$ for any three variables $a, b, c \in S$, then the following conditions are equivalent to one another.

- a) left semi-normal
- b) left semi-regular
- c) right semi-normal
- d) right semi-regular
- e) regular
- f) normal
- g) left quasi-normal
- h) right quasi-normal

- *Theorem:* A semigroup S with an identity $abc = ac$, for any $a, b, c \in S$ is left (right) semi-normal if and only if it is left (right) semi-regular.

Proof: Let S be a semigroup with an identity $abc = ac$ for any $a, b, c \in S$.

Assume that S be a left semi-normal.

Now we show that S is left semi-regular

$$\begin{aligned} \text{Since } S &\text{ is left semi-normal we have } abca = acbca \Rightarrow abca = (ac)bca \\ \Rightarrow abca &= abcbca \quad (ac = abc) \Rightarrow abca = a(bc)bca \Rightarrow abca = abacbca \quad (bc = bac) \\ \Rightarrow abca &= aba(cb)ca \Rightarrow abca = abacabca \quad (cb = cab) \Rightarrow abca = abacabca. \end{aligned}$$

$\therefore S$ is left semi-regular.

Conversely, let S be left semi regular then $abca = abacabca$

$$\Rightarrow abca = a(bac)abca \Rightarrow abca = abcabca \quad (bac = bc) \Rightarrow abca = (abc)abca$$

$\Rightarrow abca = acabca \quad (abc = ac) \Rightarrow abca = a(cab)ca \Rightarrow abca = acbca \quad (cab = cb) \quad abca = acbca.$

Hence S is left semi-normal.

- *Theorem:* An idempotent semigroup S with an identity $abc = ac$ for any a, b, c in S is left (right) semi-regular if and only if it is regular.

Proof: Let S be an idempotent semigroup with an identity $abc = ac$ for any a, b, c in S . Assume that S be a regular semigroup then $abca = abaca$

$\Rightarrow abca = ab(ac)a \Rightarrow abca = ababca \quad (ac = abc) \Rightarrow abca = a(ba)bca \Rightarrow abca = abcabca \quad (ba = bca)$

$\Rightarrow abca = a(bc)abca \Rightarrow abca = abacabca \quad (bc = bac) \Rightarrow abca = abacabca$

Hence S is left semi-regular.

Conversely, let S be left semi-regular then $abca = abacabca \Rightarrow abca = abac(abc)a$

$\Rightarrow abca = abacaca \quad (abc = ac) \Rightarrow abca = abac(aca) \Rightarrow abca = abaca.a \quad (aca = a.a)$

$\Rightarrow abca = abaca \quad (a.a = a) \Rightarrow abca = abaca$

Hence S is regular

- *Theorem:* A semigroup S with an identity $abc = ac$ for any a, b, c in S is left (right) semi-regular if and only if it is normal.

Proof: Let S be a semigroup with an identity $abc = ac$ for any $a, b, c \in S$ and assume that S be normal then $abca = acba \Rightarrow abca = (ac)ba \Rightarrow abca = abcbca \quad (ac = abc) \Rightarrow abca = abc(ba)$

$\Rightarrow abca = abcbca \quad (ba = bca) \Rightarrow abca = a(bc)bca \Rightarrow abca = abacbca \quad (bc = bac)$

$\Rightarrow abca = aba(cb)ca \Rightarrow abca = abacabca \quad (cb = cab) \Rightarrow abca = abacabca$

S is left semi-regular

Conversely, assume that S is left semi-regular then $abca = a(bac)abca$

$\Rightarrow abca = abcabca \quad (bac = bc) \Rightarrow abca = (abc)abca \Rightarrow abca = acabca \quad (abc = ac)$

$\Rightarrow abca = a(cab)ca \Rightarrow abca = acbca \quad (cab = cb) \Rightarrow abca = ac(bca) \Rightarrow abca = acba \quad (bca = ba)$

$\Rightarrow abca = acba$

Hence S is normal.

- *Theorem:* A semigroup S with an identity $abc = ac$ where a, b, c in S is regular if and only if it is left (right) semi-normal

Proof: Let S be a semigroup with an identity $abc = ac$ for all a, b, c in S .

Assume that S is regular then $abca = (ab)aca \Rightarrow abca = acbaca \quad (ab = acb) \Rightarrow abca = ac(bac)a$

$\Rightarrow abca = acbca \quad (bac = bc) \Rightarrow abca = acbca$

S is left semi-normal

Conversely, let S be left semi-normal. Then $abca = (ac)bca \Rightarrow abca = abcbca \quad (ac = abc) \Rightarrow abca = a(bc)bca \Rightarrow abca = abacbca \quad (bc = bac) \Rightarrow abca = abac(bca) \Rightarrow abca = abacba \quad (bca = ba)$

$\Rightarrow abca = aba(cba) \Rightarrow abca = abaca \quad (cba = ca)$

Hence S is regular.

- *Lemma:* A semigroup S satisfying the property $abc = ac$ for any $a, b, c \in S$ is left (right) semi-regular if and only if it is right(left) semi-regular.

- *Lemma:* A semigroup S with an identity $abc = ac$ is left(right) semi-normal if and only if it is right (left) semi-normal
- *Note:* A semigroup S with an identity $abc = ac$ where $a, b, c \in S$ is left(right) quasi-normal if and only if it is right (left) quasi-normal.
- *Lemma:* A semigroup S with the property $abc = ac$ for all $a, b, c \in S$ is normal if and only if it is left(right) semi-normal
- *Theorem:* A semigroup with an identity $abc = ac$, for any $a, b, c \in S$ is normal if and only if it is left(right) quasi-normal.

Proof: Let S be a semigroup with an identity $abc = ac$ for any $a, b, c \in S$

Let S be a left quasi-normal. Then, $abc = acbc \Rightarrow abca = acbca \Rightarrow abca = ac(bca)$

$\Rightarrow abca = acba$ ($bca = ba$) $\Rightarrow abca = acba$

S is normal.

Conversely, let S be normal. Then, $abca = acba \Rightarrow abcac = acbac$

$\Rightarrow a(bca)c = acbac \Rightarrow abac = acbac$ ($bca = ba$) $\Rightarrow a(bac) = acbac$

$\Rightarrow abc = acbac$ ($bac = bc$) $\Rightarrow abc = ac(bac) \Rightarrow abc = acbc$.

$\therefore S$ is a left quasi-normal.

Similarly, we can prove that a semigroup S with an identity $abc = ac$ for any $a, b, c \in S$ is regular if and only if it is left (right) quasi-normal.

- *Theorem:* A semigroup S with an identity $abc = ac$ for all $a, b, c \in S$ is regular if and only if it is normal.

Proof: Let s be a semigroup with an identity $abc = ac$ for any $a, b, c \in S$.

Let S be a normal semigroup then $abca = acba \Rightarrow abca = (ac)ba$

$\Rightarrow abca = abcba$ ($ac = abc$) $\Rightarrow abca = abc(ba) \Rightarrow abca = abcbca$ ($ba = bca$)

$\Rightarrow abca = a(bc)bca \Rightarrow abca = abacbca \Rightarrow abca = abac(bca) \Rightarrow abca = abacba$ ($bca = ba$)

$\Rightarrow abca = aba(cba) \Rightarrow abca = abaca$ ($cba = ca$) $\Rightarrow abca = abaca$.

Hence S is a regular.

Conversely, let S be a regular semigroup then $abca = abaca \Rightarrow abca = (ab)aca$

$\Rightarrow abca = acbaca$ ($ab = acb$) $\Rightarrow abca = ac(bac)a \Rightarrow abca = acbca$ ($bac = bc$)

$\Rightarrow abca = ac(bca) \Rightarrow abca = acba$ ($bca = ba$) $\Rightarrow abca = acba$.

$\therefore S$ is normal.

III. SEMIGROUP SATISFIES THE IDENTITY $AB = A$ ($AB = B$)

we present some results on semigroup with an identity $ab = a$ ($ab = b$) for all a, b in a semigroup S . We prove that the necessary and sufficient conditions for a semigroup S to be regular, normal, left (right) normal, left (right) semi-normal, right(left) semi-regular, left (right) regular, left (right) quasi-normal.

- *Theorem:* A semigroup S with an identity $ab = a$ for any $a, b \in S$ is normal if and only if it is regular.

Proof: Let S be a semigroup satisfying the identity $ab = a$ for all $a, b \in S$.

Assume that S is normal. Then $abca = (a)cba \Rightarrow abca = abcba$ ($a = ab$)

$\Rightarrow abca = a(b)cba \Rightarrow abca = abacba$ ($b = ba$) $\Rightarrow abca = aba(cb)a \Rightarrow abca = abaca$ ($cb = c$)

$$\Rightarrow abca = abaca.$$

Therefore S is regular.

Conversely, let S be a regular semigroup then $abca = abaca \Rightarrow abca = (ab)aca$
 $\Rightarrow abca = aaca$ ($ab = a$) $\Rightarrow abca = (aa)ca \Rightarrow abca = aca$ ($aa = a$) $\Rightarrow abca = a(c)a$
 $\Rightarrow abca = acba$ ($c = cb$) $\Rightarrow abca = acba$.

$\therefore S$ is normal.

- *Theorem:* A semigroup S satisfying the identity $ab = a$ for any a, b in S is left (right) regular if and only if it is regular.
- *Theorem:* A semigroup S with an identity $ab = a$ for all a, b in S is left(right) semi-normal if and only if it is right(left) semi-normal.
- *Theorem:* A semigroup S with an identity $ab = a$ for any a, b in S is left (right) semi-normal if and only if it is regular.

Proof: Let S be a semigroup with an identity $ab = a$ for any $a, b \in S$.

Let S be left semi-normal then $abca = acbca \Rightarrow abca = ac(bc)a \Rightarrow abca = acba$ ($bc = b$)

$\Rightarrow abca = (a)cba \Rightarrow abca = abcba$ ($a = ab$) $\Rightarrow abca = a(b)cba \Rightarrow abca = abacba$ ($b = ba$)

$\Rightarrow abca = aba(cb)a \Rightarrow abca = abaca$ ($cb = c$) $\Rightarrow abca = abaca$

Hence S is regular.

Conversely, let S be regular then $abca = abaca \Rightarrow abca = (ab)aca \Rightarrow abca = aaca$ ($ab = a$)

$\Rightarrow abca = (aa)ca$ ($aa = a$) $\Rightarrow abca = aca \Rightarrow abca = a(c)a \Rightarrow abca = acba$ ($c = cb$)

$\Rightarrow abca = ac(b)a$

$\Rightarrow abca = acbca$ ($b = bc$) $\Rightarrow abca = acbca$

$\therefore S$ is left semi-normal.

- *Lemma:* A semigroup S with an identity $ab = a$ for any a, b in S is left(right) semi-normal if and only if it is normal.
- *Theorem:* Let S be a semigroup and assume that S satisfies the identity $ab = a$ then S is left(right) normal if and only if it is normal
- *Theorem:* A semigroup S satisfying the identity $ab = a$ for any a, b in S is left(right) semi-regular if and only if it is right(left) semi-regular
- *Theorem:* A semigroup S with an identity $ab = a$, for any $a, b \in S$ is left(right) semi-regular if and only if it is normal.

Proof: Let S be a semigroup with an identity $ab = a$ for any a, b in S

Assume that S be left semi-regular then $abca = abacabca \Rightarrow abca = (ab)acabca$

$\Rightarrow abca = aacabca$ ($ab = a$) $\Rightarrow abca = (aa)cabca \Rightarrow abca = acabca$ ($aa = a$) $\Rightarrow abca = a(ca)bca$

$\Rightarrow abca = acbca$ ($ca = c$) $\Rightarrow abca = ac(bc)a \Rightarrow abca = acba$ ($bc = b$) $\Rightarrow abca = acba$.

$\therefore S$ is normal.

Conversely, let S be normal then $abca = acba \Rightarrow abca = (a)cba \Rightarrow abca = abcba$ ($a = ab$)

$\Rightarrow abca = a(b)cba \Rightarrow abca = abacba$ ($b = ba$) $\Rightarrow abca = aba(c)ba \Rightarrow abca = abacaba$ ($c = ca$)

$\Rightarrow abca = abaca(b)a \Rightarrow abca = abacabca \quad (b = bc) \Rightarrow abca = abacabca.$
Therefore S is left semi-regular.

- *Theorem:* A semigroup S with an identity $ab = a$ for all a, b in S is left(right) semi-regular if and only if it is left(right) semi-normal

Proof: Let S be a semigroup with the identity $ab = a$ for any $a, b \in S$.

Assume that S be a left semi-regular semigroup then, we have $abca = abacabca$

$\Rightarrow abca = (ab)acabca \Rightarrow abca = aacabca \quad (a = ab) \Rightarrow abca = (aa)cabca \Rightarrow abca = acabca \quad (aa = a)$

$\Rightarrow abca = a(ca)bca \Rightarrow abca = acbca \quad (ca = c) \Rightarrow abca = acbca.$

Hence S is left semi-normal.

Conversely, let S be left semi-normal then $abca = acbca \Rightarrow abca = (a)cbca$

$\Rightarrow abca = abcbca \quad (a = ab) \Rightarrow abca = a(b)cbca \Rightarrow abca = abacbc \quad (b = ba) \Rightarrow abca = aba(c)bca$

$\Rightarrow abca = abacabca \quad (c = ca) \Rightarrow abca = abacabca.$

Hence S is left semi-regular.

- *Theorem:* Let S be a semigroup and assume that S satisfy the left singular property then S is left(right) quasi-normal if and only if it is normal
- *Theorem:* A semigroup S with an identity $ab = a$ for any a, b in S is left (right) quasi-normal if and only if it is left(right) semi-regular.

Proof: Let S be a semigroup and it satisfies the identity $ab = a$ for all $a, b \in S$

Assume that S be left quasi-normal then $abc = acbc \Rightarrow ab(c) = acb(c)$

$\Rightarrow abca = acbca \quad (c = ca) \Rightarrow abca = (a)cbca \Rightarrow abca = abcbca \quad (a = ab) \Rightarrow abca = a(b)cbca$

$\Rightarrow abca = abacbc \quad (b = ba) \Rightarrow abca = aba(c)bca \Rightarrow abca = abacabca \quad (c = ca)$

$\Rightarrow abca = abacabca$

$\therefore S$ is left semi-regular.

Conversely, let S be left semi-regular then $abca = abacabca \Rightarrow ab(ca) = abacab(ca)$

$\Rightarrow abc = abacabc \quad (ca = a) \Rightarrow abc = (ab)acabc \Rightarrow abc = aacabc \quad (ab = a) \Rightarrow abc = (aa)cabc$

$\Rightarrow abc = acabc \quad (aa = a) \Rightarrow abc = a(ca)bc \Rightarrow abc = acbc \Rightarrow abc = acbc.$

Hence S is left quasi-normal.

- *Note:* Similarly, we can prove that,
 - a semigroup S with an identity $ab = a$ for any a, b , in S is left(right) semi-regular if and only if it is right(left) semi-normal.
 - a semigroup S satisfies the identity $ab = a$, for any $a, b \in S$ is left(right) quasi-normal if and only if it is any one of the following:
 - (1) Regular.
 - (2) Left(right) semi-normal.
 - (3) Left(right) semi-regular.
 - (4) Left(right) regular.
 - (5) Left(right) normal.

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Notes





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Linear Elliptic Systems with Nonlinear Boundary Conditions without Landesman-Lazer Conditions

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Abstract- The boundary value problem is examined for the system of elliptic equations of from $-\Delta u + A(x)u =$ in Ω , where $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{k \times k}$, and $\frac{\partial u}{\partial \nu} + g(u) = h(x)$ on $\partial\Omega$. It is assumed that $g \in C(\mathbb{R}^k, \mathbb{R}^k)$ is a bounded function which may vanish at infinity. The proofs are based on Leray-Schauder degree methods.

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Linear Elliptic Systems with Nonlinear Boundary Conditions without Landesman-Lazer Conditions

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Abstract- The boundary value problem is examined for the system of elliptic equations of from $-\Delta u + A(x)u =$ in Ω , where $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{k \times k}$ and $\frac{\partial u}{\partial \nu} + g(u) = h(x)$ on $\partial\Omega$. It is assumed that $g \in C(\mathbb{R}^k, \mathbb{R}^k)$ is a bounded function which may vanish at infinity. The proofs are based on Leray-Schauder degree methods.

1. INTRODUCTION

Let $\mathbb{R}^k$ be real k -dimensional space, if $w \in \mathbb{R}^k$, then $|w|_E$ denotes the Euclidean norm of w . Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$ is a bounded domain with boundary $\partial\Omega$ of class C^∞ . Let $g \in C^1(\mathbb{R}^k, \mathbb{R}^k)$, $h \in C(\partial\Omega, \mathbb{R}^k)$, and the matrix

$$A(x) = \begin{bmatrix} a_{11}(x) & a_{12}(x) & \cdots & a_{1k}(x) \\ a_{21}(x) & a_{22}(x) & \cdots & a_{2k}(x) \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1}(x) & a_{k2}(x) & \cdots & a_{kk}(x) \end{bmatrix}.$$

Verifies the following conditions:

- (A1) The functions $a_{ij} : \Omega \rightarrow \mathbb{R}$, $\forall i, j \in \{1, \dots, k\}$.
- (A2) $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{k \times k}$, almost everywhere $x \in \Omega$, and $A(x)$ is positive definite on a set of positive measure with $a_{ij} \in L^p(\Omega) \forall i, j \in \{1, \dots, k\}$ for $p > \frac{N}{2}$ when $N \geq 3$, and $p > 1$ when $N = 2$.

We will study the solvability of

$$\begin{aligned} -\Delta u + A(x)u &= 0 \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + g(u) &= h(x) \quad \text{on } \partial\Omega. \end{aligned} \tag{1.1}$$

The interest in this problem is the resonance case at the boundary with a bounded nonlinearity, we will assume that g a bounded function, and there is a constant $R > 0$ such that

$$|g(w(x))|_E \leq R \quad \forall w \in \mathbb{R}^k \text{ \& } x \in \partial\Omega. \tag{1.2}$$

Our assumptions allow that g is not only bounded, but also may be vanish at infinity i.e.;

$$\lim_{|w|_E \rightarrow \infty} g(w) = 0 \in \mathbb{R}^k. \tag{1.3}$$

Condition (1.3) is not required by our assumptions, but allowing for it is the main result of this paper.

In case of the scalar equation i.e.; $k = 1$ and g doesn't satisfy condition (1.3) but satisfying the Landesman-Lazer condition

$$g_- < \bar{h} < g^+$$

where $\lim_{w \rightarrow -\infty} g(w) = g_-$, $\bar{h} = \frac{1}{|\partial\Omega|} \int_{\partial\Omega} h \, dx$, $\lim_{w \rightarrow \infty} g(w) = g^+$,

and $A(x) = 0 \in \mathbb{R}^{k \times k}$. Then it is well known that there is a solution for (1.1). The first results when the nonlinearity in the equation in scalar case was done by Landesman and Lazer [1] in 1970. Their work led to great interest and activity on boundary value problems at resonance which continues to this day. A particularly interesting extension of Landesman and Lazer's work to systems was done by Nirenberg [2], [3] in case of system and the nonlinearity in the equation was done by Ortega and Ward [4], in the scalar case without Landesman-Lazer condition was done by Iannacci and Nkashama [5], Ortega and Sánchez [6], more completely the case for periodic solutions of the system of ordinary differential equations with bounded nonlinear g satisfying Nirenberg's condition. They studied periodic solutions

$$u'' + cu' + g(u) = p(t),$$

for $u \in \mathbb{R}^k$.

In case $c = 0$ was done by Mawhin [7]. In case the nonlinear terms vanish at infinity, as in (1.3), the Landesman-Lazer conditions fail. We would like to know what we can do in this case, and what conditions on a bounded nonlinearity that vanishes at infinity might replace that ones of the Landesman-Lazer type. Several authors have considered the case when the nonlinearity $g: \partial\Omega \times \mathbb{R} \rightarrow \mathbb{R}$ is a scalar function satisfies Carathéodory conditions i.e.;

- i: $g(\cdot, u)$ is measurable on $\partial\Omega$, for each $u \in \mathbb{R}$,
- ii: $g(x, \cdot)$ is continuous on $\mathbb{R}$, for a.e. $x \in \partial\Omega$,
- iii: for any constant $r > 0$, there exists a function $\gamma_r \in L^2(\partial\Omega)$, such that

$$|g(x, u)| \leq \gamma_r(x), \quad (1.4)$$

for a.e. $x \in \partial\Omega$, and all $u \in \mathbb{R}$ with $|u| \leq r$,

was done by Fadlallah [8] and the others have considered the case when the nonlinearity does not decay to zero very rapidly. For example in case the nonlinearity in the equation if $g = g(t)$ is a scalar function, the condition

$$\lim_{|t| \rightarrow \infty} tg(t) > 0. \quad (1.5)$$

and related ones were assumed in [9], [10], [11], [12], [13], [14], [15], [16], [17]. These papers all considered scalar problem, but also considered the Dirichlet (Neumann) problem at resonance (non-resonance) at higher eigenvalues (Steklov-eigenproblems). The work in some of these papers makes use of Leray-Schauder degree arguments, and the others using critical point theory both the growth restrictions like (1.5) and Lipschitz conditions have been removed (see [15], [17]). In this paper we study systems of elliptic boundary value problems with nonlinear boundary conditions Neumann type and the nonlinearities at boundary vanishing at the infinity. We do not require the problem to be in variational form.

Let S^{k-1} be the unit sphere in $\mathbb{R}^k$. We will assume that $S^{k-1} \cap \partial\Omega \neq \emptyset$ and Let $\mathbb{S} = S^{k-1} \cap \partial\Omega$.

1.1. Assumptions

G1: $g \in C^1(\mathbb{R}^k, \mathbb{R}^k)$ and g is bounded with $g(w) \neq 0$ for $|w|_E$ large.

G2: For each $z \in \mathbb{S}$ the $\lim_{r \rightarrow \infty} \frac{g(rz)}{|g(rz)|_E} = \varphi(z)$ exists, and the limits is uniform for $z \in \mathbb{S}$. It follows that $\varphi \in C(\mathbb{S}, \mathbb{S})$ and the topological degree of φ is defined.

G3: $\deg(\varphi) \neq 0$

Ref

1. E. Landesman and A. Lazer: *Nonlinear perturbations of linear elliptic boundary value problems at resonance*, *J. Math. Mech.* 19 (1970), 609-623.

1.2. Notations

- Let $\langle \cdot, \cdot \rangle_{L^2}$ denote the inner product in $L^2 := L^2(\Omega, \mathbb{R}^k)$ where L^2 is Lebesgue space
- Let $\langle \cdot, \cdot \rangle_E$ denote the standard inner product in $\mathbb{R}^k$
- Assume that ((A1)-(A2)) holds, then define

$$E(u, v) := \sum_{i=1}^k \langle \nabla u_i, \nabla v_i \rangle_{L^2} + \langle a_{ij}(x) u_i, v_i \rangle_{L^2}, \quad j = 1, \dots, k,$$

for $u, v \in H^1 = H^1(\Omega, \mathbb{R}^k)$ where H^1 the Sobolev space.

We note that it follows from the assumptions G1 : - G3 : that on large balls

$$B(R) := \{y : |y|_E \leq R\},$$

the $\deg(g, B(R), 0) \neq 0$ see [18],[19].

We modify the Lemma 1 and Theorem 1 in [4] to fit our problem.

Lemma 1.1. Assume that G1 : and G2 : hold and $C > 0$ is a given constant. Then there exists $R > 0$ such that

$$\int_{\partial\Omega} g(u(x)) \, dx \neq 0,$$

for each function $u \in C(\partial\Omega, \mathbb{R}^k)$ (we can write $u = \bar{u} + \tilde{u}$ where $\bar{u} = \int_{\partial\Omega} u(x) \, dx = 0$, and $\bar{u} \perp \tilde{u}$) with $|\bar{u}|_E \geq R$ and $\|u - \bar{u}\|_{L^\infty(\partial\Omega)} \leq C$

Proof. By the way of contradiction. Assume that for some $C > 0$ there is exist a sequence of functions $\{u_n\}_{n=1}^\infty \in C(\bar{\Omega}, \mathbb{R}^k)$, with

$$|\bar{u}_n|_E \rightarrow \infty, \quad \|u_n - \bar{u}_n\|_{L^\infty(\partial\Omega)} \leq C$$

and

$$\int_{\partial\Omega} g(u_n(x)) \, dx = 0. \quad (1.6)$$

We constructed a subsequence of u_n one can assume that $\bar{z}_n = \frac{\bar{u}_n}{|\bar{u}_n|_E}$ converges to some point $z \in \mathbb{S}$. The uniform bound on $u_n - \bar{u}_n$ implies that also $\frac{u_n}{|u_n|_E}$ converges to z and this convergence is uniform with respect to $x \in \bar{\Omega}$. It follows from the assumption G2 : that

$$\lim_{n \rightarrow \infty} \frac{g(u_n(x))}{|g(u_n(x))|_E} = \varphi(z)$$

uniformly in $\bar{\Omega}$. Since $\varphi(z)$ is in the unit sphere one can find an integer n_0 such that if $n \geq n_0$ and $x \in \bar{\Omega}$, then

$$\left\langle \frac{g(u_n(x))}{|g(u_n(x))|_E}, \varphi(z) \right\rangle_E \geq \frac{1}{4}$$

Define

$$\gamma_n(x) = |g(u_n(x))|_E.$$

By G1 : clearly $\gamma_n > 0$ everywhere. For $n \geq n_0$

$$\begin{aligned} \left\langle \int_{\partial\Omega} g(u_n(x)) \, dx, \varphi(z) \right\rangle_E &= \int_{\partial\Omega} \langle g(u_n(x)), \varphi(z) \rangle_E \, dx \\ &= \int_{\partial\Omega} \gamma_n(x) \left\langle \frac{g(u_n(x))}{\gamma_n(x)}, \varphi(z) \right\rangle_E \, dx \geq \frac{1}{4} \int_{\partial\Omega} \gamma_n(x) \, dx > 0 \end{aligned}$$

Therefore, $\int_{\partial\Omega} g(u_n(x)) \, dx > 0$. Now we have contradiction with (1.6)

The proof completely of the lemma. $\square$

II. MAIN RESULT

Let

$$Qu = Nu. \quad (2.1)$$

Be linear elliptic equation with nonlinear boundary condition. Suppose N is continuous and bounded (i.e.; $|Nu|_E \leq C$ for all u). If Q has a compact inverse Q^{-1} then by Leray-Schauder theory (2.1) has a solution. On the other hand if Q is not invertible the existence of a solution depends on the behavior of N and its interaction with the null space of Q see [19].

Theorem 2.1. Suppose $g \in C^1(\mathbb{R}^k, \mathbb{R}^k)$ satisfies $G1$:, $G2$:, and $G3$:. If $h \in C(\partial\Omega, \mathbb{R}^k)$, satisfies $\bar{h} = 0$. Then, (1.1) has at least one solution.

Proof. Define

$$J : H^1(\Omega) \rightarrow R$$

be continuous map in $H^1(\Omega)$ with the $L^2(\Omega)$ norm

$$J(v) = E(u, v)$$

Define

$$Dom(L) := \{u \in H^1(\Omega) : -\Delta u + A(x)u = 0\}$$

Define an operator L on $L^2 = L^2(\Omega, \mathbb{R}^k)$ for $u \in Dom(L)$ and each $v \in H^1(\Omega)$ by

$$E(u, v) = \langle Lu, v \rangle_{L^2(\Omega)},$$

we use the embedding theorem see [20] since you know that $H^1(\Omega) \hookrightarrow L^2(\Omega)$ and the trace theorem ($H^1 \rightarrow L^2(\partial\Omega)$). Thus, $L : Dom(L) \subset L^2(\partial\Omega) \rightarrow L^2(\partial\Omega)$ then the equation

$$E(u, v) = \langle h, v \rangle_{L^2(\partial\Omega)} \quad \forall v \in H^1(\partial\Omega),$$

if and only if

$$Lu = h.$$

The latter equation is solvable if and only if

$$Ph := \frac{1}{|\partial\Omega|} \int_{\partial\Omega} h = 0.$$

Now if $h \in L^\infty(\partial\Omega, \mathbb{R}^k)$ and $Ph = 0$. Then, each solution $u \in H^1(\Omega)$ is Hölder continuous, so $u \in C^\gamma(\bar{\Omega}, \mathbb{R}^k)$ for some $\gamma \in (0, 1)$. Since we know that there is constant $r_1 > 0$ such that

$$\|u\|_\gamma \leq r_1 (\|u\|_{L^2(\partial\Omega)} + \|h\|_{L^\infty(\partial\Omega)}).$$

When $Ph = 0$ there is a unique solution $Kh = \tilde{u} \in H^1(\Omega)$ with $P\tilde{u} = 0$ to

$$Lu = h,$$

and if $h \in C(\partial\Omega) = C(\partial\Omega, \mathbb{R}^k)$ then

$$\|Kh\|_\gamma \leq r_1 (\|Kh\|_{L^2(\partial\Omega)} + \|h\|_{L^\infty(\partial\Omega)}) \leq r_2 \|h\|_{C(\partial\Omega)}$$

and K maps $C(\partial\Omega)$ into itself take compact set to compact set i.e.; compactly.

Let Q be the restriction of L to $L^{-1}(C(\partial\Omega)) = KC(\partial\Omega) + \mathbb{R}^k$. We define $N : C(\partial\Omega) \rightarrow C(\partial\Omega)$ by

$$N(w)(x) := h(x) - g(w(x)) \quad \forall w \in C(\partial\Omega)$$

is continuous. Now (1.1) can be written as

$$Qu = Nu$$

Ref

19. J. Mawhin; Topological degree methods in nonlinear boundary-value problems, in NSFCBMS Regional Conference Series in Mathematics, American Mathematical Society, Providence, RI, 1979.

and $\ker Q = \operatorname{Im} P$, $\operatorname{Im} Q = \ker P$. The linear map Q is a Fredholm map (see [16]) and N is Q -compact (see [19]). Now we define the Homotopy equation as follows Let $\lambda \in [0, 1]$ such that

$$Qu = \lambda Nu. \quad (2.2)$$

The a priori estimates (i.e.; the possible solutions of (2.2) are uniformly bounded in $C(\partial\Omega)$). Now we show that the possible solutions of (2.2) are uniformly bounded in $C(\partial\Omega)$ independent of $\lambda \in [0, 1]$ Since we know that $u = \bar{u} + \tilde{u}$ where $\bar{u} = Pu$. Then

$$\|\tilde{u}\|_\gamma = \|\lambda K Nu\|_\gamma \leq r_2 \|Nu\|_{C(\partial\Omega)} \leq R_1,$$

where R_1 is a constant (g is bounded function). It remains to show that $\bar{u} \in \mathbb{R}^k$ is bounded, independent of $\lambda \in [0, 1]$. By the way of contradiction assume is not the case (i.e.; $\bar{u}$ unbounded). Then there are sequence $\{\lambda_n\} \subset [0, 1]$, and $\{u_n\} \subset \operatorname{Dom}(Q)$ with $\|\tilde{u}_n\|_\gamma \leq R_1$,

$$Qu_n = \lambda_n Nu_n \text{ and } |\bar{u}_n|_E \rightarrow \infty,$$

we get that

$$PNu_n = PN(\tilde{u}_n + \bar{u}_n) = - \int_{\partial\Omega} g(\tilde{u}_n(x) + \bar{u}_n(x)) dx = 0.$$

Now $u_n = \tilde{u}_n + \bar{u}_n$ so $\|u_n - \bar{u}_n\|_{L^\infty(\partial\Omega)} = \|\tilde{u}_n\|_{L^\infty(\partial\Omega)} \leq R_1$ and $\|\bar{u}_n\|_{L^\infty(\partial\Omega)} \rightarrow \infty$. It follows from Lemma 1.1 that for all sufficiently large n

$$\int_{\partial\Omega} g(u_n(x)) dx \neq 0.$$

We have reached a contradiction, and hence all possible solutions of (2.2) are uniformly bounded in $C(\partial\Omega)$ independent of $\lambda \in [0, 1]$

Let $\bar{B}(0, r) = \{x : |x|_E \leq r\}$ denote the ball in $C(\partial\Omega, \mathbb{R}^k)$. Now you can apply Leray-Schauder degree theorem see ([18],[19]), the only thing left to show is that

$$\deg(PN, \bar{B}(0, r) \cap \ker Q, 0) \neq 0,$$

for large $r > 0$. So $\deg(PN, \bar{B}(0, r) \cap \ker Q, 0) = \deg(g, \bar{B}_r, 0)$, where $\bar{B}_r$ is the ball in $\mathbb{R}^k$ of radius r . Since for $|x|_E$ large, and $\deg(\varphi) \neq 0$ we have that $\deg(g, \bar{B}_r, 0) \neq 0$ for large r . Therefore $\deg(PN, \bar{B}(0, r) \cap \ker Q, 0) \neq 0$ By Leray-Schauder degree theorem equation (2.2) has a solution when $\lambda = 1$. Therefore, equation (1.1) has at least one solution. This proves the theorem. $\square$

We will give one example.

Example 2.1. Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$ is a bounded domain with boundary $\partial\Omega$ of class C^∞ . Let

$$\begin{aligned} -\Delta u + A(x)u &= 0 \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + \frac{u}{1 + |u|_E^2} &= h(x) \quad \text{on } \partial\Omega \end{aligned} \quad (2.3)$$

where $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{2 \times 2}$, and where $u = (u_1, u_2) \in \mathbb{R}^2$ and h real valued function and continuous on $\partial\Omega$, and $\int_{\partial\Omega} h(x) dx = 0$ and $g(u) = \frac{u}{1 + |u|_E^2}$

$$\lim_{|u|_E \rightarrow \infty} g(u) = \lim_{|u|_E \rightarrow \infty} \frac{u}{1 + |u|_E^2} = 0$$

$g(u)$ vanishes at infinity, clearly $g \in C^1(\mathbb{R}^2, \mathbb{R}^2)$ and bounded with $g(u) \neq 0$, for $|u|_E$ large. Therefore g satisfies G1 :.

$$\frac{g(ru_1, ru_2)}{|g(ru_1, ru_2)|} = \frac{g(ru)}{|g(ru)|} = \frac{\frac{ru}{1 + |ru|_E^2}}{\left| \frac{ru}{1 + |ru|_E^2} \right|} = \frac{u}{|u|_E} = u$$

For all u in $\mathbb{S}$ and $r > 0$. Therefore G_2 : holds.

And $\varphi(u) = u$ so that $\deg(\varphi) \neq 0$. Therefore G_3 : holds. By Theorem 2.1. Then, equation (2.3) has at least one solution.

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Applications of Semigroups

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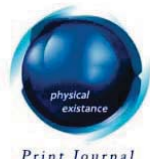
Abstract- This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc.

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Applications of Semigroups

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Abstract- This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc.

I. INTRODUCTION

The concept of a semigroup is relatively young, the first, often fragmentary, studies were carried out early in the twentieth century. Then the necessity of studying general transformations, rather than only invertible transformations (which played a large role in the development of group theory) became clear. During the past few decades connection in the theory of semigroups and the theory of machines became of increasing importance, both theories enriching each other. In association with the study of machines and automata, other areas of applications such as formal languages and the software use the language of modern algebra in terms of Boolean algebra, semigroups and others. But also parts of other areas, such as biology, psychology, biochemistry and sociology make use of semigroups.

The theory of automata has its origins in the work by Turing (Shannon1948, and Heriken 1994.). Turing developed the theoretical concept of what is now called Turing machines, in order to give computability a more concrete and precise meaning. Hannon investigated the analysis and synthesis of electrical contact circuits using switching algebra. The work of McCullon and pitts centers on neuron models to explain brain functions and neural networks by using finite automata. Their work was continued by Kleene. The development of technology in the areas of electromechanical and machines and particularly computers had a great influence on automata theory which traces back to the mid-1950s. Many different parts of pure mathematicians are used as tools such as abstract algebra, universal algebra, lattice theory, category theory, graph theory, mathematical logic and the theory of algorithms. In turn automata theory can be used in economics, linguistics and learning processes.

The beginning of the study of formal languages can be traced to Chomsky, who introduced the concept of a context-free language in order to model natural languages in 1957. Since then late 1960 there has been considerable activity in the theoretical development of context-free languages both in connection with natural languages and with the programming languages. Chomsky used semi-thue systems to define languages, which can be described as certain subsets of finitely generated free monoids. Chomsky (1957) details a revised approach in the light of experimental evidence and careful consideration of semantic and syntactic structures of sentences. For a common approach to formal languages and the theory of automata we refer to Eilenberg (1974).

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Semigroups can be used in biology to describe certain aspects in the crossing of organisms, in genetics and in consideration of metabolisms. The growth of plants can be described algebraically in Hermann and Rosenberg (1975). Further material on this subject is contained in Holcombe (1982). Details on the use of semiautomata in metabolic pathways and the aid of a computer therein, including a theory of scientific experiments, can be found in Krohn, Langer and Rhodes (1976). Rosen (1973) studies ways in which environmental changes can affect the repair capacity of biological systems and considers carcinogenesis and reversibility problems. Language theory is used in cell – development problems, as introduced by Lindenmayer (1968), Hermann and Rosendalg (1975). Suppes (1969) Kiesras (1976) develop a theory of learning in which a subject is instructed to behave like a semiautomaton.

The study of kinship goes back to a study by A. Weil in response to an inquiry by the anthropologist C. Levi-Strauss in 1949. White (1963), Kim and Breiger (1979) and Breiger, Boorman and Sable (1975) are also developed the elementary structure of kinship. Ballonoff (1974) presents several fundamental papers on kinship. Carlso (1980) gives elementary examples of applications of groups in anthropology and sociology. Rudolf Lidl and Guter Pilz were started with a selected set X of basic relations such that the set of all their relation products yields all remaining kinship relations. In this way they arrive at the concept of a free (hence infinite) semigroups over X . Sociology includes the study of human interactive behavior in group situations, in particular, in underlying structures of societies. Such structures can be revealed by mathematical analysis. This indicates how algebraic techniques may be introduced into studies of this kind.

II. SEMIGROUPS AND ITS APPLICATIONS

Now a days the theory of semigroups has been expanded greatly due to its applications to computer science and we also finds its usage in biological science and sociology. In this section we discuss some applications of semigroups in different areas.

a) *Semigroups –Automaton*

The algebraic theory of automata, which uses algebraic concepts to formalize and study certain types of finite-state machines. One of the main algebraic tools used to do this is the theory of semigroups. Automaton is an abstract model of computing device. Using this models different types of problems can be solved. We discuss what is common to all automata by describing an abstract model will be amenable to mathematical treatment and see that there is a close relationship between automata and semigroup. We can establish a correspondence between automata and monoids.

The problem may be identifying or adding two integers etc. i.e., we will be encountering automata in several forms such as calculating machines, computers, money changing devices, telephone switch boards and elevator or left switchings. All the above have one aspect in common namely a “box” which can assume various states. These states can be transformed into other states by outside influence and process “outputs” like results of computations.

- i. *Semi automata:* A semi automaton is a triple $Y = (Z, A, \delta)$ consisting of two non empty sets Z and A and a function $\delta : Z \times A \rightarrow Z$. Z is called the set of states, A is the set of input alphabet and δ the “next – state function” of Y .

- ii. *Automata:* An automaton is a quintuple $\check{A} = (Z, A, B, \delta, \lambda)$ where (Z, A, δ) is a semi automaton, B is a non empty set called the output alphabet and $\lambda : Z \times A \rightarrow B$ is the “output function.”

If $z \in Z$ and $a \in A$ then we interpret $\delta(z, a) \in Z$ as the next state into which z is transformed by the input a . $\lambda(z, a) \in B$ is the output of z resulting from the input a . Thus if the automaton is in the stage z and receives input a , then it changes to state $\delta(z, a)$ with out put $\lambda(z, a)$. A(semi)-automaton is finite, if all the sets Z , A and B are finite, finite automata are also called mealy automata.

Examples: 1) Let $A = \{a_1, a_2, \dots, a_n\}$, $B = \{b_1, b_2, \dots, b_m\}$ and $Z = \{z_1, z_2, \dots, z_k\}$

Description by tables:

Input table

δ	a_1	$a_2, \dots, a_n$
z_1	$\delta(z_1, a_1)$	$\delta(z_1, a_2) \quad \delta(z_1, a_n)$
z_2	$\delta(z_2, a_1)$	$\delta(z_2, a_2) \quad \delta(z_2, a_n)$
.	.	.
.	.	.
.	.	.
.	.	.
z_k	$\delta(z_k, a_1)$	$\delta(z_k, a_2) \quad \delta(z_k, a_n)$

Out put table

λ	a_1	$a_2, \dots, a_n$
z_1	$\lambda(z_1, a_1)$	$\lambda(z_1, a_2) \quad \lambda(z_1, a_n)$
z_2	$\lambda(z_2, a_1)$	$\lambda(z_2, a_2) \quad \lambda(z_2, a_n)$
.	.	.
.	.	.
.	.	.
.	.	.
z_k	$\lambda(z_k, a_1)$	$\lambda(z_k, a_2) \quad \lambda(z_k, a_n)$

Description by graphs

We depict $z_1, z_2, \dots, z_k$ as “discs” in the plane and draw an arrow labeled a_i from z_r to z_s , if $\delta(z_r, a_i) = z_s$. In case of an automaton we denote the arrow also by $\lambda(z_r, a_i)$. This graph is called the state graph

Example: (marriage Automation)

Let us consider the following situation in a household . The husband is angry or bored or happy: the wife is quite or shouts or cooks his favorite dish. Silence on her part does not change the husband’s mood, shouting “lowers” it by one “degree “(if he is already angry, then no change), cooking of his favorite dish creates general happiness for him. We try to describe this situation in terms of a semi-automaton $Y = (Z, A, \delta)$. We define $Z = (z_1, z_2, z_3)$ and $A = \{a_1, a_2, a_3\}$ with

z_1 = husband is angry

z_2 = husband is bored

z_3 = husband is happy

a_1 = wife is quite

a_2 = wife is shout

a_3 = wife cooks

The following is the description of δ

δ	a_1	a_2	a_3
z_1	z_1	z_1	z_3
z_2	z_2	z_1	z_3
z_3	z_3	z_2	z_3

Notes

For this situation add the output $B = \{b_1, b_2\}$ with the interpretation b_1 = husband shouts, b_2 = husband quite.

Let us assume that the husband is only shouts if he is angry and his wife shouts. Otherwise he is quite even in state z_3 . We shall define the output function λ by using the following output table.

λ	a_1	a_2	a_3
z_1	b_2	b_1	b_2
z_2	b_2	b_2	b_2
z_3	b_2	b_2	b_2

Let A^1 be the free monoid on A , i.e., $A^1 = F_A^{(1)}$, the monoid of all finite (including the empty) sequences of elements of the set A . Let us denote the identity (the empty sequence) in A^1 be Λ . We shall extend the next state function δ and the output function λ from $Z \times A$ to $Z \times A^1$ as follows:

For any $z \in Z$ and $a_1 a_2, \dots, a_n \in A^1$, define

$$\delta^1(z, a_1 a_2, \dots, a_n) = \begin{cases} z, & \text{if } n = 0 \\ \delta(\delta^1(z, a_1 a_2, \dots, a_{n-1}), a_n), & \text{if } n > 0. \end{cases}$$

and

$$\lambda^1(z, a_1 a_2, \dots, a_n) = \begin{cases} \Lambda, & \text{if } n = 0 \\ \lambda(z, a_1) \lambda^1(\delta(z, a_1), a_2, \dots, a_n), & \text{if } n > 0. \end{cases}$$

In other words, $\delta^1: Z \times A^1 \rightarrow Z$ and $\lambda^1: Z \times A^1 \rightarrow B^1$ are defined inductively as given above. Note that B^1 is the free monoid on B ; i.e., B^1 is the set of all finite (including the empty) sequences of elements in B .

Now we establish a correspondence between monoids and automata and discuss certain examples. If $(S, \cdot)$ is a monoid and we define $\delta: S \times S \rightarrow S$; $\lambda: S \times S \rightarrow S$ by $\delta(s, t) = s \cdot t$ and $\lambda(s, t) = s \quad \forall s, t \in S$. Then $(S, S, S, \delta, \lambda)$ is an automata and if we define $f_a: S \rightarrow S$ by $f_a(z) = \delta^1(z, a)$ for any $a \in A$ and $z \in S$, then $\{f_a / a \in A^1\}$ is a monoid (under the composition of mappings).

Let us consider the semi automaton given in above marriage automata. First we construct the table for $f_\Lambda, f_{a1}, f_{a2}, f_{a3}$ indicating their actions on the states z_1, z_2, z_3 .

*	f_Λ	f_{a1}	f_{a2}	f_{a3}
$z_1 \rightarrow$	z_1	z_1	z_1	z_3
$z_2 \rightarrow$	z_2	z_2	z_1	z_3
$z_3 \rightarrow$	z_3	z_3	z_2	z_3

Since $f_\Lambda = f_{a1}$, we delete f_{a1} and check whether $\{f_\Lambda, f_{a2}, f_{a3}\}$ forms a monoid. The operation “o” is given in the following table.

o	f_Λ	f_{a2}	f_{a3}
f_Λ	f_Λ	f_{a2}	f_{a3}
f_{a2}	f_{a2}	f_{a2a2}	f_{a3a2}
f_{a3}	f_{a3}	f_{a3}	f_{a3}

o	f_{a2a2}	f_{a3a2}
$z_1 \rightarrow$	z_1	z_2
$z_2 \rightarrow$	z_1	z_2
$z_3 \rightarrow$	z_1	z_2

Therefore $\{f_\Lambda, f_{a2}, f_{a3}\}$ is not a monoid. Now, we extend it to $\{f_\Lambda, f_{a2}, f_{a3}, f_{a3a2}, f_{a3a2}\}$ and verify that this set is a monoid. This is the monoid M corresponding to the automaton A^1 .

b) Semigroups- Formal languages

A formal language is an abstraction of general characterization of programming language. A grammar is model to describe a language. Given an alphabet or vocabulary (constructing of letters, words, symbols.....) we have a method (grammar) for constructing meaningful words or sentences from this alphabets. This immediately reminds us of the term “word semigroup” and indeed these free (word) semigroup well play a major role in constructing language. The formal language L constructed will be a subset of the free semigroup A^* or the free monoid A^* on the alphabet A.

We are familiar with notion of natural languages such as English, Telugu. English language consists of sentences like “Ram walked quickly, Gita ate slowly...”. The question is how to define a language, we need some basics. A finite non empty set of symbols denoted by A. Finite sequence of symbols from the alphabet, for example abab, abba are strings over alphabet $A = \{a, b\}$.

Concatenation of two strings u and v is the string obtained by appending symbols of v to the right end of u. $u = abab, v = abba$ then $uv = abababba$. The length of a string u is the number of symbols in the string and denoted by $|u|$. The string with no symbols is denoted by Λ and its length is zero.

If u and v are two strings then $|uv| = |u| + |v|$.

If A is an alphabet, A^* is the set of all strings obtained by concatenation zero or more symbols from A, and $A^* = A^+ - \{\Lambda\}$.

If $A = \{a, b\}$ then $A^* = \{\Lambda, a, b, ab, aa, \dots\}$.

In general a language is defined as a subset of A^* .

Grammar: A model to describe language is known as grammar. There are essentially three ways to construct a language.

I) Approach via grammar

II) Approach via automata

III) Algebraic approach

Example: Let $A = \{a\}$ and $G = \{g_0\}$, $g_0 \neq a$ and $\rightarrow = \{g_0 \rightarrow a, g_0 \rightarrow a g_0\}$, $G^1 = \{A, G, \rightarrow, g_0\}$. Then $g_0 =_z$ implies $z = a$ or $z = a g_0$. Again there is no x with $a \rightarrow x$, $a g_0 \rightarrow y$ is only possible for $y = aa$ or $y = aa g_0$. Thus $aa g_0 \rightarrow aaa$ and $aa g_0 \rightarrow aaa g_0$ etc.

$$\begin{array}{c}
 g_0 \rightarrow a \\
 \downarrow \\
 ag_0 \rightarrow aa \\
 \downarrow \\
 aag_0 \rightarrow aaa \\
 \downarrow \\
 aaag_0 \rightarrow \dots \\
 \vdots
 \end{array}$$

Cancellation of elements outside A^* gives us the result $L(G) = \{a, aa, aaa, \dots\} = A^*$

c) Semigroups in Biology

Semigroup can be used in biology to describe certain aspects in the crossing of organisms, in genetics, and in consideration of metabolisms.

Example: In breeding a strain of cattle, which can be black or brown monochromatic or spotted, it is known that black is dominant and brown recessive and that monochromatic is dominant over spotted. Thus there are four possible types of cattle in this herd.

a = Black monochromatic

c = Brown monochromatic

b = black spotted

d = brown spotted.

Due to dominance, in crossing a black spotted one with a brown monochromatic one, we expect a black monochromatic one. This can be symbolized by “ $b * c = a$ ”. The operation ‘ $*$ ’ can be studied for all possible pairs to obtain the table.

$*$	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

Then $S = \{a, b, c, d\}$ is a semigroup with identity element d .

In general, the table for breeding operations is more complicated.

We can ask for connections between hereditary laws and the corresponding semigroups. Of course, we need $SoS = S$ for such semigroups S , since $s \in S \rightarrow s^2$ would vanish after the first generation and would not even be observed. A different biological problem which leads to algebraic problems is as follows: all genetic information in an organism is given in the so called deoxy ribonucleic acid (DNA) which consists of two strands which are combined together to form the famous double helix. Each strand is made up as a polymer of four different basic substances, the nucleotides. If the

nucleotides are denoted by n_1, n_2, n_3 and n_4 , then the stand can be regarded as a word. Over $\{n_1, n_2, n_3, n_4\}$. DNA cannot put the genetic information into effect. By means of a messenger ribonucleic acid, the information contained in the DNA is copied ("translation") and then transferred onto the protein chains ("translation"). These protein chains are polymers consisting of 21 different basic substances, the amino acids, denoted by $a_1, a_2, a_3, a_4, \dots, a_{21}$. As with the DNA each protein chain can be regarded as a word over $\{a_1, a_2, a_3, \dots, a_{21}\}$.

In general, it is assumed that the sequence of nucleotides in the DNA is uniquely determined by the sequence of amino acids in a protein chain. In other words, it is assumed that there is a monomorphism from the free semigroup $F_{21} = \{a_1, a_2, a_3, \dots, a_{21}\}^*$ into the free semigroup $F_4 = \{n_1, n_2, n_3, n_4\}^*$.

d) Semigroups in Sociology

Sociology includes the study of human interactive behavior in group situations, in particular in underlying structures of societies. Such structures can be revealed by mathematical analysis. This indicates how algebraic techniques may be introduced into studies of this kind. So the study of such relations can be elegantly formulated in the language of semigroups.

Rudolf Lidl and Gunter Pilz define relation semigroup as follows.

Definition: $(R(M), \circ)$ is called the relation semigroup on M . The operation $\circ$ is called the relation product, where M is a monoid.

It is obvious that $R \in R(M)$ is transitive if and only if $R \circ R \subseteq R$. The set of their relation products yields all remaining kinship relations. In this way they arrive at the concept of a free semigroup over X . But there are only finite people on the earth and some kinship relations like "daughter of a mother" and "daughter of a father" might be considered to be "the same".

Definition: A kinship system is a semigroup $S = \{X, R\}$ where R is a relation on X , which express equality of kinship relationships.

Examples:

- 1) Let $X = \{\text{"is father of"}, \text{"is mother of"}\}$ and $R = \emptyset$. Then the kinship system S is the semigroup $\{\text{"is father of"}, \text{"is mother of"}, \text{"is grand father on fathers side of"}, \dots\}$.
- 2) Let $F = \text{"is father of"}, M = \text{"is mother of"}, S = \text{"is son of"}, D = \text{"is daughter of"}, B = \text{"is brother of"}, C = \text{"is child of"}, Si = \text{"is sister of"}$ then $X = \{F, M, S, D, B, Si, C\}$ and $R = \{(CM, CF), (BS, S), (SiD, D), (CBM, CMM), (MC, FM), (SB, S), (Dsi, D), (MBC, MMC), \dots\}$.

For complete list of R see Boyd, Haehl and Sailer (1972) [6]. The first means that in the semigroup we have $CM = CF$, children of the mother are the same as the children of the father.

Let G be a society i.e., a non empty set of people and let $S(G)$ be the semigroup of all different kinship relationships of this society.

The semigroup $S(G)$ often have special properties, eg. "is son of" and "is father of" are nearly inverse relations. The framework for investigation of special $S(G)$ would be the theory of inverse semigroups, i.e., semigroups S such that for all $s \in S$ with $ss^1 = s$

and $s^1ss^1 = s^1$. For instance, $(\text{Mn}(\mathbb{R}), \cdot)$ is an inverse semigroup for all $n \in \mathbb{N}$. If s denotes “is son of” and s^1 denotes “is father of” then these two equations hold in most societies.

e) Further Study

This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc.

For consider the following examples:

Examples:

- 1) Let the kinship system $S = \{X, R\}$ be defined by $X = \{P = (\text{“is parent of”}), C = (\text{“is child of”})\}$, $R = \{(PP, P), (PC, CP), (CC, C)\}$. Let a, b, c be the equivalence classes of P, C and PC respectively. Now “o” is given by

$\cdot$	a	b	c
a	a	c	c
b	c	b	c
c	c	c	c

Then $(S, \cdot)$ is a semigroup

- 2) Let $X = \{F, M\}$ and $R = \{(FF, F), (MM, M), (FM, FM)\}$ then $S = \{[F], [M], [FM]\}$ with the following table is semigroup.

$\cdot$	F	M	FM
F	F	FM	FM
M	FM	M	FM
FM	FM	FM	FM

And it is clearly,

$$FFF = F, \quad MMM = M, \quad FMFMFM = FM, \quad FMFFM = FM, \quad FMMFM = FM$$

* So, S is a regular semigroup which is not an inverse semigroup.

- 3) Let $X = \{F, M\}$ and $R = \{(FFF, F), (MM, M), (FM, MF)\}$ with the operatable given below is semigroup.

$\cdot$	F	FF	M	FM	FFM
F	FF	F	FM	FFM	FM
FF	F	FF	F	FM	FFM
M	FM	FFM	M	FM	FFM
FM	FFM	FM	FM	FFM	FM
FFM	FM	FFM	FF	FM	FFM

- 4) Let $X = \{S, M, F\}$ and $R = \{(SS, S), (MM, M), (FF, F), (SM, S), (SF, S), (MF, M), (MS, M)\}$ then $S = \{[S], [M], [F]\}$ is an inverse semigroup with the operation table as shown below

·	S	M	F
S	S	S	S
M	M	M	M
F	F	F	F

* 5) Let Si = sister and D = daughter then

$X = -Si, D$ and $R = -(SiD, D), (DSi, D), (SiSi, Si), (DD, D)$ then $S = -[Si], [D]$ is regular semigroup but not inverse semigroup with the following operation table.

·	Si	D
Si	Si	D
D	D	Si

6) Let $X = \{F, M\}$ and $R = \{(FF, F), (MM, M), (FM, MF)\}$ then $S = \{[F], [M], [MF], [FM]\}$ with the following operation table is semigroup.

·	F	M	FM	MF
F	F	FM	FM	MF
M	MF	M	FM	MF
FM	MF	FM	FM	MF
MF	MF	MF	FM	MF

Further we want to study some more structures of semigroups which will find applications in different areas.

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Kaluza-Klein Barotropic Cosmological Models with Varying Gravitational Constant G in Creation Field Theory of Gravitation

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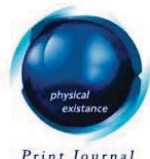
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Abstract- Kaluza Klein cosmological models with varying G in Hoyle Narlikar creation field theory of gravitation for barotropic fluid distribution have been investigated. The solution of the field equations have been obtained by assuming that $G=A^l$, where A is a scale factor and l is a constant. The physical properties of the model are studied.

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1. INTRODUCTION

From the Astronomical observations in the late eighties, it is concluded that the predictions of FRW type models do not always meet our requirements as was believed earlier (Smooth *et al.* (1992)). Hence alternatives theories were proposed of which the most well-known theory was steady state theory by Bondi and Gold (1948). The main approach of this theory is the universe does not have any singular beginning nor an end on the cosmic time scale where the matter density is constant throughout. They have considered a very slow but continuous creation of the matter in contrast to explosive creation of standard model for maintaining constancy of matter density. But the theory was discarded for not giving any physical justification about continuous creation of matter. To overcome this difficulty Hoyle and Narlikar (1964 a, b, c) adopted a field theoretic approach introducing a massless and charge-less scalar field in the Einstein Hilbert action to account for a matter creation. Bali and Saraf (2013a, b, c) have investigated cosmological models with varying (λ) in Creation field theory of gravitation. Narlikar (1973) stated that introduction of negative energy C-field solved the horizon and flatness problem faced by big bang model. Chatterjee and Banerjee (2004) have extended the study of Hoyle Narlikar theory in higher dimensional space times. Singh and Chaubey (2009) have studied Kantowski-Sachs and Bianchi type universes in creation field theory. Adhav *et al.* (2010) have obtained Bianchi type-I universe with cosmological models in Creation field theory of gravitation with different contexts. Recently Ghate *et al.* (2014 a) have investigated LRS Bianchi type-V dust filled universe with varying $\Lambda(t)$ in creation field theory of gravitation.

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In Einstein's general theory of relativity, the gravitational constant G plays the role of coupling constant between geometry and matter. The concept of gravitational constant G was first proposed by Dirac (1937). Pochoda and Schwarzschild (1963) and Gamow (1967) have studied the solar evolution in the presence of a time varying gravitational constant. Barrow (1978) assumed that $G\alpha^{-n}$ and obtained from Helium abundance for $5.9 \times 10^{-13} < n < 7 \times 10^{-13}$, $\left| \frac{\dot{G}}{G} \right| < (2 \pm .93) \times 10^{-12} \text{ yr}^{-1}$, by assuming a flat Universe.

Subsequently alternative theories of gravity especially Brans and Dicke (1961), Canuto *et al.* (1977) were developed to generalized Einstein's general theory of relativity by including variable G and satisfying conservation equation. Bali and Kumawat (2011, 12) have investigated cosmological models with variable G in C -field cosmology.

The Kaluza-Klein theory was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory by adding the fifth dimension. Kaluza (1921) has demonstrated that GR when interpreted as a vacuum 5D theory contains four-dimensional GR in the presence of electromagnetic field, together with Maxwell's electromagnetism. To do so, Kaluza supposed that - (i) model should maintain Einstein's vision that nature is purely geometric (ii) GR mathematics is not modified but just extended to five dimensions and (iii) there is no physical dependence on the fifth dimension. Klein (1926) suggested the compactification of fifth dimension. Number of relativists have studied Kaluza-Klein cosmological models with different contexts [Leon (1988), Chi (1990), Fukui (1993), Coley (1994), Liu & Wesson (1994)]. Alvarez *et al.* (1983), Ranjibar-Daemi *et al.*, (1984) and Marciano (1984) suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. Recently Ghate *et al.* (2014 b, c) have investigated Kaluza-Klein dust filled universe with time dependent Λ in creation field cosmology.

In this paper, Kaluza-Klein cosmological models with variable G in Hoyle Narlikar's creation field theory of gravitation for barotropic fluid distribution have been investigated. The solution have been obtained by assuming that $G = A^l$, where A is a scale factor and l is a constant ($l = -1$). The physical properties of the model are studied.

II. HOYLE-NARLIKAR THEORY

The Einstein field equations are modified by Hoyle and Narlikar [3-5] through the introduction of a massless scalar field usually called Creation field *viz.* C -field. The modified field equations are

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G \left[T_{(m)}^j + T_{(c)}^j \right], \quad (1)$$

where $T_{(m)}^j$ is a matter tensor for perfect fluid of Einstein's theory given by

$$T_{(m)}^j = (\rho + p) v_i v^j - p g_i^j \quad (2)$$

and $T_{(c)}^j$ is a matter tensor due to C -field given by

Ref

3. Alvarez, E. and Gavelo, M. B.: Phys. Rev. Lett., **51**, 931 (1983).

$$T_{(c)}^j = -f \left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha \right). \quad (3)$$

Here ρ is the energy density of massive particles and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1, v_\alpha = 0, \alpha = 1, 2, 3$. $f > 0$ is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

As T^{00} has negative value (i.e. $T^{00} < 0$), the C -field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Thus the energy conservation law reduces to

$${}^{(m)}T^{ij}{}_{;j} = -{}^{(c)}T^{ij}{}_{;j} = f C^i C^j{}_{;j}, \quad (4)$$

i.e. the matter creation through a non-zero left hand side is possible while conserving the overall energy and momentum.

The above equation is identical with

$$m g_{ij} \frac{dx^i}{ds} - C_j = 0, \quad (5)$$

which indicates the 4-momentum of the created particle is compensated by 4-momentum of the C -field. In order to maintain the balance, the C -field must have negative energy.

Further, the C -field satisfies the source equation

$$f C^i{}_{;i} = J^i{}_{;i} \quad \text{and} \quad J^i = \rho \frac{dx^i}{ds} = \rho v^i, \quad (6)$$

where ρ is the homogeneous mass density.

The conservation equation for C -field is given by

$$(8\pi G T_i^j)_{;j} = 0. \quad (7)$$

The physical quantities in cosmology are the expansion scalar θ , the mean anisotropy parameter Δ , the shear scalar σ^2 and the deceleration parameter q defined as

$$\theta = 3H, \quad (8)$$

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2, \quad (9)$$

$$\sigma^2 = \frac{1}{2} \left(\sum H_i^2 - 3H^2 \right) = \frac{3}{2} \Delta H^2, \quad (10)$$

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2}, \quad (11)$$

where H is a Hubble parameter.

III. METRIC AND FIELD EQUATIONS

Kaluza-Klein metric is considered in the form

$$ds^2 = dt^2 - A^2 [dx^2 + dy^2 + dz^2] - B^2 du^2, \quad (12)$$

where A, B are scale factors and are functions of cosmic time t and u is a space-time co-ordinate.

It is assumed that creation field C is a function of time only *i.e.* $C(x, t) = C(t)$ and

$$T_{(m)}^j = (\rho, -p, -p, -p, -p).$$

The field equations (1) for metric (12) with the help of equations (2) and (3) are given by

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = 8\pi G \left(\rho - \frac{1}{2} f \dot{C}^2 \right) \quad (13)$$

$$\text{and} \quad 2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{B}}{B} + 2 = 8\pi G \left(\frac{1}{2} f \dot{C}^2 - p \right), \quad (14)$$

where the overdot $\left(\frac{\cdot}{\cdot} \right)$ denotes partial differentiation with respect to t .

The conservation equation (7) for metric (12) is

$$8\pi G \left(\rho - \frac{1}{2} f \dot{C}^2 \right) + 8\pi G \left[\dot{\rho} - f \dot{C} \ddot{C} + \rho \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) - f \dot{C}^2 \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) + p \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) \right] = 0. \quad (15)$$

IV. SOLUTIONS OF FIELD EQUATIONS

The field equations (13) and (14) are two independent equations in five unknowns A, B, p, ρ and G . Hence two additional conditions may be used to obtain the solution.

We assume that the expansion θ is proportional to shear scalar σ . This condition leads to

$$B = A^n, \quad n \neq 1 \quad (16)$$

where n is a proportionality constant.

The motive behind assuming condition is explained with reference to Thorne (1967), the observations of the velocity red-shift relation for extra galactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent (Kantowski and Sachs (1966), Kristian and Sachs (1966)). To put more precisely, red shift place the limit $\frac{\sigma}{\theta} \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* (1980) have pointed that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

With the help of (16), field equations (13), (14) and (15) take the form

$$3(n+1)\frac{\dot{A}^2}{A^2} = 8\pi G \left(\rho - \frac{1}{2} f \dot{C}^2 \right), \quad (17)$$

$$(n+2)\frac{\ddot{A}}{A} + (n^2 + n + 1)\frac{\dot{A}^2}{A^2} = 8\pi G \left(\frac{1}{2} f \dot{C}^2 - p \right), \quad (18)$$

$$\text{and} \quad 8\pi G \left(\rho - \frac{1}{2} f \dot{C}^2 \right) + 8\pi G \left[\dot{\rho} - f \dot{C} \ddot{C} + \rho(n+3)\frac{\dot{A}}{A} - f \dot{C}^2(n+3)\frac{\dot{A}}{A} + p(n+3)\frac{\dot{A}}{A} \right] = 0. \quad (19)$$

Following Hoyle and Narlikar, the source equation of C -field: $C_{;i}^i = 0$ leads to $C = t$ for large r . Thus $\dot{C} = 1$.

Using $\dot{C} = 1$, equation (17) leads to

$$8\pi G\rho = (3n+1)\frac{\dot{A}^2}{A^2} + 4\pi Gf. \quad (20)$$

Using $\dot{C} = 1$ and barotropic condition $p = \gamma\rho$ in (18), we have

$$(n+2)\frac{\ddot{A}}{A} + (n^2 + n + 1)\frac{\dot{A}^2}{A^2} = 4\pi Gf - 8\pi G\gamma\rho, \quad (21)$$

where $0 \leq \gamma \leq 1$.

Multiplying equation (20) by γ and adding (21) gives

$$(n+2)\frac{\ddot{A}}{A} + \left[(n^2 + n + 1) + 3(n+1)\gamma\right]\frac{\dot{A}^2}{A^2} = (1-\gamma)4\pi Gf. \quad (22)$$

To obtain the deterministic solution, we assume

$$G = A^l, \quad (23)$$

where l is a constant and A is the scale factor.

Using equation (23) in equation (22), we get

$$2\ddot{A} + 2\left[\frac{n^2 + (1+3\gamma)n + (1+3\gamma)}{n+2}\right]\frac{\dot{A}^2}{A^2} = \frac{2(1-\gamma)4\pi f}{(n+2)}A^{l+1}. \quad (24)$$

Let $\dot{A} = F(A)$,

Thus $\ddot{A} = FF'$ with $F' = \frac{dF}{dA}$.

Using this in equation (24), it reduces to

$$\frac{dF^2}{dA} + 2\left[\frac{n^2 + n(1+3\gamma) + (1+3\gamma)}{n+2}\right]\frac{F^2}{A} = \frac{2(1-\gamma)4\pi f}{(n+2)}A^{l+1}, \quad (25)$$

which on simplification gives

$$F^2 = 2\frac{4\pi f(1-\gamma)}{(n+2)} \cdot \frac{A^{l+2}}{l+2 + 2\left[\frac{n^2 + n(1+3\gamma) + (1+3\gamma)}{n+2}\right]}. \quad (26)$$

The integration constant has been taken zero for simplicity.

Equation (26) leads to

$$\frac{dA}{\sqrt{A^{l+2}}} = \sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[l+2 + 2\left(\frac{n^2 + n(1+3\gamma) + (1+3\gamma)}{n+2}\right)\right]}} dt. \quad (27)$$

To obtain the determinate value of A in terms of cosmic time t , let $l = -1$.

Putting $l = -1$ in (27), we have

$$\frac{dA}{\sqrt{A}} = \sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[1 + 2\left(\frac{n^2 + n(1+3\gamma) + (1+3\gamma)}{n+2}\right)\right]}} dt. \quad (28)$$

On integration equation (28) gives

$$A = (at + b)^2, \quad (29)$$

$$\text{where } a = \frac{1}{2} \sqrt{\frac{2(1-\gamma)4\pi f}{(n+2) \left[1 + 2 \left(\frac{n^2 + n(1+3\gamma) + (1+3\gamma)}{n+2} \right) \right]}}. \quad (30)$$

$$\text{and } b = \frac{N}{2}. \quad (31)$$

Here N is a constant of integration.

$$\text{Thus we have } G = A^{-1} = (at + b)^{-2}. \quad (32)$$

Using equations (29) and (32), equation (20) simplifies to

$$8\pi\rho = 12a^2(n+1) + 4\pi f. \quad (33)$$

With the help of (29), the metric (12) leads to

$$ds^2 = dt^2 - (at + b)^4 [dx^2 + dy^2 + dz^2] - (at + b)^{4n} du^2. \quad (34)$$

Using $p = \gamma\rho$, equation (19) leads to

$$8\pi(G\dot{\rho} + \dot{G}\rho) - 4\pi\dot{G}f\ddot{C} - 8\pi Gf\ddot{C} - 8\pi G(n+3)f\dot{C}^2 \frac{\dot{A}}{A} + 8\pi G(n+3)\rho \frac{\dot{A}}{A}(1+\gamma) = 0. \quad (35)$$

With the help of equations (29) and (32), equation (33) yields

$$\frac{d\dot{C}^2}{dt} + (2n+5) \frac{2a}{(at+b)} \dot{C}^2 = \frac{a[(2n+4) + (2n+6)\gamma]}{(at+b)} \left[\frac{3a^2(n+1)}{\pi f} + 1 \right]. \quad (36)$$

To reach the deterministic value of $\dot{C}$, we assume $a=1$ and $b=0$.

Thus (36) leads to

$$\frac{d\dot{C}^2}{dt} + \frac{2(2n+5)}{t} \dot{C}^2 = \frac{[(2n+4) + (2n+6)\gamma]}{t} \left[\frac{3(n+1)}{\pi f} + 1 \right]. \quad (37)$$

On integration equation (37) reduces to

$$\dot{C}^2 t^{4n+10} = [(2n+4) + (2n+6)\gamma] \left[\frac{3(n+1)}{\pi f} + 1 \right] \int \frac{1}{t} t^{4n+10} dt. \quad (38)$$

Simplifying equation (38), we get

$$\dot{C} = \sqrt{\frac{(n+2) + (n+3)\gamma}{4(2n+5)} \left[\frac{3(n+1)}{\pi f} + 1 \right]}. \quad (39)$$

On integration of (39), we get

$$C = t \sqrt{\frac{(n+2) + (n+3)\gamma}{4(2n+5)} \left[\frac{3(n+1)}{\pi f} + 1 \right]}. \quad (40)$$

Taking $\pi f = \left[\frac{3(n+1)}{\frac{4(2n+5)}{(n+2)+(n+3)\gamma} - 1} \right]$, we find $\dot{C} = 1$, which agrees with the value used in

the source equation. Thus creation field C is proportional to time t and the metric (12) for the constraints mentioned above, leads to

$$ds^2 = dt^2 - t^4(dx^2 + dy^2 + dz^2) - t^{4n} du^2. \quad (41)$$

For the model (41), the physical parameters homogeneous mass density (ρ), Gravitational constant (G), the scale factor (A) and the deceleration parameter (q) are given by

Energy density (ρ),

$$8\pi\rho = 12(n+1) \left[1 + \frac{(n+2)+(n+3)\gamma}{(7n+18)-(n+3)\gamma} \right]. \quad (42)$$

Gravitational constant G ,

$$G = t^{-2}, \quad (43)$$

Scale factor A ,

$$A = t^2, \quad (44)$$

Spatial Volume,

$$V = t^{2n+6}$$

Expansion Scalar (θ),

$$\theta = \frac{2(n+3)}{t}, \quad (45)$$

Anisotropic parameter (Δ),

$$\Delta = \frac{3[(n+2)^2 + 3]}{4(n+3)^2}, \quad (46)$$

Shear Scalar (σ),

$$\sigma^2 = \frac{6[(n+2)^2 + 3]}{t^2}, \quad (47)$$

Deceleration parameter (q),

$$q = -\frac{1}{2}. \quad (48)$$

Physical Behavior of the Model:

From equations (16) and (29), the spatial scale factors are finite at the initial epoch $t=0$, hence the model has a point type singularity (MacCallum (1971)). From equations (45) and (46), we observe that the spatial volume is zero and expansion scalar is infinite at $t=0$ which show that the universe starts evolving with zero volume at $t=0$ which is big bang scenario.

In fig. 1, the plot of Gravitational constant versus cosmic time t has shown. We observed that, the gravitational constant G is initially infinite for the model. The Gravitational constant G is a decreasing function of time and approaches to zero for large values of t . In most variable G cosmologies (Weinberg (1972), Norman (1986)) G is a decreasing function of time. But the possibility of an increasing G has also been suggested by several authors (Abdel-Rahaman, A. M. M. (1990), Pradhan *et al.* (2007), Singh *et al.* (2008), Singh and Kale (2009), Singh *et al.* (1998)).

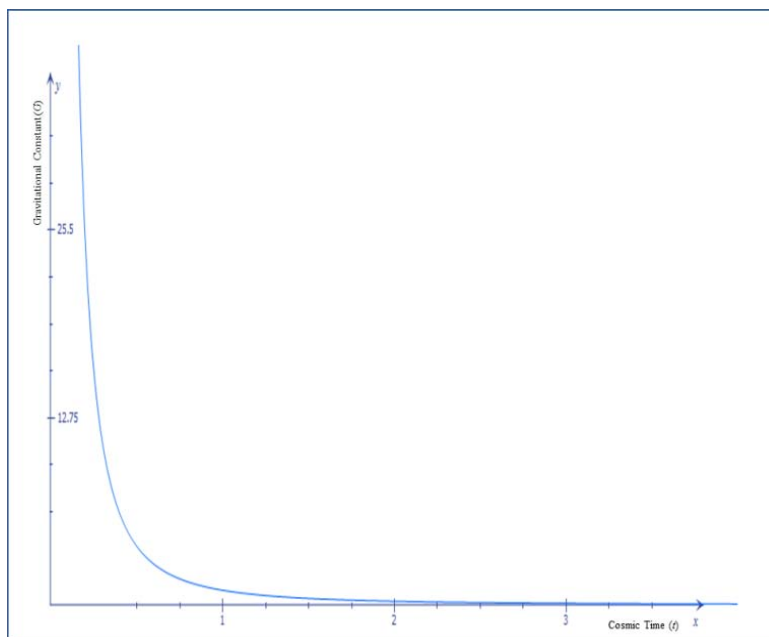


Figure 1 : Plot of Gravitational Constant (G) versus Cosmic Time (t)

From equations (46) and (48), the mean anisotropy parameter Δ is constant and $\frac{\sigma^2}{\theta^2} = \text{constant } t (\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy).

From equation (42), it is observed that the energy density is constant throughout the evolution of the universe hence the model represents steady state universe.

From equation (48), the value of deceleration parameter is $q = -\frac{1}{2}$, which lies between -1 and 0, hence the model is accelerating throughout the evolution.

V. CONCLUSION

Kaluza-Klein cosmological models have been investigated in Hoyle Narlikar's Creation field theory of gravitation. The source of energy momentum tensor is considered as barotropic fluid with varying gravitational constant G . It is worth to mention that the model obtained is point type singular, expanding, shearing. The mean anisotropic parameter is constant and $\frac{\sigma^2}{\theta^2} = \text{constant } (\neq 0)$, hence the model is anisotropic throughout the evolution of the universe. (i.e. the model is not isotropic for the complete evolution). The deceleration parameter is constant $\left(q = -\frac{1}{2}\right)$, which lies between -1 and 0, hence the model is accelerating throughout the evolution.

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General Definition of the Mean for Non-Additive Variables

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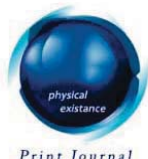
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General Definition of the Mean for Non-Additive Variables

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I. INTRODUCTION

a) General observation

If we sum some deterministic variable, we obtain the same result as if we substitute all of the variable by its arithmetical mean. In stochastic case it is true approximately and with a big probability if we sum big enough number of independent and identically distributed variables with finite second moment as central limit theorem show us (see [3], [5]). But in life we do not deal with summing only. If we will multiply them, for instance, and if we substitute the values by arithmetical mean, the result will differ more if the values of the variables will vary from the mean more. But if we used geometrical mean except of arithmetical ones, the situation will be exactly as in previous case of summation. This is an observation which brings us to our simple, but very useful concept.

There are a lot of other reasons why we need to represent a big set of data by only one number. Expected value or arithmetical mean is the most popular way how to do it, but not the best in all of the cases.

To make the problem of mean value more clear let us consider investment funds. They can measure the profit in different years by the rate of return, but rate of return is not additive variable too, this means, that the sum of the interest rate is not quantity with the sense. What information gives to us the arithmetical mean of the rates of return per different year. The answer is not very heartwarming.

Let us suppose that the number of time periods (say of years to be in the common practical situation) is N , and rate of profit in i -th period is ξ_i . let us suppose, that mean value of rates is equal to K , hence:

$$\frac{1}{N} \sum_{i=1}^N \xi_i = K \quad (1)$$

overall rate of profit by all of the N time periods is

$$\zeta = \prod_{i=1}^N (1 + \xi_i) - 1 \quad (2)$$

If $N \geq 2$ equations (1) and (2) does not determine ζ — which is the value we are actually interested in — in unique way. On the set of all $(\xi_i)_{i=1}^N$ fulfilling (1) take the function ζ different values: its maximum is $(1 + K)^N - 1$, and it is not bounded below indeed, hence: *The knowledge of arithmetical mean admit*

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to bounded the rate of the whole return above but not below. With the same arithmetical mean the rate of the whole return can be arbitrarily small.

1.1 Proof: If $N = 2$ then if ξ_1 a ξ_2 are rates of return in a two successive time periods and ζ rate of return per two period and if K is the arithmetical mean of ξ_1 a ξ_2 we have:

$$(1 + \xi_1) \cdot (1 + \xi_2) = 1 + \zeta$$

$$\frac{1}{2} \xi_1 + \frac{1}{2} \xi_2 = K$$

hence

$$\xi_2 = -\xi_1 + 2K$$

$$\zeta = 2K - \xi_1^2 + 2\xi_1 K \quad (3)$$

The last equation gives attitude toward actual rate of return per two periods and rate of return per one of the period in the case of constant average rate of return K . The dependence is analytical so that we can find the maximum as the zero point of the derivative:

$$\frac{\partial}{\partial \xi_1} \zeta = -2\xi_1 + 2K = 0 \iff \xi_1 \xi_2 = K \quad (4)$$

and

$$\frac{\partial^2}{\partial \xi_1^2} \zeta = -2 \quad (5)$$

so that if average rate of return is constant equal to K then:

- rate of return ζ have maximum $\zeta = (K + 1)^2 - 1$ if $\xi_1 = \xi_2 = K$, i. e. if all of the rates are equal.
- If one of the rate is equal to zero, $\xi_1 = 0$, and the second is equal to $\xi_2 = 2K$ then the whole rate of return is $\zeta = 2K$.
- If one of the rate $\xi_1 = -1$ i. e. we lost all of the capital, it is enough, to second rate of return be $\xi_2 = 1 + 2K$ and the mean value will stay K , while whole rate of return will be $\zeta = -1$. It means that the investor lost all of the invested resources, but arithmetical mean of rates of the return is still K . This fact show the marketing potential of arithmetical mean in the case. You can report the average rate of return 10% while you bereave your clients for all of the money.
- further if ξ_1 goes to $-\infty$, then ξ_2 goes to $-\infty$ and total rate of return goes to $-\infty$ but we can still keep the positive average.

A similar situation occurs also in the case when the number of time periods is greater. If we know arithmetical mean K of the rates of return the rate of return per N time periods is

$$\left(1 + N K - \left(\sum_{i=1}^{N-1} \xi_i\right)\right) \left(\prod_{i=1}^{N-1} (1 + \xi_i)\right) = 1 + \zeta \quad (6)$$

and the function ζ , with arguments made of $N - 1$ rates of returns ξ_i has again maximum

$$\zeta = (1 + K/N)^N - 1 \quad (7)$$

in the point

$$(\xi_i)_{i=1}^N = (K)_{i=1}^n \quad (8)$$

and ζ is not bounded above.

- Present value of invested capital should be equal to 0 independently on the arithmetical mean of rates of return. It is necessary and sufficient if one of the rates is equal to -1 .
- If one of a fund has (arithmetical) average of rate returns higher than other, it does not necessary mean, that it has higher total rate of return per all of the periods (that it brings higher profit to its savers). For the marketing point of view to show mean o rates of returns is better advertisement for the funds which have higher diversity between rates of return.

We can see, that arithmetical mean does not gives to us any substantive information in that case.

Similar situation is, of course, nowadays popular average temperature of the earth (see [4]). What rush people make about it notwithstanding this quantity is not satisfactorily defined. The number, which

should be a result of some integration does not realize themselves anywhere. This fact will goes more clear, if we imagine that we calculated the average temperatures of stars that we see. The result should not be the temperature of any place in the universe. /it There exists average of temperatures, but not average temperature.

In the case of investment funds, one can say, that we only used bad mean. That usage of the geometrical mean, brings to us much more better result. If we should compute the mean using the rule:

$$\zeta = \left(\prod_{i=1}^N (1 + \xi_i) \right)^{\left(\frac{1}{N}\right)} - 1 \quad (9)$$

expect of

$$\zeta = \frac{1}{N} \sum_{i=1}^N \xi_i \quad (10)$$

we should obtain something, connected with the rate of return by the unique and the simple way: $\left(\prod_{i=1}^N (1 + \xi_i) \right) = (1 + \zeta)^N$. Particularly funds with higher mean rate of return should have higher whole return.

In the history there are more cases of attempts to generalize the notion of mean value. Two of them I find the most interesting and the most deep.

II. TWO GENERAL CONCEPTIONS OF MEAN

With one generalization come János Dezs Aczél, the great Hungarian mathematician living in Canada. As an expert on the functional equations, he watch the question: how big family of possible means of two values one can obtain if he compute arithmetical mean of images of this values by some mapping and then he returned back with inverse mapping. To be more precise on one side we can imagine the mean as a binary operation $\circ$ and on the other hand as a value

$$(x, y) \mapsto k^{-1} \left(\frac{k(x) + k(y)}{2} \right) \quad (11)$$

General properties of this mean are described in [Azzel p. 229]. Main result, which shows quite general character of this definition is:

2.2. Theorem: There exists a continuous and strictly monotonic function k which gives a value of a mean; (11) holds if, and only if, $\circ: I^2 \mapsto I$ is continuous and strictly increasing in both variables, idempotent, commutative (symmetric) and medial (bisymmetric) which means:

$$(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w) \quad (12)$$

It is rely beautiful result but not practical enough.

On the other way, in the Book Means and Their Inequalities, P. S. Bullen, D. S. Mitrinović P. M. Vasić (see [2] p. 372) attempt to axiomatic definition of the mean:

- it is symmetric,
- homogeneous of degree 1,
- reflexive
- associative, $f(a_1, \dots, a_n) = f(f(a_1, \dots, a_p), \dots, f(a_1, \dots, a_p), a_{p+1}, \dots, a_n)$
- increasing in each variable.

This definition is very good of generalization of the term, but not good enough for us, from a lot of reasons. One of the reason should be that a average interest rate of saving is not symmetrical (and the symmetry is not disrupt only by adding of some weights). By latest interest are interested all of the deposits, but by the first one only the first deposit.

The drawback of these conceptions is, that the mean is related only to the values, it is computed from but not to their meaning, which means to this what we are going to do with the values. This fact has a deep impact. There are lot of money ion the

(For instance: arithmetical mean has sense only for additive variables it means variables we are going to sum but common definition does not respect this fact.) Our conception is at once more simple,

more general. It is, in some way, the generalization of the Azzel's conception. The central role plays the *objective function* in it. This function characterize what are the values, we deal with, relevant for.

III. GENERAL DEFINITION OF THE MEAN

3.3. Definition: Z is the mean value of $(z_i)_{i=1}^n$ with respect to function $F: \prod_{i \in I} X^i \rightarrow Y$ if

$$F((z_i)_{i=1}^n) = F((Z)_{i=1}^n) \quad (13)$$

(i. e. $F(z_1, z_2, \dots, z_n) = F(Z, Z, \dots, Z)$).

If F is injection, then the mean value is determined by unique way.

If $\text{Im}(f) = \text{Im}(f|_{\Delta})$, where Δ is diagonal: $\Delta = \{(x)_{i=1}^n | x \in X\}$, mean exists for every input values.

3.4. Example: If F is summation, we obtain arithmetical mean. If F is multiplying, we obtain geometrical mean.

In the case of an election, the votes are summed. Arithmetical mean of number of votes corresponds to proportional representation in parliament.

Rates of inflation does multiply to each other. If

$$\iota := [.01, .03, .02, .01, .03] \quad (14)$$

is the rate of inflation per 5 time periods, the rate of inflation per all of the time is

$$\left(\prod_{j=1}^5 (1 + \iota_j) \right) - 1 = .10386857 \quad (15)$$

and if the rate of inflation should be the same per all of the time subintervals and should be equal to

$$\kappa := \left(\prod_{j=1}^5 (1 + \iota_j) \right)^{1/5} - 1 = .019960783 \quad (16)$$

then the rate of inflation per all of the time should be the same:

$$\left(\prod_{j=1}^5 (1 + \kappa) \right) - 1 = .10386857 \quad (17)$$

Hence if the objective function will be

$$\iota \mapsto \left(\prod_j (1 + \iota_j) \right) - 1 \quad (18)$$

then the mean with respect to this function is (except for some addition of unit) geometrical mean.

IV. MEAN RATE OF RETURN OF PENSION FUNDS

Let us suppose, that we know rate of return of pension funds. We are looking for mean rate of return, which admits to us compare this funds.

Let us suppose, that deposits of the savers are constant for a long of the time. We can suppose, that, if the deposit per year (together with state grants) are equal to x and if the rate of return, in consequence of dividing of profit in the year i is equal to ξ_i saver should have after the time N :

$$x \left(\sum_{j=1}^N \prod_{i=1}^j (1 + \xi_{N-i+1}) \right) \quad (19)$$

For saver is important mean rate of return with respect to the function

$$(\xi)_{i=1}^T \rightarrow \left(\sum_{j=1}^N \prod_{i=1}^j (1 + \xi_{N-i+1}) \right) \quad (20)$$

using our definition, this rate of the return is ζ , fulfilling the equation

$$K = \sum_{j=1}^N \left(\prod_{i=1}^j (1 + \xi_{N-i+1}) \right) = \sum_{j=1}^N \left(\prod_{i=1}^j (1 + \zeta) \right) = -\frac{(1 + \zeta)^{(N+1)} + 1 + \zeta}{\zeta} \quad (21)$$

It is an algebraical equation of the degree N .

If the time period is only one, the mean rate of return is equal to the rate of return per this period:

$$\zeta = -1 + K \quad (22)$$

If the number of time periods is equal to 2, the equation has two solutions, in general.

If the total remittance will be higher than $\frac{1}{4}$ of all saved remittances all of the solution will be real. In the case — it seem to be reasonable — when total remittance will be higher than double of saved remittances one solution will be positive and will be equal to

$$\zeta = -\frac{3}{2} + \frac{1}{2} \sqrt{1 + 4K} \quad (23)$$

In case of 3 periods, there is the only one real mean rate of return:

$$\zeta = 1/6 \frac{(28 + 108K + 12\sqrt{9 + 42K + 81K^2})^{2/3} - 8 - 8\sqrt[3]{28 + 108K + 12\sqrt{9 + 42K + 81K^2}}}{\sqrt[3]{28 + 108K + 12\sqrt{9 + 42K + 81K^2}}} \quad (24)$$

If the number of time periods is higher than four, we are not able to solve the equation (21) algebraically, and we have to solve it numerically.

4.5. Example: There was some pension funds in Czech republic during the five year between the years 1999 and 2003. This was an period with very wildy changed rates of return of the pension funds. We will concentrate onto these:

(1) ČSOB Progres, (2) Zemský PF, (3) PF KB, (4) ING, (5) Credit Suisse, (6) PF ČP, (7) Allianz, (8) Generali, (9) Nový ČP, (10) ČSOB Stabilita, (11) PF ČS, (12) Hornický PF; The following table show us their rate of return:

Funds	rate of return (in %)				
	1999	2000	2001	2002	2003
ČSOB Progres	7.7	5.6	3.9	4.3	4.3
Zemský PF	7.0	5.0	4.6	4.1	4.01
PF KB	7.2	4.9	4.4	4.6	3.4
ING	6	4.4	4.8	4	4
Credit Suisse	6.5	4.1	4.3	3.4	3.4
PF ČP	6.6	4.5	3.8	3.2	3.1
Allianz	6	3.8	4.4	3.7	3
Generali	5.3	3.6	4.6	4.1	3
Nový ČP	5.6	3.8	4.1	3.5	3.34
ČSOB Stabilita	6.1	4.2	3.2	3.0	2.34
PF ČS	4.4	4.2	3.8	3.5	2.64
Hornický PF	4.4	2	2.8	3.2	2.44

(25)

Českomoravská stavební spořitelna published for agents middleman papers for information. They ordered the pension funds by the arithmetical mean of their rate of return, which are, as we have explained, irrelevant (second row of following table, values are in percents). If we computed the mean with respect to objective function (ξ_i^k is rate of return of k -th fund at i -th time period):

$$\Phi: (\xi^k)_{i=1}^5 \mapsto \sum_{j=1}^5 \prod_{i=1}^j (1 + \xi_{5-i+1}^k) \quad (26)$$

(third column of following table, values are in percents) we can see, that it happens in some cases, that funds with higher rate of return are behind funds with smaller rate of return:

Funds	mean values of the rates of return	
	arithemtical (<i>inappropriate</i>)	vith respect to the Φ (<i>appropriate</i>)
ČSOB Progres	5.16	4.638
Zemský PF	4.94	4.501
PF KB	4.90	4.394
ING	4.64	4.358
Credit Suisse	4.34	3.895
PF ČP	4.24	† 3.704
Allianz	4.18	† 3.787
Generali	4.12	† 3.857
Nový ČP	4.06	3.757
ČSOB Stabilita	3.76	‡ 3.203
PF ČS	3.70	‡ 3.438
Hornický PF	2.96	2.789

(27)

(†) We can see, that the fund Allianz, which stay on the 7th place had the rate of return better, than PF ČP which is on the 6th place and better rate of return than this two had thwe fund Generali, which stay behind booth others. Booth are in the papers presented as the funds with the same awarage rate of return equal to 4.2.

(‡) The fund ČSOB Stabilita, the papers in question are written for its propagation too, is presented as the fund with mean rate of return (arithmetic mean 3.8) higher than rate of return of fund PF ČS (arithmetic mean 3.7) but it should be presented (with mean 3.2) behind the fund PF ČS (mean 3.4)!

If we are going to interpreted the result, we have to determine the sensitivity on the change of interest rate. We can substitute the rates of return by mean rates of return we have computed. The relative change of saved money after time T if the interest rate is changed from ξ onto ζ , (i. e.. rate of the profit or loss on the whole saved quantity) is

$$\frac{(1 + \zeta)^T \xi - \xi - (1 + \xi)^T \zeta + \zeta}{((1 + \xi)^T - 1) \zeta} \quad (28)$$

In our case is $T = 5$ and the values for differen funds are in the same order as in the previous tables, they are in percents:

0	-0.27	-0.49	-0.56	-1.5	-1.8	-1.7	-1.5	-1.7	-2.8	-2.4	-3.6
0.27	0	-0.21	-0.29	-1.2	-1.6	-1.4	-1.3	-1.5	-2.6	-2.1	-3.4
0.49	0.21	0	-0.072	-0.99	-1.4	-1.2	-1.1	-1.3	-2.4	-1.9	-3.2
0.56	0.29	0.072	0	-0.92	-1.3	-1.1	-1.0	-1.2	-2.3	-1.8	-3.1

1.5	1.2	1.0	0.93	0	-0.38	-0.21	-0.075	-0.27	-1.4	-0.91	-2.2
1.9	1.6	1.4	1.3	0.38	0	0.17	0.31	0.11	-1.0	-0.53	-1.8
1.7	1.4	1.2	1.1	0.21	-0.17	0	0.14	-0.061	-1.2	-0.70	-2.0
1.6	1.3	1.1	1.0	0.075	-0.31	-0.14	0	-0.20	-1.3	-0.83	-2.1
1.8	1.5	1.3	1.2	0.28	-0.11	0.061	0.20	0	-1.1	-0.64	-1.9
2.9	2.6	2.4	2.3	1.4	1.0	1.2	1.3	1.1	0	0.47	-0.83
2.4	2.1	1.9	1.9	0.92	0.53	0.70	0.84	0.64	-0.47	0	-1.3
3.8	3.5	3.3	3.2	2.2	1.8	2.0	2.2	2.0	0.83	1.3	0

(29)

In the i -th row j -th column the percentage of how much should be the whole amount higher if we invested into j -th fund except of into i -th fund is written. So in the proper choose of the fund we should have approximately 4 percent more than in the improper choose.

V. MEAN RATE OF RETURN OF SIMULTANEOUS SAVINGS

Suppose, that we have two accounts, each interested with different interest rate ξ_1 and ξ_2 . We divide the capital in amount $x_1 + x_2$ between them this way, than we will have in the first capital in amount x_1 and in the second x_2 . Objective function — future value in time t is the function $\Psi_{x_1, x_2, t}: (\xi_1, \xi_2) \mapsto x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t$ and it depends on three parameters. Mean interest rate with respect to this function is the solution ζ of the equation:

$$x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t = (x_1 + x_2) (1 + \zeta)^t \quad (30)$$

hence

$$\zeta = \left(\frac{x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)} - 1 \quad (31)$$

And it is generalized exponential mean. Worthy to note is its dependence on the time t , especially the limits $t \rightarrow \infty$.

$$\begin{aligned} \lim_{t \rightarrow \infty} \left(\frac{x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)} - 1 &= \lim_{x \rightarrow \infty} e^{\ln \left(\frac{x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)}} - 1 = \\ &= \lim_{x \rightarrow \infty} e^{\frac{\ln \left(x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t \right) - \ln (x_1 + x_2)}{t}} - 1 \end{aligned} \quad (32)$$

using the L'Hospital rule we obtain

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{\ln \left(x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t \right) - \ln (x_1 + x_2)}{t} &= \\ &= \lim_{t \rightarrow \infty} \frac{x_1 (1 + \xi_1)^t \ln (1 + \xi_1) + x_2 (1 + \xi_2)^t \ln (1 + \xi_2)}{x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t} \end{aligned} \quad (33)$$

and after canceling by the term $(1 + \xi_2)^t$ we obtain:

$$\lim_{t \rightarrow \infty} \frac{x_1 \ln (1 + \xi_1) \left(\frac{1 + \xi_1}{1 + \xi_2} \right)^t + x_2 \ln (1 + \xi_2)}{x_1 \left(\frac{1 + \xi_1}{1 + \xi_2} \right)^t + x_2} \quad (34)$$

Supposing: $\xi_1 < \xi_2$

$$\lim_{t \rightarrow \infty} \left(\frac{x_1 (1 + \xi_1)^t + x_2 (1 + \xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)} - 1 = \xi_2, \quad (35)$$

we obtain the greatest of both number. It looks mysteriously: If we save divide capital into two parts and save booth with some interest rates per unit of time ξ_1 and ξ_2 respectively, we obtain the same result as to save booth parts with interest rate per unit of time ζ , and if the time goes to infinity the difference between ζ and the maximum of ξ_1 and ξ_2 goes to the zero. On the other hand:

$$\lim_{t \rightarrow 0} \zeta = t (1 + \xi_1)^{\frac{x_1}{x_1 + x_2}} (1 + \xi_2)^{\frac{x_2}{x_1 + x_2}} - 1 \quad (36)$$

which is generalized geometrical mean.

The same result we obtain also in the case of more than two accounts. If the rates of interest will be (ξ_i) and the initial state of accounts will be (x_i) then the state function will be

$$(\xi_i) \mapsto \sum_{i=1}^n x_i (1 + \xi_i)^t \quad (37)$$

and mean value (ξ_i) with respect to the function will be solution ζ of the equation

$$\sum_{i=1}^n x_i (1 + \xi_i)^t = \sum_{i=1}^n x_i (1 + \zeta)^t = (1 + \zeta)^t \sum_{i=1}^n x_i \quad (38)$$

hence

$$\zeta = \left(\frac{\sum_{i=1}^n x_i (1 + \xi_i)^t}{\sum_{i=1}^n x_i} \right)^{\frac{1}{t}} - 1 \quad (39)$$

and if $\xi_n = \max_i (\xi_i)$ we can show in the same way, that

$$\lim_{t \rightarrow \infty} \zeta = e^{\left(\lim_{t \rightarrow \infty} \frac{\left(\sum_{i=1}^{n-1} x_i \ln(1 + \xi_i) \left(\frac{1 + \xi_i}{1 + \xi_n} \right)^t \right) + x_n \ln(1 + \xi_n)}{\left(\sum_{i=1}^{n-1} x_i \left(\frac{1 + \xi_i}{1 + \xi_n} \right)^t \right) + x_n} \right)} - 1 = \xi_n = \max_i (\xi_i) \quad (40)$$

a

$$\lim_{t \rightarrow 0} \zeta = \prod_{i=1}^n (1 + \xi_i)^{\sum_{j=1}^n \frac{x_i}{x_j}} \quad (41)$$

Aggregate interest rate per unit of time converges to maximal interest rate per unit of time with time going to infinity.

The similar situation is the saving annuities:

VI. MEAN INTEREST RATE OF SAVING

Suppose, savings with annuities x_i in the moments $t \in \mathbb{N}$ with constants interest rates ξ_i . Objective function is sum of future values of savings with different interest rates:

$$\Phi_{(x_i)_{i=1}^k, N} := \left((\xi_i)_{i=1}^k \right) \mapsto \sum_{i=1}^k \left(\sum_{j=0}^{N-1} x_i (1 + \xi_i)^j \right) = \sum_{i=1}^k \frac{x_i \left((1 + \xi_i)^N - 1 \right)}{\xi_i} \quad (42)$$

saved amount, $(x_i)_{i=1}^k$ and number of savings with different interest rates k are parameters Φ .

Mean interest rate of savings $\Xi = \Xi((x_i), N)$ with respect to function $\Phi_{(x_i)_{i=1}^k, N}$ is solution of equation:

$$xxx := \frac{\sum_{i=1}^k x_i}{\Xi} \left((1 + \Xi)^N - 1 \right) = \sum_{i=1}^k \frac{x_i \left((1 + \xi_i)^N - 1 \right)}{\xi_i} \quad (43)$$

We will investigate dependence on parametr N , i. e. on the number of saved ammounts. And again:

$$\lim_{t \rightarrow \infty} \Xi = \max_i (\xi_i) \quad (44)$$

6.6. Proof: Let

$$\eta = \eta(\xi_i, x_i, n, T) \quad (45)$$

be mean interest rate with respect to the function

$$\Phi := \xi \rightarrow \sum_{\tau=1}^n \left(\sum_{j=1}^k x_j (1 + \xi_j)^{(T-\tau)} \right) \quad (46)$$

i. e. solution of the equation

$$\sum_{\tau=1}^n \left(\sum_{j=1}^k x_j (1 + \xi_j)^{(T-\tau)} \right) = \left(\sum_{j=1}^k x_j \right) \sum_{\tau=1}^n (1 + \eta)^{(T-\tau)} \quad (47)$$

and let

$$\zeta(\xi_i, x_i, \tau, T) \quad (48)$$

be a mean interest rate with respect to the function

$$\Psi := \xi \rightarrow \sum_{j=1}^k x_j (1 + \xi_j)^{(T-\tau)} \quad (49)$$

i. e. solution of the equation

$$\sum_{j=1}^k x_j (1 + \xi_j)^{(T-\tau)} = (1 + \zeta)^{(T-\tau)} \left(\sum_{j=1}^k x_j \right) \quad (50)$$

It is clear, that

$$\min(\zeta(\xi_i, x_i, \tau, T))_{\tau=1}^n \leq \eta(\xi_i, x_i, n, T) \leq \max(\zeta(\xi_i, x_i, \tau, T))_{\tau=1}^n \quad (51)$$

And using (40) we have

$$\lim_{T \rightarrow \infty} \min(\zeta(\dots, \tau, T))_{\tau=1}^n = \lim_{T \rightarrow \infty} \max(\zeta(\dots, \tau, T))_{\tau=1}^n = \max_i (\xi) \quad (52)$$

hence

$$\lim_{T \rightarrow \infty} \eta(\xi_i, x_i, n, T) \quad (53)$$

that is why

$$\lim_{T \rightarrow \infty} \eta(\xi_i, x_i, T, T) = \lim_{n \rightarrow \infty} \lim_{T \rightarrow \infty} \eta(\xi_i, x_i, n, T) = \lim_{n \rightarrow \infty} \max_i (\xi_i) = \max_i (\xi_i) \quad (54)$$

Q. e. d.

Mean interest rate of saving is less than mean interest rate of interesting, but it has the same limit $t \rightarrow 0$ a $t \rightarrow \infty$.

The previous example was the mean, which has already a name. Let us finish with quite exotic mean, the mean which *is not increasing function in all of its arguments*.

6.7. Example: One of the specific bank in Czech republic, which offer mortgages is so called building societies. The common product they offer has three parts, one used at once (it's happening in order to accommodate certain Czech laws): saving with one interest rate — saved money is then used for the redemption of debt — so called bridging credit with the second interest — this credit is used at the time of saving — and credit with third interest rate, which is used for amortization of the rest of the

debt. Unnecessarily complicated? Mabe. And now imagine, that someone, who want to buy a house will comes to the mortgage market try to compare all of the mortgages by their interest rate and here he has three interest rates except of one. Of course, he need to substitute them by only one comparable number and I think that this number should be called, in accordance with tradition, mean of this three. But it is clear, that while customer wishes the interest rate of the loan and bridge loan to be the as low as possible he wants to have the interest rate of savings by contrast the greatest possible. So the mean will be increasing in the first two interest rates, but in the third will be decreasing.

Let us choose some denotation: suppose, that we pay the debt by payments x_1 with interest rate ξ_1 and at the same time we save money (as saving or insurance,...) by remittance at quantity x_2 with the interest rate ξ_2 both the time of duration N . After that we use the saved money to amortize part of the depth, than we pay the rest of the depth with interest rate ξ_3 by payments x_3 time of the length K .

For the evaluation of an expediency of a product like this is good to know the suitable mean value of the values ξ_1 , ξ_2 and ξ_3 . Present value of all of the payments, we are going to pay is

$$PV = x_1 \sum_{t=0}^{N-1} (1 + \xi_1)^t (1 + \xi_2)^{-N} + x_2 \sum_{t=1}^N (1 + \xi_2)^{-t} + x_3 \sum_{t=1}^K (1 + \xi_3)^{-t} (1 + \xi_2)^{-N} =$$

$$= \frac{(1 + \xi_2)^{-N} x_1 \left((1 + \xi_1)^N - 1 \right)}{\xi_1} - \frac{x_2 \left(\left((1 + \xi_2)^{-1} \right)^N - 1 \right)}{\xi_2} - \frac{\left(\left((1 + \xi_3)^{-1} \right)^K - 1 \right) (1 + \xi_2)^{-N} x_3}{\xi_3}$$

hence we are interested in mean value with respect to objective function

$$(\xi_1, \xi_2, \xi_3) \mapsto PV$$

and this is the solution of the equation

$$x_1 \sum_{t=0}^{N-1} (1 + \xi_1)^t (1 + \xi_2)^{-N} + x_2 \sum_{t=1}^N (1 + \xi_2)^{-t} + x_3 \sum_{t=1}^K (1 + \xi_3)^{-t} (1 + \xi_2)^{-N} =$$

$$= x_1 \sum_{t=0}^{N-1} (1 + \zeta)^t (1 + \zeta)^{-N} + x_2 \sum_{t=1}^N (1 + \zeta)^{-t} + x_3 \sum_{t=1}^K (1 + \zeta)^{-t} (1 + \zeta)^{-N}$$

If no one of the interest rate is equal to 0, the equation is equivalent to

$$\frac{(1 + \xi_2)^{-N} x_1 \left((1 + \xi_1)^N - 1 \right)}{\xi_1} - \frac{x_2 \left(\left((1 + \xi_2)^{-1} \right)^N - 1 \right)}{\xi_2} - \frac{\left(\left((1 + \xi_3)^{-1} \right)^K - 1 \right) (1 + \xi_2)^{-N} x_3}{\xi_3} =$$

$$= \frac{(1 + \zeta)^{-N} x_1 \left((1 + \zeta)^N - 1 \right)}{\zeta} - \frac{x_2 \left(\left((1 + \zeta)^{-1} \right)^N - 1 \right)}{\zeta} - \frac{\left(\left((1 + \zeta)^{-1} \right)^K - 1 \right) (1 + \zeta)^{-N} x_3}{\zeta}$$

We are not able solve this equation algebraically in general so we cannot write the explicit formula for mean, we are looking for, but for all possible choice of parameters we can solve it numerically.

There is an interesting fact, that this mean is reflective (mean value of (ζ, ζ, ζ) is (ζ)), but *it is not increasing in the first variable* (ξ_1): no other conception mentioned above reckon on that fact!

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Notes

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A Class of Regression Estimator with Cum-Dual Product Estimator As Intercept

By F. B. Adebola & N. A. Adegoke

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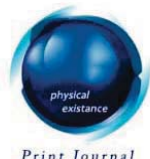


ACLASOFGRESSIONESTIMATORWITHCUMDUALPRODUCTESTIMATORASINTERCEPT

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A Class of Regression Estimator with Cum-Dual Product Estimator As Intercept

F. B. Adebola ^α & N. A. Adegoke ^σ

Abstract- This paper examines a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X. We obtained the bias and the mean square error (MSE) of the proposed estimator. We also obtained MSE of its asymptotically optimum estimator (AOE). Theoretical and numerical validation of the proposed estimator was done to show its superiority over the usual simple random sampling estimator and ratio estimator, product estimator, cum-dual ratio and product estimator. It was found that the asymptotic optimal value of the proposed estimator performed better than other competing estimators and performed in exactly the same way as the regression estimator, when compared with the usual simple random estimator for estimating the average sleeping hours of undergraduate students of the department of statistics, Federal University of Technology Akure, Nigeria.

Keywords: difference estimator, auxiliary variable, cum-dual product estimator, bias, mean square error, efficiency, simple random sampling.

I. INTRODUCTION

In estimating the mean of the study variable Cochran (1940), used the auxiliary information X at the estimation phase to increase the efficiency of the study variable. To estimate the ratio estimator of the population mean or total of the study variable Y, he used additional knowledge on the auxiliary variable X which was positively correlated with Y. When the relationship between the study variable Y and the auxiliary variable X is linear through the origin and Y proportional to X, the ratio estimator will be more efficient than the normal Simple Random Sampling (SRS) Sanjib Choudhury et al (2012). Also, Robson (1957) proposed product estimator and showed that when the relationship between the study variable Y and the auxiliary variable X is linear through the origin and Y is inversely proportional to X, the product estimator will be more efficient than the usual SRS. Murthy 1964 suggested the use of ratio estimator $\bar{y}_p$ when $\frac{\rho C_y}{C_x} > \frac{1}{2}$ and unbiased estimator $\bar{y}$ when $-\frac{1}{2} \leq \rho \frac{C_y}{C_x} \leq \frac{1}{2}$, where C_y , C_x and ρ are coefficients of variation of y, x and correlation between y and x respectively.

Suppose that SRSWOR of n units is drawn from a population of N units to estimate the population mean $\bar{Y} = \frac{1}{N} \sum_{i=1}^N y_i$ of the study variable Y. All the sample units are observed for the variables Y and X. Let (y_i, x_i) where $i = 1, 2, 3, \dots, n$ denote the set of the

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observation for the study variable Y and X . Let the sample means $(\bar{x}, \bar{y})$ be unbiased of the population means of the auxiliary variable $\bar{X}$ and study variable $\bar{Y}$ based on the n observations.

The usual product estimator of $\bar{y}$ is given as $\bar{y}_p = \frac{\bar{y}}{\bar{X}} \bar{x}$ and the usual regression estimator is given as $\bar{y}_{reg} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$, where $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$, and $\hat{\beta} = \frac{s_{xy}}{s_x^2}$ is the estimate slope of regression coefficient of Y and X . Cum-Dual Product estimator given as $\bar{y}_{dp} = \bar{y} \frac{\bar{X}}{\bar{x}^*}$, where $\bar{x}^*$ is the un-sampled auxiliary variable in X given as $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N - n}$ was obtained by Bandyopadhyay (1980). The use of auxiliary information in sample surveys was extensively discussed in well-known classical text books such as Cochran(1977), Sukhatme and Sukhatme(1970), Sukhatme, Sukhatme, and Asok (1984), Murthy(1967) and Yates(1960) among others. Recent developments in ratio and product methods of estimation along with their variety of modified forms are lucidly described in detail by Singh (2003).

II. THE PROPOSED CLASS OF ESTIMATOR

The proposed a class of regression estimator with dual product as the intercept for estimating population mean $\bar{Y}$, given as;

$$\bar{y}_{pd}^* = \bar{y} \frac{\bar{X}}{\bar{x}^*} + \alpha(\bar{X} - \bar{x}^*) \quad \dots\dots\dots (1)$$

Where, α is a suitable scalar.

We obtained the bias and MSE of the proposed estimator $\bar{y}_{pd}^*$ up to the first order approximation, this is obtained by substituting $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N - n}$ into equation (1),

$$\bar{y}_{pd}^* = \frac{\bar{y}\bar{X}(N - n)}{N\bar{X} - n\bar{x}} + \alpha\left(\frac{n\bar{x} - n\bar{X}}{N - n}\right)$$

We write $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$.

The bias $\bar{y}_{pd}^*$ is given as

$$Bias(\bar{y}_{pd}^*) = \frac{(1-f)}{n} (g^2 \bar{Y}^2 \frac{S_x^2}{\bar{X}^2} + \bar{Y} g \frac{S_{xy}}{\bar{X}\bar{Y}}) \quad \dots\dots\dots (1.1)$$

The MSE of $\bar{y}_{pd}^*$ is given as

$$E(\bar{y}_{pd}^* - \bar{Y})^2 = \left(\frac{1-f}{n}\right) \left(S_y^2 + 2g\bar{Y} \frac{\rho S_x S_y}{\bar{X}\bar{Y}} (\bar{Y} + \alpha\bar{X}) + g^2 \frac{S_x^2}{\bar{X}^2} (\bar{Y} + \alpha\bar{X})^2 \right)$$

$$MSE(\bar{y}_{pd}^*) = \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho\bar{Y} C_x C_y (\bar{Y} + \alpha\bar{X}) + g^2 C_x^2 (\bar{Y} + \alpha\bar{X})^2) \quad \dots\dots\dots (2)$$

The optimum value of the MSE ($\bar{y}_{pd}^*$) is obtained as follows,

$$\frac{\partial}{\partial \alpha} MSE(\bar{y}_{pd}^*) = \left(\frac{1-f}{n}\right) \left(2g\bar{Y} \frac{S_{xy}}{\bar{X}\bar{Y}} (\bar{X}) + g^2 C_x^2 (2)(\bar{X})(\bar{Y} + \alpha\bar{X}) \right) \quad \dots\dots\dots (3)$$

Set equation (3) to zero; we have

$$2gS_{xy} + 2g^2C_x^2\bar{X}(\bar{Y} + \alpha\bar{X}) = 0$$

$$\alpha = -\left(R + \frac{\beta}{g}\right)$$

The optimal MSE of $\bar{y}_{pd}^*$.

$$MSE(\bar{y}_{pd}^{*opt}) = \left(\frac{1-f}{n}\right) S_y^2(1-\rho^2) \quad \dots \dots \dots (4)$$

Equation (4) shows that the MSE ($\bar{y}_{pd}^{*opt}$) is the same as the MSE of Regression Estimate of y on X.

Remark

The bias of $\bar{y}_{pd}^*$ is the same as the bias of the dual product estimator and when $\alpha=0$, the MSE ($\bar{y}_{pd}^*$) boils down to product cum estimator proposed by Bandyopadhyay (1980). The bias and MSE of $\bar{y}_p^*$ are given as

$$\text{Bias}(\bar{y}_p^*) = \left(\frac{1-f}{n}\right) (g^2 \bar{Y}^2 C_x^2 + \bar{Y} g \rho C_x C_y)$$

$$\text{MSE}(\bar{y}_p^*) = \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho\bar{Y}^2 C_x C_y + \bar{Y} g^2 C_x^2) \quad \dots \dots \dots (5)$$

III. THE EFFICIENCY COMPARISONS

In this section, we compared the MSE of $\bar{y}$ with the MSE of $\bar{y}_{pd}^*$. The MSE of $\bar{y}$ under SRS scheme is given as

$$\text{MSE}(\bar{y}) = \left(\frac{1-f}{n}\right) \bar{Y}^2 C_x^2$$

From equation (2) and (5), $\bar{y}_{pd}^*$ is better than $\bar{y}$

If $\text{MSE}(\bar{y}_{pd}^*) < \text{MSE}(\bar{y})$

That is

$$\left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho\bar{Y} C_x C_y (\bar{Y} + \alpha\bar{X}) + g^2 C_x^2 (\bar{Y} + \alpha\bar{X})^2) \leq \left(\frac{1-f}{n}\right) \bar{Y}^2 C_x^2$$

$$(\bar{Y} + \alpha\bar{X}) (2g\rho\bar{Y} C_x C_y + g^2 C_x^2 (\bar{Y} + \alpha\bar{X})) < 0$$

This holds if

(1) $\bar{Y} + \alpha\bar{X} < 0$ and $2g\rho\bar{Y} C_x C_y + g^2 C_x^2 (\bar{Y} + \alpha\bar{X}) > 0$

Or

(2) $\bar{Y} + \alpha\bar{X} > 0$ and $2g\rho\bar{Y} C_x C_y + g^2 C_x^2 (\bar{Y} + \alpha\bar{X}) < 0$

The range of α under which $\bar{y}_{pd}^*$ is more efficient than usual SRS $\bar{y}$ is

$$\min\left\{-R, -R\left(1 + \frac{2\rho C_y}{g C_x}\right)\right\}, \max\left\{-R, -R\left(1 + \frac{2\rho C_y}{g C_x}\right)\right\}.$$

We also compared $\bar{y}_{pd}^*$ with the usual ratio estimator $\bar{y}_R$. The MSE of the $\bar{y}_R$ is given as,

$$MSE(\bar{y}_R) = \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 - 2\rho \bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2)$$

It is found that $\bar{y}_{pd}^*$ will be more efficient than the usual ratio estimator $\bar{y}_R$ if $MSE(\bar{y}_{pd}^*) < MSE(\bar{y}_R)$. That is,

$$\left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho \bar{Y} C_x C_y (\bar{Y} + \alpha \bar{X}) + g^2 C_x^2 (\bar{Y} + \alpha \bar{X})^2) \leq \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 - 2\rho \bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2)$$

This holds if the following two conditions are satisfied

$$(1). (g(\bar{Y} + \alpha \bar{X}) + \bar{Y}) < 0 \text{ And } 2\rho \bar{Y} C_x C_y + C_x^2 (g(\bar{Y} + \alpha \bar{X}) - \bar{Y}) > 0.$$

or

$$(2). (g(\bar{Y} + \alpha \bar{X}) + \bar{Y}) > 0 \text{ And } 2\rho \bar{Y} C_x C_y + C_x^2 (g(\bar{Y} + \alpha \bar{X}) - \bar{Y}) < 0.$$

This condition holds if $\alpha > -R\left(\frac{N}{n}\right)$ and $\alpha < R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)$ or $\alpha < -R\left(\frac{N}{n}\right)$ and $\alpha > R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)$

$$\min\left\{-R\left(\frac{N}{n}\right), R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{-R\left(\frac{N}{n}\right), R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)\right\}.$$

We also compared $\bar{y}_{pd}^*$ with the usual product estimator y_P . The MSE of the y_P is given as

$$MSE(\bar{y}_P) = \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2\rho \bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2)$$

It is found that the proposed estimator $\bar{y}_{pd}^*$ will be more efficient than the usual ratio estimator $\bar{y}_P$ if $MSE(\bar{y}_{pd}^*) < MSE(\bar{y}_P)$. That is,

$$\left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho \bar{Y} C_x C_y (\bar{Y} + \alpha \bar{X}) + g^2 C_x^2 (\bar{Y} + \alpha \bar{X})^2) \leq \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2\rho \bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2)$$

This holds if the following two conditions are satisfied

$$(1). (g(\bar{Y} + \alpha \bar{X}) - \bar{Y}) < 0 \text{ And } 2\rho \bar{Y} C_x C_y + C_x^2 (g(\bar{Y} + \alpha \bar{X}) + \bar{Y}) > 0.$$

Or

$$(2). (g(\bar{Y} + \alpha \bar{X}) - \bar{Y}) > 0 \text{ And } 2\rho \bar{Y} C_x C_y + C_x^2 (g(\bar{Y} + \alpha \bar{X}) + \bar{Y}) < 0.$$

This condition holds if $\alpha > R\left(\frac{N-2n}{n}\right)$ and $\alpha < -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)$ or $\alpha < R\left(\frac{N-2n}{n}\right)$ and $\alpha > -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)$

$$\min\left\{R\left(\frac{N-2n}{n}\right), -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{R\left(\frac{N-2n}{n}\right), -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)\right\}.$$

We compared the MSE of the proposed estimator with MSE of dual product estimator from equation (2) and (5) it is found that the proposed estimator $\bar{y}_{pd}^*$ will be more efficient than that of Bandyopadhyay (1980) estimator $\bar{y}_p^*$ if $MSE(\bar{y}_{pd}^*) < MSE(\bar{y}_p^*)$. That is

$$\left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho \bar{Y} C_x C_y (\bar{Y} + \alpha \bar{X}) + g^2 C_x^2 (\bar{Y} + \alpha \bar{X})^2) \leq \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho \bar{Y}^2 C_x C_y + \bar{Y}^2 g^2 C_x^2)$$

This holds if

$$1. \alpha\bar{X} < 0 \text{ and } 2\rho\bar{Y}C_y + gC_x(2\bar{Y} + \alpha\bar{X}) > 0$$

Or

$$2. \alpha\bar{X} > 0 \text{ and } 2\rho\bar{Y}C_y + gC_x(2\bar{Y} + \alpha\bar{X}) < 0$$

The range of α under which the proposed estimator $\bar{y}_{pd}^*$ is more efficient than $\bar{y}_p^*$ is

$$\min\left\{0, -2R\left(\frac{\rho C_y}{gC_x} + 1\right)\right\}, \max\left\{0, -2R\left(\frac{\rho C_y}{gC_x} + 1\right)\right\}$$

Lastly, we compared MSE of $\bar{y}_{pd}^*$ with that of dual to ratio estimator $\bar{y}_R^*$ proposed Srivenkataramana(1980), $\bar{y}_{pd}^*$ will be more efficient than $\bar{y}_R^*$ if

$$MSE(\bar{y}_{pd}^*) < MSE(\bar{y}_R^*)$$

$$\begin{aligned} \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y} + \alpha\bar{X}) + g^2 C_x^2 (\bar{Y} + \alpha\bar{X})^2) \\ \leq \left(\frac{1-f}{n}\right) (\bar{Y}^2 C_y^2 - 2g\rho\bar{Y}^2 C_xC_y + \bar{Y}^2 g^2 C_x^2) \end{aligned}$$

This holds if

$$1. 2\bar{Y} + \alpha\bar{X} < 0 \text{ and } 2\rho\bar{Y}C_y + gC_x(\alpha\bar{X}) > 0$$

Or

$$2. 2\bar{Y} + \alpha\bar{X} > 0 \text{ and } 2\rho\bar{Y}C_y + gC_x(\alpha\bar{X}) < 0$$

This condition holds if $-2R > \alpha$ and $\frac{-2R\rho C_y}{gC_x} < \alpha$ or $-2R < \alpha$ and $\frac{-2R\rho C_y}{gC_x} > \alpha$

Therefore, the range of α under which the proposed estimator $\bar{y}_{pd}^*$ is more efficient than dual to ratio estimator $\bar{y}_R^*$ is

$$\min\left\{-2R, -2R\left(\frac{\rho C_y}{gC_x}\right)\right\}, \max\left\{-2R, -2R\left(\frac{\rho C_y}{gC_x}\right)\right\}$$

Comparison of 'AOE' to $\bar{y}_{pd}^{OPT}$

$\bar{y}_{pd}^{OPT}$ is more efficient than the other existing estimators $\bar{y}$, the Ratio estimator $\bar{y}_R$, the product estimator $\bar{y}_p$, the dual to ratio estimator $\bar{y}_R$ and the dual to product estimator $\bar{y}_p^*$ since.

$$MSE(\bar{y}) - MSE(\bar{y}_{pd}^{OPT}) = \left(\frac{1-f}{n}\right) (\bar{Y}^2 \rho^2 C_y^2) > 0$$

$$MSE(\bar{y}_R) - MSE(\bar{y}_{pd}^{OPT}) = \left(\frac{1-f}{n}\right) \left(\bar{Y}^2 C_x^2 \left(1 - \frac{\rho C_y}{C_x}\right)^2\right) > 0$$

$$MSE(\bar{y}_p) - MSE(\bar{y}_{pd}^{OPT}) = \left(\frac{1-f}{n}\right) \left(\bar{Y}^2 C_x^2 \left(1 + \frac{\rho C_y}{C_x}\right)^2\right) > 0$$

$$MSE(\bar{y}_R^*) - MSE(\bar{y}_{pd}^{OPT}) = \left(\frac{1-f}{n}\right) \left(\bar{Y}^2 C_x^2 \left(\frac{\rho C_y}{C_x} - g\right)^2\right) > 0$$

$$MSE(\bar{y}_p^*) - MSE(\bar{y}_{pd}^{OPT}) = \left(\frac{1-f}{n}\right) \left(\bar{Y}^2 C_x^2 \left(\frac{\rho C_y}{C_x} + g \right)^2 \right) > 0$$

Hence, we conclude that the proposed class of estimator $\bar{y}_{pd}^*$ is more efficient than other estimator in case of its optimality.

IV. NUMERICAL VALIDATION

To illustrate the efficiency of the proposed estimator over the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}_{R^*}$ and $\bar{y}_p^*$. Data on the ages and hours of sleeping by the undergraduate students of the Department of Statistics Federal University of Technology Akure, Ondo State, Nigeria were collected. A sample of 150 out of 461 students of the department was obtained using simple random sampling without replacement. The information on the age of the students was used as auxiliary information to increase the precision of the estimate of the average sleeping hours. The estimate of the average hours of sleeping of the students were obtained and also the 95% confidence intervals of the average hours of sleeping were obtained for the proposed estimator and the other estimators. Table 1.0, gives the estimates of the average sleeping hours and the 95% confidence Interval. As shown in Table 1.0, the proposed estimator performed better than the other estimators, also the width of the confidence interval of the proposed estimator is smallest than the other competing estimators.

Table 1.0 : Average Sleeping Hours and 95% confidence intervals for Different Estimators for the undergraduate Students of Department of Statistics, Federals University of Technology Akure. Nigeria

Estimator	Average Sleeping Hours	LCL	UCL	WIDTH
$\bar{y}$	6.08	5.930386531	6.229613469	0.299226939
$\bar{y}_R$	6.210472103	6.042844235	6.378099971	0.335255737
$\bar{y}_p$	5.952268908	5.778821411	6.125716404	0.346894993
$\bar{y}_{R^*}$	6.141606636	5.988421023	6.294792249	0.306371226
$\bar{y}_p^*$	6.01901342	5.862732122	6.175290562	0.31255844
$\bar{y}_{PD}^*$	6.072287	5.927857	6.216717	0.28886

The proposed estimator performed the same way as the regression estimator when compare with the usual estimator $\bar{y}$. The average Sleeping Hours and 95% confidence intervals for the proposed estimator and the regression estimator is given below, the two estimators have the same confidence Interval width.

Table 2.0 : Average Sleeping Hours and 95% confidence intervals for the proposed estimators and regression estimators for the undergraduate Students of Department of Statistics, Federals University of Technology Akure. Nigeria

Estimator	Average Sleeping Hours	LCL	UCL	WIDTH
$\bar{y}_{PD}^*$	6.072287	5.927857	6.2167177	0.28886
$\bar{y}_{REG}^*$	6.089652737	5.945222793	6.234082681	0.288859888

To examine the gain in the efficiency of the proposed estimator $\bar{y}_{pd}^*$ over the estimator $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}_R^*$ and $\bar{y}_p^*$, we obtained the percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$ in Table 2.0. The proposed estimator $\bar{y}_{pd}^*$ performed better than the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}_R^*$ and $\bar{y}_p^*$ and performed exactly the same way as regression estimator.

Table 3.0. : The percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$

ESTIMATOR	PERCENTAGE RELATIVE EFFICIENCY
$\bar{y}$	100
$\bar{y}_R$	79.66158486
$\bar{y}_p$	74.40554745
$\bar{y}_R^*$	95.39056726
$\bar{y}_p^*$	91.65136111
$\bar{y}_{REG}^*$	107.3067159
$\bar{y}_{pd}^*$	107.3067159

V. CONCLUSION

We have proposed a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X as in equation (1) and obtained 'AOE' for the proposed estimator. Theoretically, we have demonstrated that proposed estimator is always more efficient than other under the effective ranges of α and its optimum values.

Table 1.0 shows that the proposed estimator performed better than the other estimators as the width of the confidence interval of the proposed estimator is smallest than the other competing estimators. The percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$ in Table 2.0 shows that the proposed estimator $\bar{y}_{PD}^*$ performed better than the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}_R^*$ and $\bar{y}_p^*$ and performed exactly the same way as regression estimator when compared to the usual estimator $\bar{y}$. Hence, it is preferred to use the proposed class of estimator in practice.

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On Special Pairs of Pythagorean Triangles

By M. A. Gopalan, S. Vidhyalakshmi, N. Thiruniraiselvi & R. Presenna

Shrimati Indira Gandhi College, India

Abstract- We search for pairs of Pythagorean triangles such that, in each pair, twice the difference between their perimeters is expressed in terms of special polygonal numbers.

Keywords: pair of pythagorean triangles, special polygonal numbers.

GJSFR-F Classification : FOR Code : MSC 2010: 11D09, 11Y50.



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On Special Pairs of Pythagorean Triangles

M. A. Gopalan ^α, S. Vidhyalakshmi ^σ, N. Thiruniraiselvi ^ρ & R. Presenna ^ω

Abstract- We search for pairs of Pythagorean triangles such that, in each pair, twice the difference between their perimeters is expressed in terms of special polygonal numbers.

Keywords: pair of pythagorean triangles, special polygonal numbers.

Notations used:

$$t_{m,n} = n \left(1 + \frac{(n-1)(m-2)}{2} \right)$$

$$s_n = 6n(n-1) + 1$$

$$cs_n = n^2 + (n-1)^2$$

I. INTRODUCTION

Number is the essence of mathematical calculation. Varieties of numbers have variety of range and richness, many of which can be explained very easily but extremely difficult to prove. One of the varieties of numbers that have fascinated mathematicians and the lovers of maths is the pythagorean number as they provide limitless supply of exciting and interesting properties. In other word, the method of obtaining three non-zero integers x,y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians. For an elaborate review of various properties are may refer [1-17]. In [18], the author proves existence of an infinite family of pairs of dissimilar Pythagorean triangles that are pseudo smarandache related. For problems on pairs of Pythagorean triangles one may refer [1,5].

In this communication concerns with the problem of determining pairs of Pythagorean triangles wherein each pair 2-times the difference between the perimeters is expressed interms of special polygonal numbers.

II. METHOD OF ANALYSIS

Let $T_1(\alpha_1, \beta_1, \gamma_1)$ and $T_2(\alpha_2, \beta_2, \gamma_2)$ be two distinct Pythagorean triangles where $\alpha_1 = 2mq, \beta_1 = m^2 - q^2, \gamma_1 = m^2 + q^2$ and $\alpha_2 = 2pq, \beta_2 = p^2 - q^2, \gamma_2 = p^2 + q^2, m > q > 0; p > q > 0$

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Let $P_1 = 2m(m+q)$, $P_2 = 2p(p+q)$ be their perimeters respectively. We illustrated below the process of obtaining pair of Pythagorean triangles such that 2 times the difference between their perimeters is expressed in terms of a special polygonal number.

Illustration 1

The assumption $2(P_1 - P_2) = (3\alpha - 1)^2$ (1)

gives $(2m+q)^2 = (2p+q)^2 + (3\alpha - 1)^2$ (2)

which is in the form of well know Pythagorean equation and it is satisfied by

$$3\alpha - 1 = 2RS, 2p+q = R^2 - S^2, 2m+q = R^2 + S^2, R > S > 0 \quad (3)$$

Solving the above system of equations (3), we have

$$\alpha = \left(\frac{2RS+1}{3} \right), m = p + S^2, q = R^2 - S^2 - 2p$$

As our main thrust is on integers, note that α is an integer when

$$R = 3(k_1 + k_2) - 2, S = 3k_2 - 2$$

$$\alpha = (k_1 + k_2)(6k_2 - 4) - 4k_2 + 3$$

and thus $m = p + (3k_2 - 2)^2$ (4)

$$q = 9k_1^2 + 18k_1k_2 - 12k_1 - 2p$$

The conditions $m > q > 0$, $p > q > 0$ lead to

$$2p < 9k_1^2 + 18k_1k_2 - 12k_1 < 3p \quad (5)$$

Choose k_1, k_2, p such that (5) is satisfied. Substituting the corresponding values of k_1, k_2, p in (4) and in the value of m, q, α are obtained. Thus, it is observed that

$$2(P_1 - P_2) = \begin{cases} 3t_{8,\alpha} + 1 \\ s_\alpha + 3t_{4,\alpha} \end{cases}$$

A few examples are given below in Table -1

k_1	k_2	p	m	q	α	$2(P_1 - P_2)$	$3t_{8,\alpha} + 1$	$s_\alpha + 3t_{4,\alpha}$
1	1	7	8	1	3	64	$3(21) + 1$	$37 + 3(3)^2$
1	2	16	32	1	19	3136	$3(1045) + 1$	$2053 + 3(19)^2$
1	2	15	31	3	19	3136	$3(1045) + 1$	$2053 + 3(19)^2$
1	2	14	30	5	19	3136	$3(1045) + 1$	$2053 + 3(19)^2$
1	2	13	29	7	19	3136	$3(1045) + 1$	$2053 + 3(19)^2$
1	2	12	28	9	19	3136	$3(1045) + 1$	$2053 + 3(19)^2$
1	3	25	74	1	47	19600	$3(6533) + 1$	$12973 + 3(47)^2$
1	3	24	73	3	47	19600	$3(6533) + 1$	$12973 + 3(47)^2$

1	3	23	72	5	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	22	71	7	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	21	70	9	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	20	69	11	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	19	68	13	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	18	67	15	47	19600	$3(6533)+1$	$12973+3(47)^2$

It is worth to note that there are only finitely many pairs of Pythagorean triangles such that in each pair, twice the difference between their perimeters is expressed in terms of a special polygonal number.

Illustration- 2

Assume $2(P_1 - P_2) = (5\alpha - 2)^2$

Proceeding as in **illustration 1**, it is seen that there are 3 values for α satisfying the equation $\alpha = \left(\frac{2(RS+1)}{5} \right)$ and thus, there are 3 sets of triples (m, q, α) which are represented as follows

$$\begin{aligned} \alpha &= 2(k + k_1)(5k_1 - 4) - 12k_1 + 10 \\ \text{SET 1:} \quad m &= p + (5k_1 - 4)^2 \\ q &= (10k_1 + 5k - 10)(5k - 2) - 2p \end{aligned}$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 10)(5k - 2) < 3p$

$$\begin{aligned} \alpha &= 2(k + k_1)(5k_1 - 3) - 6k_1 + 4 \\ \text{SET 2:} \quad m &= p + (5k_1 - 3)^2 \\ q &= (5k + 10k_1 - 6)(5k) - 2p \end{aligned}$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 6)(5k) < 3p$

$$\begin{aligned} \alpha &= 2(k + k_1)(5k_1 - 2) - 4k_1 + 2 \\ \text{SET 3:} \quad m &= p + (5k_1 - 2)^2 \\ q &= (5k + 10k_1 - 4)(5k) - 2p \end{aligned}$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 4)(5k) < 3p$

From each of the above 3 sets it is observed that $2(P_1 - P_2) = 5t_{12, \alpha} + 4$

Illustration 3

Assume $2(P_1 - P_2) = (\alpha - 1)^2$

For this choice, the values for m, q, α satisfying the equation

$$(2m + q)^2 = (2p + q)^2 + (\alpha - 1)^2 \text{ are given by}$$

$$\alpha = 2rs + 1, m = s^2 + p, q = r^2 - s^2 - 2p$$

The conditions $m > q > 0, p > q > 0$ lead to

$$2p < r^2 - s^2 < 3p \quad (6)$$

Choose r, s, p satisfying (6), it is seen that

$$2(P_1 - P_2) = (cs_\alpha - t_{4,\alpha})$$

III. CONCLUSION

In this paper, we have obtained finitely many pairs of Pythagorean triangles where each pair connects 2-times the difference between the perimeters with special polygonal numbers. To conclude, one may search for the connections between special numbers and the other characterizations of Pythagorean triangle.

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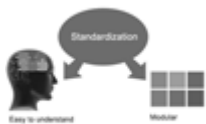
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Title Page:

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Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript-- must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
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Approach:

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Approach:

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Approach:

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- Resources and methods are not a set of information.
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The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently. You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.



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- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
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Approach

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- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

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<i>References</i>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



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