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Mathematics and Decision Sciences

Highlights

- Integral Operator Associated Multipliers of Distributions Spaces
- Analytic and Numeric Solution Development of Discrete Version

Discovering Thoughts, Inventing Future

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Pathway Integral Operator Associated with Aleph-Function and General Polynomials

By Dr. Rinku Jain & Dr. Kirti Arekar
K.J. Somaiya Institute of Management Studies & Research, India

Abstract - This paper is devoted to the study of a pathway fractional integral operator associated with the pathway model and pathway probability density for the ℵ- function and a generalized polynomial in the kernel. By specializing the coefficients and various parameters in the generalized polynomials and ℵ- function, our main theorem would readily yield several interesting results.

Keywords : pathway fractional integral operator, aleph function (ℵ- function), generalized polynomial.

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PATHWAY INTEGRAL OPERATOR ASSOCIATED WITH ALEPH-FUNCTION AND GENERAL POLYNOMIALS

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Pathway Integral Operator Associated with Aleph-Function and General Polynomials

Dr. Rinku Jain α & Dr. Kirti Arekar σ

Abstract - This paper is devoted to the study of a pathway fractional integral operator associated with the pathway model and pathway probability density for the ℵ- function and a generalized polynomial in the kernel. By specializing the coefficients and various parameters in the generalized polynomials and ℵ- function, our main theorem would readily yield several interesting results.

Keywords : *pathway fractional integral operator, aleph function (ℵ-function), generalized polynomial.*

I. Introduction

In the last three decades several authors have made significant contribution in the field of fractional calculus. Fractional calculus has been applied to almost every field of science, engineering, and Mathematics. The most popular one, we are based on here, is the Riemann-Liouville fractional integral operator [7]. The Pathway fractional integral operator, as an extension of Riemann-Liouville fractional integral operator, introduced by Nair [8] is defined in the following manner

$$(P_{0+}^{(\eta,\alpha)} f)(x) = x^{\eta} \int_0^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt \qquad (1.1)$$

Where f(x) ∈ L (a,b), η ∈ ℂ, Re(η) > 0, a > 0 and 'pathway parameter' α < 1.

The Pathway model is introduced by Mathai [1] and studied further by Mathai and Haubold[2], [3]. For real scalar α, the Pathway model for scalar random variables is represented by the following probability density function

$$f(x) = c\ |x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{\beta}{1-\alpha}}, \qquad (1.2)$$

$-\infty < x < \infty$, $\delta > 0$, $\beta \geq 0$, $\left[1-a(1-\alpha)|x|^{\delta}\right] > 0$, $\gamma > 0$, where c is the normalizing constant and α is called the pathway parameter. For real α, the normalizing constant is as follows:

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Ref.

7. S.G. Samko, A.A. Kilbas and O.I. Marichev, Fractional Integrals and Derivatives, Theory and Applications, Gordon and Breach, New York (1993).

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$$c = \frac{1}{2}\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\Gamma\left(\frac{\gamma}{\delta}+\frac{\beta}{1-\alpha}+1\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{1-\alpha}+1\right)}, \quad for\ \alpha<1, \tag{1.3}$$

$$= \frac{1}{2}\frac{\delta[a(\alpha-1)]^{\frac{\gamma}{\delta}}\Gamma\left(\frac{\beta}{1-\alpha}\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{\alpha-1}-\frac{\gamma}{\delta}\right)}, \quad for\ \frac{1}{\alpha-1}-\frac{\gamma}{\delta} > 0,\ \alpha>1, \tag{1.4}$$

$$= \frac{1}{2}\frac{\delta[a\beta]^{\frac{\gamma}{\delta}}}{\Gamma\left(\frac{\gamma}{\delta}\right)}, \quad for\ \alpha\to 1. \tag{1.5}$$

For $\alpha<1$, it is a finite range density with $\left[1-a(1-\alpha)|x|^{\delta}\right] > 0$ and (1.2) remains in the extended generalized type-1 beta family. The Pathway density in (1.2), for $\alpha < 1$, includes the extended type-1 beta density, the triangular density, the uniform density and many other p.d.f.

For $\alpha>1$, we have

$$f(x) = c\ |x|^{\gamma-1}\left[1+a(\alpha-1)|x|^{\delta}\right]^{-\frac{\beta}{\alpha-1}}, \tag{1.6}$$

$-\infty < x < \infty$, $\delta > 0$, $\beta \geq 0$, $\left[1-a(\alpha-1)|x|^{\delta}\right] > 0$, $\gamma > 0$, which is extended generalized type-2 beta model for real x. t includes the type-2 beta density, the F density, the Student-t density, and Cauchy density and many more.

Here, we consider only the case of Pathway parameter $\alpha < 1$. For $\alpha\to 1$ both (1.2) and (1.6) take the exponential form, since

$$\lim_{\alpha\to 1} c\ |x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{\beta}{1-\alpha}} = c\ |x|^{\gamma-1}\left[1+a(\alpha-1)|x|^{\delta}\right]^{-\frac{\beta}{\alpha-1}}$$
$$= c\ |x|^{\gamma-1}e^{-\alpha\eta|x|^{\delta}} \tag{1.7}$$

This includes the generalized Gamma-, the Weibull-, the Chi-Square the Laplace and other related densities. For more details on the Pathway model, the reader is referred to the recent papers of [2], [3].

II. Pathway Integral Operator of An ℵ - Function

The Aleph ℵ-function introduced by Sudland et al. [6] which is defined as a contour integral of Mellin Barnes Type:

Ref.

2. A.M. Mathai and H.J. Haubold, On generalized distributions and pathways, Physics Letters 372 (2008), 2109-2113.

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$$\aleph[Z]=\aleph_{p_i,q_i,\tau_i;r}^{m,n}[Z]=\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[z\left(\frac{x}{y}\right)^q\left|\begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,pi}\\(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,qi}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\omega}\int_L\Omega_{p_i,q_i,\tau_i;r}^{m,n}(s)z^{-s}ds \tag{2.1}$$

for all z≠o, $\omega=\sqrt{-1}$ and

$$\Omega_{p_i,q_i,\tau_i;r}^{m,n}(s)=\frac{\prod_{j=1}^{m}\Gamma(b_j+B_js)\prod_{j=1}^{n}\Gamma(1-a_j-A_js)}{\sum_{i-1}^{r}\tau_i\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}s)\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}s)} \tag{2.2}$$

The integration path $\ell=\ell_{i\gamma\infty},\gamma\in$R extends from γ-i∞ to γ+i∞, and is such that the poles, assumed to be simple, of $\Gamma(1-a_j-A_js),j=\overline{1,n}$ do not coincide with the pole $\Gamma(b_j+B_js),j=\overline{1,m}$. The parameters pi, qi are non-negative integers satisfying 0 ≤ n ≤ pi, 1≤ m ≤ qi , $\tau_i>0$ for $i=\overline{1,r}$. The parameters Aj, B_j, A_{ji}, $B_{ji}>0$ and aj, bj a_{ji}, $b_{ji}\in$ C. The empty product in (1.3) is interpreted as unity. The existence conditions for the defining integral (1.1) are given below:

$$\psi_l>0,\quad |\arg(z)|<\frac{\pi}{2}\psi_l\quad l=\overline{1,r}; \tag{2.3}$$

(4):

$$\psi_l\geq 0,\quad |\arg(z)|<\frac{\pi}{2}\psi_l\quad and\ \Re\{\varsigma_l\}+1<0, \tag{2.4}$$

Where

$$\psi_l=\sum_{j=1}^{n}A_j+\sum_{j=1}^{m}B_j-\tau_l\left(\sum_{j=n+1}^{p_l}A_{jl}+\sum_{j=m+1}^{q_l}B_{jl}\right) \tag{2.5}$$

$$\varsigma_l=\sum_{j=1}^{m}b_j-\sum_{j=1}^{n}a_j+\tau_l\left(\sum_{j=m+1}^{q_l}b_{jl}-\sum_{j=n+1}^{p_l}a_{jl}\right)+\frac{1}{2}(p_l-q_l)\quad l=\overline{1,r}. \tag{2.6}$$

The general polynomials of R variables given by Srivastava [5] defined and represented as:

Ref.

5. H. M. Srivastava, A multilinear generating function for the Konhausar sets of biortogonal polynomials suggested by the Laguerre polynomials, pacific J.Math. 177 (1985), 183-191.

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$$S_{n_1,\ldots,n_R}^{m_1,\ldots,m_R}[x_1,\ldots x_R] = \sum_{s_1=0}^{\left[n_1/m_1\right]} \cdots \sum_{s_R=0}^{\left[n_R/m_R\right]} \frac{(-n_1)_{m_1 s_1}}{\angle s_1} \cdots \frac{(-n_R)_{m_R s_R}}{\angle s_R} \tag{2.7}$$

$$A\left[n_1, s_1;\ldots;n_R s_R\right] x_1^{s_1} \ldots x_R^{s_R}$$

Notes

Where m_i is an arbitrary positive integer and coefficients $A\left[n_1, s_1;\ldots;n_R s_R\right]$ are arbitrary constants, real or complex.

Theorem1. With the set of sufficient conditions (2.3), (2.4), (2.5) and (2.6), let (η, u, $u_1,\ldots,u_R$, $\beta \in \mathbb{C}$), $\operatorname{Re}\left(1+\frac{\eta}{1-\alpha}\right)>0$, $\alpha<1$, Re(η, u, $u_1,\ldots,u_R$, β) > 0 and m_i is an arbitrary positive integer and coefficients $A\left[n_1, s_1;\ldots;n_R s_R\right]$ are arbitrary constants, real or complex, then

$$P_{0+}^{(\eta,\alpha)}\left[x^{u-1} S_{n_1,\ldots,n_R}^{m_1,\ldots m_R}\left[x^{u_i},\ldots,x^{u_R}\right] \aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[x^{\beta}\right]\right]$$

$$= \sum_{s_1=0}^{\left[n_1/m_1\right]} \cdots \sum_{s_R=0}^{\left[n_R/m_R\right]} \frac{(-n_1)_{m_1 s_1}}{\angle s_1} \cdots \frac{(-n_R)_{m_R s_R}}{\angle s_R} A\left[n_1, s_1;\ldots;n_R s_R\right]$$

$$\frac{x^{\eta+u+u_i k_i+\ldots+u_R k_R}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{u+u_i k_i+\ldots+u_R k_R}}$$

$$\aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n+1}\left[\frac{x^{\beta}}{[a(1-\alpha)]^{\beta}}\left|\begin{matrix}[1-u-(u_i s_1+\ldots+u_R s_R);\beta],\ (a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,pi}\\ (b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,qi},[-u-(u_i s_1+\ldots+u_R s_R);\beta]\end{matrix}\right.\right] \tag{2.8}$$

Proof: The Theorem 1 can be evaluated by using the definitions (1.1), (2.1) and (2.7) then by interchange the order of integrations and summations (which is permissible under the conditions stated above), evaluate the inner integral by making use of beta function formula, we arrive at the desired result.

Theorem2. Let (η, u, $u_1,\ldots,u_R$, $\beta \in \mathbb{C}$), $\operatorname{Re}\left(1+\frac{\eta}{1-\alpha}\right)>0$, $\alpha<1$, Re(η, u, $u_1,\ldots,u_R$, β) > 0 and m is an arbitrary positive integer and coefficients $A\left[n_1, s_1;\ldots;n_R s_R\right]$ are arbitrary constants, real or complex, then

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$$P_{0+}^{(\eta,\alpha)}\left[x^{u-1}S_{n_1,\ldots,n_R}^{m_1,\ldots m_R}\left[x^{u_i},\ldots,x^{u_R}\right]H_{p,q,}^{m,n}\left[x^{\beta}\right]\right]$$

$$=\sum_{s_1=0}^{\left[n_1/m_1\right]}\cdots\sum_{s_R=0}^{\left[n_R/m_R\right]}\frac{(-n_1)m_1s_1}{\angle s_1}\cdots\frac{(-n_R)m_Rs_R}{\angle s_R}A\left[n_1,s_1;\ldots;n_Rs_R\right]$$

$$\frac{x^{\eta+u+u_ik_i+\ldots+u_Rk_R}\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\left[a(1-\alpha)\right]^{u+u_ik_i+\ldots+u_Rk_R}}$$

$$H_{p+1,q+1,}^{m,n+1}\left[\frac{x^{\beta}}{\left[a(1-\alpha)\right]^{\beta}}\left|\begin{matrix}[1-u-(u_is_1+\ldots+u_Rs_R);\beta],\ (a_j,A_j)_{1,n},\ldots,[(a_j,A_j)]_{n+1,p}\\(b_j,B_j)_{1,m},\ldots,[(b_j,B_j)]_{m+1,q},[-u-(u_is_1+\ldots+u_Rs_R);\beta]\end{matrix}\right.\right],$$

Where $H_{p,q,}^{m,n}\left[x\right]$ is the Fox's H- Function [4]. (2.9)

Proof: The result in (2.2) can be derived from Theorem 1 by taking $\tau_1=\ldots=\tau_r=1$ and $r=1$. We have the required result.

Theorem3. Suppose that the conditions corresponding to Theorem 2 are satisfied. Then

$$P_{0+}^{(\eta,\alpha)}\left[x^{u-1}x^{\frac{u_1n_1}{2}}H_{n_1}\left[\frac{1}{2\sqrt{x^{u_1}}}\right]\aleph_{p_i,q_i,\tau_i;r}^{m,n}\left[x^{\beta}\right]\right]=\sum_{s_1=0}^{\left[n_1/2\right]}\frac{(-n_1)2s_1}{\angle s_1}(-1)^{s_1}$$

$$\frac{x^{\eta+u+u_1s_1}\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\left[a(1-\alpha)\right]^{u+u_1k_1}}$$

$$\aleph_{p_i+1,q_i+1,\tau_i;r}^{m,n+1}\left[\frac{x^{\beta}}{\left[a(1-\alpha)\right]^{\beta}}\left|\begin{matrix}[1-u-u_is_1);\beta],\ (a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,pi}\\(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,qi},[-u-u_is_1;\beta]\end{matrix}\right.\right],$$

where $H_n(x)$ is the Hermite polynomials. (2.10)

Ref.

4. Fox, C. The *G* and *H*-functions as symmetrical Fourier kernels. *Trans. Amer. Math. Soc.* 98, (1961), 395-429.

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Proof: In Theorem 1, if we take R =1, m_1 =2 and $A_{n_1, s_1} = (-1)^{s_1}$, then we get the desired result.

III. Special Cases

1. Letting R =1 in the result (2.9), we get the result recently obtained by Chaurasia and Ghiya [9] for ρ, ρ_1, and $\rho_2 \to 0$.
2. Letting $n_{i'} \to 0$, i' = 1,…, R, in the result (2.9), we get the result obtained by Nair in [8].

Notes

References Références Referencias

1. A.M. Mathai, A pathway to matrix-variate gamma and normal densities, Linear Algebra and Its Applications 396 (2005), 317-328.
2. A.M. Mathai and H.J. Haubold, On generalized distributions and pathways, Physics Letters 372 (2008), 2109-2113.
3. A.M. Mathai and H.J. Haubold, Pathway models, superstatistics, Trallis statistics and a generalized measure of entropy, Physica A 375 (2007), 110-122.
4. Fox, C. The *G* and *H*-functions as symmetrical Fourier kernels. *Trans. Amer. Math. Soc.* 98, (1961), 395-429.
5. H. M. Srivastava, A multilinear generating function for the Konhausar sets of biortogonal polynomials suggested by the Laguerre polynomials, pacific J.Math. 177 (1985), 183-191.
6. N Südland, B Baumann and TF Nonnenmacher. Open problem: who knows about the Aleph (ℵ)–functions?. *Fract. Calc. Appl. Anal.* **1**(4): 401–402, 1998
7. S.G. Samko, A.A. Kilbas and O.I. Marichev, Fractional Integrals and Derivatives, Theory and Applications, Gordon and Breach, New York (1993).
8. S. S. Nair, Pathway fractional Integration operator, Fractional Calculus and Applied Analysis 12(3) (2009), 237-252.
9. B. L. Chaurasia and Neeti Ghiya, Pathway fractional integral Operator pertaining to special functions, Global J. Sci., Front. Res., 10(6)(Ver.1.0),(2010), 79-83.

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Generalizations of 2D-Canonical Sine-Sine Transform

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Abstract - Integral transform, fractional integral transform is a flourishing filed of active research due to its wide range of application. Fourier transform, fractional Fourier transform is probably the most intensively studied among all fractional transforms, similarly 2D canonical sine-sine transforms, and 2D canonical cosine-cosine is a powerful mathematical tool for processing images. In this paper the canonical 2D sine-sine transform is define in generalized sense. And various testing functions spaces defined by using Gelfand-shilov technique. Also uniqueness theorem, modulation theorems are proved.

Keywords : 2D canonical sine-sine transform, testing function space, generalized functions, fourier transform.

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GENERALIZATIONS OF 2D-CANONICAL SINE-SINE TRANSFORM

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1. Chavhan S.B. BorkarV.C.,"Operator of Two Dimensional Generalized Canonical Sine Transform. IJERD.Vol.4 Issue 2. (2012). 10-14

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I. Introduction

Integral transforms have been successfully used for almost two centuries is solving many problems in mathematical physical, applied mathematics and engineering science. Historically origin of the integral transform is P.S. Laplace and J.Fourier. Laplace transform is useful, for evaluating certain definite integral [2].

The definition of canonical sine-sine transform as follows [1].

$$\{2DCSST\, f(t,x)\}(s,w) = \langle f(t,x), K_{s_1}(t,x) K_{s_2}(x,w) \rangle$$

In the present paper, 2D sine-sine transform is extended in the distribution sense. The plan of the paper is as follows. The definitions are given in section 2. In section 3, testing functionspace is defined by Galfand-shilovtechnique [3],[4].Section 4 some result on countable union space are proved. In section5, inversion and uniqueness theorem are stated. In section 6, modulations theorems are given. The notations and terminology as per zemanian [5],[6].

II. Definition Two Dimensional Canonical Sine-Sine Transform

Let $E'(R\times R)$ denote the dual of $E(R\times R)$. Therefore the generalized canonical sine transform of $f(t,x)\in E'(R\times R)$ is defined as

$$\{2DCST\, f(t,x)\}(s,w)$$

$$= (-1)\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2} f(t,x)\,dxdt$$

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Where $$\gamma_{E,k}\left\{K_{s_1}(t,s)K_{s_2}(x,w)\right\} = \sup_{\substack{-\infty<t<\infty \\ -\infty<x<\infty}} \left| D_t^k D_x^l K_{s_1}(t,s) K_{s_2}(x,w) \right| < \infty$$

III. Different S-Type Testing Function Spaces

In this section we have defined s-type testing function spaces by imposing conditions not only on the decreases of the fundamental functions at infinity, but also on the growth of their derivatives as the order of derivative increases. Clearly, $SS^{a,b}$ space will be extension of testing function space D, so that these spaces have been successfully, used in pseudo differential operator theory.

a) The space $SS_{\gamma}^{a,b}$:

It is given by

$$SS_{\gamma}^{a,b} = \left\{ \phi : \phi \in E_+ / \sigma_{l,k,p}\phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \le C_{kp} A^l . l^{l\gamma} \right\} \tag{3.1}$$

The constant $C_{k,p}$ and A depend on ϕ.

b) The space $SS^{ab,\beta}$:

$SS^{a,b,\beta}$ this space is given by

$$SS^{a,b,\beta} = \left\{ \phi . \phi \in E_+ / \rho_{l,k,p}\phi(t,x) = \sup \left| t^l D_t^k D_x^p \phi(t,x) \right| \le C_{l,p} B^k k^{k\beta} \right\} \tag{3.2}$$

The constants $C_{l,p}$ and B depend on ϕ.

c) The space $SS_{\gamma}^{a,b,\beta}$

This space is formed by combining the condition (3.1) and (3.2)

$$SS_{\gamma}^{a,b,\beta} = \left\{ \phi : \phi \in E_+ / \xi_{l,k,p}\phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \le C\, A^l l^{l\gamma} B^k k^{k,\beta} \right\} \tag{3.3}$$

$l, k, p = 0,1,2.....$ Where A,B,C depend on ϕ.

d) The space $SS_{\gamma,m}^{a,b,\beta}$

It is defined as,

$$SS_{\gamma,m}^{a,b} = \left\{ \phi : \phi \in E_+ / \sigma_{l,k,p}\phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \le C_{k,p,\mu} (m+\mu)^l\, l^{l\gamma} \right\} \tag{3.4}$$

For any $\mu > 0$ where m is the constant, depending on the function ϕ.

e) The space $SS^{a,b,\beta,n}$

This space is given by

$$SS^{a,b,\beta,n} = \left\{ \phi : \phi \in E_+ / \rho_{l,k,p}\phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \le C_{l,p,\delta} (n+\delta)^k\, k^{k\beta} \right\} \tag{3.5}$$

For any $\delta > 0$ where n the constant is depends on the function ϕ.

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f) The space $SS_{\gamma,m}^{a,b,\beta,n}$

This space is defined by combining the conditions in (3.4) and (3.5).

$$SS_{\gamma,m}^{a,b,\beta,n} = \left\{\phi : \phi \in E_+ \,/\, \xi_{l,k,p}\phi(t,x) =_{I_1}^{\sup} \left| t^l D_t^k D_x^p \phi(t,x) \right| \right.$$
$$\left. \le C_{\mu\delta} (m+\mu)^l (n+\delta)^k .l^{l\gamma} k^{k\beta} \right\} \tag{3.6}$$

IV. Results on Countable Unions-Type Space

Proposition 4.1: If $m_1 < m_2$ then $SS_{\gamma,m_1}^{a,b} \subset SS_{\gamma,m_2}^{a,b}$. The topology of $SS_{\gamma,m_1}^{a,b}$ is equivalent to the topology induced on $SS_{\gamma,m_1}^{a,b}$ by $SS_{\gamma,m_2}^{a,b}$

$$i.e\ T_{\gamma,m_1}{}^{a,b} \sim T_{\gamma,m_2}{}^{a,b} \,/\, SS_{\gamma,m_1}^{a,b}$$

Proof: For $\phi \in SS_{\gamma,m_1}^{a,b}$ and $\delta_{l,k,p}(\phi) \le C_{k,\mu}(m_1+\mu)^l l^{l\gamma}$

$$\le C_{k,\mu,p}(m_2+\mu)^l l^{l\gamma} \quad \text{Thus, } SS_{\gamma,m_1}^{a,b} \subset ss_{\gamma,m_2}^{a,b}$$

The space $SS_{\gamma,}^{a,b}$ can be expressed as union of countable normed spaces.

Proposition 4.2: $SS_{\gamma}^{a,b} = \overset{\infty}{\underset{i=1}{U}} SS_{\gamma,m_1}^{a,b}$ and if the space $SS_{\gamma}^{a,b}$ is equipped with strict inductive limit topology $S_{a,b,m}$ defined by injective map from $SS_{\gamma,m_1}^{a,b}$ to $SS_{\gamma}^{a,b}$ then the sequence $\{\phi_n\}$ in $SS_{\gamma}^{a,b}$ converges to zero.

Proof: we show that $SS_{\gamma}^{a,b} = \overset{\infty}{\underset{i=1}{U}} SS_{\gamma.m_1}^{a,b}$

Clearly $\overset{\infty}{\underset{i=1}{U}} SS_{\gamma.m_1}^{a,b} \subset SS_k^{a,b}$ for proving the other inclusion, let $\phi \in SS_{\gamma}^{a,b}$ then

$$\delta_{l,k,p}(\phi(t,x)) =_{I_1}^{\sup} \left| t^l D_t^k D_x^p \ \phi_n(t,x) \right|$$
$$\le C_{k,p} A^l l^{l\gamma}, \tag{4.1}$$

where A is some positive constant, choose an integer $m = m_A$ and $\mu = 0$ such that $C_{k,p} A^l \le C_{k,p}(m+\mu)^l$.

Then (4.1) we get $\phi \in SS_{\gamma,m_1}^{a,b}$ implying that $SS_{\gamma}^{a,b} = \overset{\infty}{\underset{i=1}{U}} SS_{\gamma,m_1}^{a,b}$

Proposition 4.3: If $\gamma_1 < \gamma_2$ and $\beta_1 < \beta_2$ then $SS_{\gamma_1}^{a,b,\beta_1} \subset SS_{\gamma_2}^{a,b,\beta_2}$ and the topology of $SS_{\gamma_i}^{a,b,\beta_i}$ is equivalent to the topology induced on $SS_{\gamma_1}^{a,b,\beta_1}$ by $SS_{\gamma_2}^{a,b,\beta_2}$.

Proof: Let $\phi \in SS_{\gamma_1}^{a,b,\beta_1}$

$$\xi_{l,k,p}(\phi) =_{I_1}^{\sup} \left| t^l D_t^k D_x^p \phi(t,x) \right|$$

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$$\leq CA^l l^{l\gamma_1} B^k k^{k\beta_1}$$

$$\leq CA^l l^{l\gamma_2}.B^p k^{p,\beta_2} \qquad \text{where} l, k, p = 0,1,2,3$$

Hence $\phi \in SS_{\gamma_2}^{a,b,\beta_2}$. Consequently, $SS_{\gamma_1}^{a,b,\beta_1} \subset SS_{\gamma_2}^{a,b,\beta_2}$. The topology of $SS_{\gamma_1}^{a,b,\beta_1}$ Is equivalent to the topology $T_{\gamma_2}^{a,b,\beta_2} / SS_{\gamma_2}^{a,b,\beta_2}$

It is clear from the definition of topologies of these spaces.

Proposition 4.4: $SS^{a,b} = \bigcup_{\gamma_i\beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$ and if the space $SS^{a,b}$ is equipped with the strict $SS^{a,b}$ inductive limit topology defined by the injective maps from $SS_{\gamma_i}^{a,b,\beta_i}$ to $SS^{a,b}$ then the sequence $\{\phi_n\}$ in $SS^{a,b}$ converges to zero iff $\{\phi_n\}$ is contained in some $SS_{\gamma_i}^{a,b,\beta_i}$ and converges to zero.

Proof: $SS^{a,b} = \bigcup_{\gamma_i\beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,,\beta_i}$

Clearly $$\bigcup_{\gamma_i\beta_i\ 1=}^{\infty} SS_{\gamma_i}^{a,b,\beta_i} \subset SS^a$$

For proving other inclusion, let $\phi(t,x) \in SS^{a,b}$ then

$$\eta_{l,k,p}(\phi) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right|,$$

is bounded by some number. We can choose integers γ_m and β_m such that

$$\eta_{l,k,p}(\phi) \leq C\, A^l l^{l,\gamma} B^{k,m}, k^{k,\beta,m}$$

$\therefore \phi \in SS_{\gamma_i}^{a,b,\beta_i}$ for some integer γ_i and β_i

Hence $SS^{a,b} \subset \bigcup_{\gamma_i\beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$ Thus $SS^{a,b} = \bigcup_{\gamma_i\beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$

V. Inversion and Uniqueness Theorems

Theorem 5.1: (Inversion) If $\{2DCSST\ f(t,x)\}(s,w)$ is canonical sine-sine transform of $f(t,x)$ then inverse of transform is given by

$$f(t,x)$$
$$= -\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\, e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2} e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right) e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2} \{2DCSST\ f(t,x)\}(s,w)\, dsdw$$

Theorem 5.2: (Uniqueness) If $\{2DCSST\ f(t,x)\}(s,w)$ and $\{2DCSST\ g(t,x)\}(s,w)$ are 2D canonical sine-sine transform and $\sup pf \subset s_a$ and s_b also $\sup p\ g \subset s_a and\ \ s_b$

Where $s_a = \{t : t \in R^n,\ \ |t| \leq a, a > 0\}$ and $s_b = \{x : x \in R^n,\ \ |x| \leq b, b > 0\}$

If $\{2DCSST\ f(t,x)\}(s,w) = \{2DCSST\ g(t,x)\}(s,w)$

then, $f = g$ in the sense of equality in $D'(I)$

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Proof: By inversion theorem

$$f-g=$$
$$-e^{\frac{-i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{-i}{2}\left(\frac{a}{b}\right)x^2}\left[\left(\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)\{2DCSST\ f(t,x)\}(s,w)\,dsdw,\right)\right.$$

$$\left.-\left(\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{\frac{-i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{-i}{2}\left(\frac{d}{b}\right)w^2}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)\{2DCSST\ g(t,x)\}(s,w)\,dsdw,\right)\right]$$

$$\therefore f-g=-\sqrt{\frac{2\pi i}{b}}\sqrt{\frac{2\pi i}{b}}e^{-\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{-\frac{i}{2}\left(\frac{a}{b}\right)x^2}\int_{-\infty}^{\infty}e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{-\frac{i}{2}\left(\frac{d}{b}\right)s^2}\sin\left(\frac{s}{b}x\right)\sin\left(\frac{w}{b}x\right)$$
$$\left[\{2DCSST\,f(t,x)\}-\{2DCSST\,g(t,x)\}\right]dsdw$$

Thus $f=g$ in $D'(I)$

VI. Modulation Theorems for Canonical Sine-Sine Transform

Theorem 6.1: If $\{2DCSST\ f(t,x)\}(s,w)$ is canonical sine-sine transform of $f(t,x)$ then

$$\{2DCSST\cos\mu t\ f(t,x)\}(s,w)$$
$$=\frac{e^{-\frac{i}{2}\left(\mu^2bd\right)}}{2}\left[e^{-i(s\mu d)}\{2DCSST\ f(t,x)\}(s+\mu b,w)+e^{i(\mu sd)}\{2DCSST\ f(t,x)\}(s-\mu b,w)\right]$$

Proof: Definition of two dimensional canonical sine-sine transform $f(t,x)$ is

$$\{2DCSST\ f(t,x)\}(s,w)$$
$$=-\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$\{2DCSST\cos\mu t\ f(t,x)\}(s,w)$$
$$=-\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s}{b}t\right)\sin\left(\frac{w}{b}x\right)\cos\mu t\,e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt$$

$$=-\frac{1}{2}\left[\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s+\mu b}{b}\right)t\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt\right.$$

$$\left.+\frac{1}{\sqrt{2\pi ib}}\frac{1}{\sqrt{2\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}e^{\frac{i}{2}\left(\frac{d}{b}\right)w^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sin\left(\frac{s-\mu b}{b}\right)t\sin\left(\frac{w}{b}x\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}e^{\frac{i}{2}\left(\frac{a}{b}\right)x^2}f(t,x)\,dxdt\right]$$

$$=\frac{1}{2}\left[e^{-i(s\mu d)}e^{\frac{-i}{2}\left(u^2bd\right)}\{2DCSST\ f(t,x)\}(s+ub,w)+e^{i(s\mu d)}e^{\frac{-i}{2}\left(\mu^2bd\right)}\{2DCSST\ f(t,x)\}(s-ub,w)\right]$$

$$\{2DCSST\cos\mu t\ f(t,x)\}(s,w)$$
$$=\frac{e^{-\frac{i}{2}\left(\mu^2bd\right)}}{2}\left[e^{-i(s\mu d)}\{2DCSST\ f(t,x)\}(s+\mu b,w)+e^{i(\mu sd)}\{2DCSST\ f(t,x)\}(s-\mu b,w)\right]$$

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Theorem 6.2 If $\{2DCSST\ f(t,x)\}(s,w)$ is canonical sine-sine transform of $f(t,x)$ then

$$\{2DCSST \sin\mu t\ f(t,x)\}(s,w) =$$

$$\frac{ie^{-\frac{i}{2}(\mu^2 bd)}}{2}\left[e^{-i(s\mu d)}\{2DCCST\ f(t,x)\}(s+\mu b,w) - e^{i(\mu sd)}\{2DCCST\ f(t,x)\}(s-\mu b,w)\right]$$

Theorem 6.3 If $\{2DCSST\ f(t,x)\}(s,w)$ is canonical sine-sine transform of $f(t,x)$ then

$\{2DCSSTe^{i\mu t}\ f(t,x)\}(s,w)$

$$= \frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2}\left[e^{-i(s\mu d)}\left(\{2DCSST\ f(t,x)\}(s+\mu b,w) - \{2DCCST\ f(t,x)\}(s+\mu b,w)\right)\right.$$

$$\left. + e^{i(s\mu d)}\left(\{2DCSST\ f(t,x)\}(s-\mu b,w) + \{2DCCST\ f(t,x)\}(s-\mu b,w)\right)\right]$$

Proof: Since $\{2DCSST\, e^{i\mu t}\ f(t,x)\}(s,w) = \{2DCSST(\cos\mu t + i\sin\mu t)\, f(t,x)\}(s,w)$

$$\{2DCSST\, e^{i\mu t}\ f(t,x)\}(s,w) = \{2DCSST \cos\mu t\ f(t,x)\}(s,w) + i\{2DCSST \sin\mu t\, f(t,x)\}(s,w)$$

$$-\frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2}\left[e^{-i(s\mu d)}\{2DCCST\ f(t,x)\}(s+\mu b,w) - e^{i(s\mu d)}\{2DCCST\ f(t,x)\}(s-\mu b,w)\right]$$

$$\frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2}\left[e^{-i(s\mu d)}\{2DCSST\ f(t,x)\}(s+\mu b,w) + e^{i(s\mu d)}\{2DCSST\ f(t,x)\}(s-\mu b,w)\right.$$

$$\left. -e^{-i(s\mu d)}\{2DCCST\ f(t,x)\}(s+\mu b,w) + e^{i(s\mu d)}\{2DCSCT\ f(t,x)\}(s-\mu b,w)\right]$$

$$\{2DCSSTe^{i\mu t}\ f(t,x)\}(s,w) = \frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2}\left[e^{-i(s\mu d)}\left(\{2DCSST\ f(t,x)\}(s+\mu b,w) - \{2DCCST\ f(t,x)\}(s+\mu b,w)\right)\right.$$

$$\left. + e^{i(s\mu d)}\left(\{^2DCSST\ f(t' x)\}(s-\mu b' w) + \{^2DCCST\ f(t' x)\}(s-\mu b' w)\right)\right]$$

VII. Conclusion

In this paper 2D canonical sine–sine transform is generalized in the distributional sense. Uniqueness theorem is proved and various testing functions specs defined by using Gelfand-shilov technique, topology properties are discussed. And lastly modulation theorems are proved.

References Références Referencias

1. Chavhan S.B. BorkarV.C.,"Operator of Two Dimensional Generalized Canonical Sine Transform. IJERD.Vol.4 Issue 2. (2012). 10-14
2. DebnathLokenath and BhattaDambaru. "Integral transforms and their applications," Chapman and Hall/CRC Taylor and Francis Group Boca Raton London New York
3. Gelfand I.M. and Shilov G.E., "Generalized Functions," Volume I Academic Press, New York, 1964.

Notes

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4. Gelfand I.M. and Shilov G.E., "Generalized Functions," Volume II, Academic Press, New York, 1967.
5. Zemanian A.H., "Distribution theory and transform analysis," McGraw Hill, New York, 1965.
6. Zemanian A.H., Generalized integral transform," Inter Science Publisher's New York, 1968.

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Some Subclasses of P-Valent Analytic Functions

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Abstract - The object of the present paper is to derive the sufficient conditions for certain subclasses of p-valent analytic functions in the open unit disk. A number of known results would follow upon specializing the parameters involved in our main results. Also, sufficient conditions are found for function to be univalent.

Keywords : *analytic functions, univalent, p-valently starlike functions, p-valently convex functions.*

GJSFR-F Classification : *MSC 2010: 11E45*

SOME SUBCLASSES OF P-VALENT ANALYTIC FUNCTIONS

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Some Subclasses of P-Valent Analytic Functions

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I. Introduction and Preliminaries

Let $\mathcal{A}(p)$ denote the class of functions of the form

$$f(z) = z^p + \sum_{n=1}^{\infty} a_{p+n} z^{p+n} \quad (p \in \mathbb{N} = \{1, 2, \dots\}),$$

which are analytic and p-valent in the open unit disk $\mathbb{U} := \{z : |z| < 1\}$. Let $\mathcal{S}$ be the class of analytic and univalent functions in $\mathbb{U}$. We note that $\mathcal{A}(1) \equiv \mathcal{S}$.

A function $f \in \mathcal{A}(p)$ is said to be in the class $\mathcal{S}^*(p, \alpha)$ of p-valently starlike of order α in $\mathbb{U}$ if and only if it satisfies the inequality

$$\mathrm{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad (z \in \mathbb{U}; 0 \le \alpha < p).$$

On the other hand, a function $f \in \mathcal{A}(p)$ is said to be in the class $\mathcal{K}(p, \alpha)$ of p-valently convex of order α in $\mathbb{U}$ if and only if it satisfies the inequality

$$\mathrm{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \quad (z \in \mathbb{U}; 0 \le \alpha < p).$$

In particular, we write $\mathcal{S}^*(1, 0) := \mathcal{S}^*, \mathcal{K}(1, 0) := \mathcal{K}$, where $\mathcal{S}^*$ and $\mathcal{K}$ are the usual subclass of $\mathcal{A}$, consisting of functions which are starlike and convex, respectively (see [1, 2]).

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Ref.

[1] P. L. Duren, *Univalent Functions*, Grundlehren der Mathematischen Wissenschaften 259, Springer- Verlag, New York Berlin, Heidelberg, and Tokyo, (1983).

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The object of the present paper is to investigate various properties of the following classes of analytic and p-valent function defined as follows.

A function $f \in \mathcal{A}(p)$ is said to be a member of the class $\mathcal{B}(\gamma, \beta, p, \alpha)$ if and only if it satisfies the inequality

$$\left| \left(\frac{\beta\gamma z^3 f'''(z) + (2\beta\gamma + \beta - \gamma) z^2 f''(z) + z f'(z)}{\beta\gamma z^2 f''(z) + (\beta - \gamma) z f'(z) + (1 - \beta + \gamma) f(z)} \right) - p \right| < p - \alpha \tag{1}$$

$$(0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}),$$

for some α, for all $z \in \mathbb{U}$.

Note that the condition (1) implies that

$$\mathrm{Re} \left\{ \frac{\beta\gamma z^3 f'''(z) + (2\beta\gamma + \beta - \gamma) z^2 f''(z) + z f'(z)}{\beta\gamma z^2 f''(z) + (\beta - \gamma) z f'(z) + (1 - \beta + \gamma) f(z)} \right\} > \alpha,$$

$$(0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}).$$

We note that $\mathcal{B}(0, \beta, p, \alpha) \equiv \mathcal{T}_\beta(p; \alpha)$ is the class studied by Irmak and Raina in [3]. The important subclasses such as $\mathcal{S}^*(p, \alpha), \mathcal{K}(p, \alpha)$, $\mathcal{S}^*$ and $\mathcal{K}$ are seen to be easily identifiable with the aforesaid class.

In recent times, Irmak et al. [3] and Prajapat [9] investigated certain subclasses of multivalent analytic functions and obtained some sufficient conditions for these classes. In this paper, motivated by the aforementioned works, we obtained sufficient conditions for functions to be a member of the class $\mathcal{B}(\gamma, \beta, p, \alpha)$. We also indicate some special cases and consequences of the main result. The other results investigated include certain inequalities for p-valent functions which characterize the properties of starlikeness and convexity in the open unit disk. Furthermore our result unifies the result for a functions belonging to the class of p-valently starlike function of order α and p-valently convex function of order α.

In order to derive our main results, we need the following Lemmas.

Lemma 1. [4] *Let $w(z)$ be the non-constant and analytic function in $\mathbb{U}$ with $w(0) = 0$. If $|w(z)|$ attains its maximum value on the circle $|z| = r < 1$ at a point z_0, then*

$$z_0 w'(z_0) = k w(z_0) \tag{2}$$

where $k \geq 1$ is a real number.

Lemma 2. [5] *Let Ω be a set in the complex plane $\mathbb{C}$ and suppose that $\Phi(z)$ is a mapping from $\mathbb{C}^2 \times \mathbb{U}$ to $\mathbb{C}$ which satisfies $\Phi(ix, y; z) \notin \Omega$ for $z \in \mathbb{U}$, and for all real x, y such that $y \leq -n(1 + x^2)/2$. If the function $q(z) = 1 + q_n z^n + q_{n+1} z^{n+1} + \cdots$ is analytic in $\mathbb{U}$ such that $\Phi(q(z), zq'(z); z) \in \Omega$ for all $z \in \mathbb{U}$, then* $\mathrm{Re}\, q(z) > 0$.

Lemma 3. [7] *Let δ be the complex number,* $\mathrm{Re}\, \delta > 0$, *and λ be a complex number, $|\lambda| \leq 1, \lambda \neq -1$ and let $h(z) = z + a_2 z^2 + \cdots$ be a regular function on $\mathbb{U}$. If*

$$\left| \lambda |z|^{2\delta} + \left(1 - |z|^{2\delta}\right) \frac{z h''(z)}{\delta h'(z)} \right| \leq 1$$

Ref.

[3] H. Irmak and R. K. Raina, The starlikeness and convexity of multivalent functions involving certain inequalities, *Rev. Mat. Complut.* (16) (2) (2003), 391–398.

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for all $z \in \mathbb{U}$, *then the function*

$$F_\delta(z) = \left(\delta \int_0^z t^{\delta-1} h'(t)dt\right)^{1/\delta}$$

$$= z + \frac{2a_2}{\delta+1}z^2 + \left(\frac{3a_3}{\delta+2} + \frac{2\delta(1-\delta)a_2^2}{(\delta+1)^2}\right)z^3 + \cdots$$

is regular and univalent in $\mathbb{U}$.

Lemma 4. [8] *Let* δ *be a complex number,* $\operatorname{Re}\delta > 0$, *and* λ *a complex number,* $|\lambda| < 1$, *and* $h \in \mathcal{A}$. *If*

$$\frac{1-|z|^{2\operatorname{Re}\delta}}{\operatorname{Re}\delta}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1-|\lambda|$$

for all $z \in \mathbb{U}$, *then for any complex number* $\eta, \operatorname{Re}\eta \geq \operatorname{Re}\delta$, *the function*

$$F_\eta(z) = \left(\eta \int_0^z t^{\eta-1} h'(t)dt\right)^{1/\eta}$$

is in the class $\mathcal{S}$.

Lemma 5. [6] *Let* $p(z)$ *be analytic in* $\mathbb{U}, p(0) = 1, p(z) \neq 0$ *in* $\mathbb{U}$ *and suppose that there exists a point* $z_0 \in \mathbb{U}$ *such that*

$$|\arg(p(z))| < \frac{\pi}{2}\alpha, \quad for\ |z| < |z_0|, \ \ |\arg(p(z_0))| = \frac{\pi}{2}\alpha,$$

where $0 < \alpha \leq 1$, *then we have*

$$\frac{z_0 p'(z_0)}{p(z_0)} = ik\alpha,$$

where

$$k \geq \frac{1}{2}\left(a + \frac{1}{a}\right) \geq 1 \ \ when \quad \arg(p(z_0)) = \frac{\pi}{2}\alpha,$$

$$k \leq -\frac{1}{2}\left(a + \frac{1}{a}\right) \leq -1 \ \ when \quad \arg(p(z_0)) = -\frac{\pi}{2}\alpha,$$

$$p(z_0)^{1/\alpha} = \pm ai, \quad (a > 0).$$

II. Main Results

By using Lemma 2, we first prove the following theorem.

Theorem 6. *Let* $f \in \mathcal{A}(p)$. *Define a function* $G_{\beta,\gamma}$ *by*

$$G_{\beta,\gamma}(z) := \beta\gamma z^2 f''(z) + (\beta-\gamma)zf'(z) + (1-\beta+\gamma)f(z), \quad (0 \leq \gamma \leq \beta \leq 1; z \in \mathbb{U}),$$

and if $G_{\beta,\gamma}(z)$ *satisfies*

$$\operatorname{Re}\left\{\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\left(2 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right)\right\} > p\left(1-\frac{n}{2}\right) + \frac{n}{2}\alpha$$

Ref.

[8] V. Pescar, Univalence criteria of certain integral operators, *Acta Cienc. Indica Math.*, (29) (1) (2003), 135–138.

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$$(0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p, n \in \mathbb{N}),$$

then $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$.

Proof. Let $f \in \mathcal{A}(p)$. Define a function $w(z)$ in $\mathbb{U}$ by

$$\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} = p + (p-\alpha)w(z), (0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}), \tag{3}$$

then the function $w(z)$ is analytic in $\mathbb{U}$, and $w(0) = 0$.
A computation using (3) shows that

$$\begin{aligned}\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\left(2 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right) &= (p-\alpha)[zw'(z) + w(z)] + p \\ &= \Phi(w(z), zw'(z); z),\end{aligned}$$

where $\Phi(r, s; z) = (p-\alpha)[s + r] + p$.
For all real x, y satisfying $y \leq -n(1+x^2)/2$, we have

$$\begin{aligned}\text{Re } \Phi(ix, y; z) &= \text{Re } \{(p-\alpha)[y + ix] + p\} \\ &\leq -\frac{n}{2}(p-\alpha)(1+x^2) + p \\ &\leq -\frac{n}{2}(p-\alpha) + p \\ &= p\left(1 - \frac{n}{2}\right) + \frac{n}{2}\alpha.\end{aligned}$$

Let $\Omega = \left\{w : \text{Re } w > p(1 - \frac{n}{2}) + \frac{n}{2}\alpha\right\}$. Then $\Phi(w(z), zw'(z); z) \in \Omega$ and $\Phi(ix, y; z) \notin \Omega$ for all real x and $y \leq -n(1+x^2)/2$, $z \in \mathbb{U}$.

By using Lemma 2, we have Re $w(z) > 0$, which implies that

$$\text{Re}\left\{\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right\} > \alpha, (0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}),$$

and hence $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$.

By setting $\gamma = \beta = 0$ in Theorem 6, we have following corollary.

Corollary 7. *If* $f \in \mathcal{A}(p)$ *satisfies*

$$\text{Re}\left\{\frac{zf'(z)}{f(z)}\left(2 + \frac{zf''(z)}{f'(z)} - \frac{zf'(z)}{f(z)}\right)\right\} > p\left(1 - \frac{n}{2}\right) + \frac{n}{2}\alpha$$
$$(0 \leq \alpha < p; p, n \in \mathbb{N}),$$

then $f(z) \in \mathcal{S}^*(p, \alpha)$.

Its further case when $\alpha = 0$ and $p = 1$, Corollary 7 reduces to Corollary 8.

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Corollary 8. *If $f \in \mathcal{A}$ satisfies*

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\left(2+\frac{zf''(z)}{f'(z)}-\frac{zf'(z)}{f(z)}\right)\right\} > 1-\frac{n}{2}$$
$$(n \in \mathbb{N}),$$

then $f(z) \in \mathcal{S}^*$.

By taking $\gamma = 0, \beta = 1$ in Theorem 6, we have the following corollary.

Corollary 9. *If $f \in \mathcal{A}_{(p)}$ satisfies*

$$\operatorname{Re}\left\{\frac{(zf'(z))'}{f'(z)}\left(1+\frac{z^2f'''(z)+2zf''(z)}{zf''(z)+f'(z)}-\frac{zf''(z)}{f'(z)}\right)\right\} > p\left(1-\frac{n}{2}\right)+\frac{n}{2}\alpha$$
$$(0 \leq \alpha < p; p, n \in \mathbb{N}),$$

then $f(z) \in \mathcal{K}(p, \alpha)$.

A further case of Corollary 9, when $\alpha = 0, p = 1$ gives the following corollary.

Corollary 10. *If $f \in \mathcal{A}$ satisfies*

$$\operatorname{Re}\left\{\frac{(zf'(z))'}{f'(z)}\left(1+\frac{z^2f'''(z)+2zf''(z)}{zf''(z)+f'(z)}-\frac{zf''(z)}{f'(z)}\right)\right\} > 1-\frac{n}{2}$$
$$(n \in \mathbb{N}),$$

then $f(z) \in \mathcal{K}$.

Theorem 11. *Let $-1 < b < a \leq 1$, $0 \leq \alpha < p$, $p \in \mathbb{N}$ such that $p(1+\alpha)+a \leq 2p(p-b)+b$. If $G_{\beta,\gamma}(z)$ satisfies the inequality*

$$\left|1+\frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)}-\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right| < \frac{p(a+b)}{(p+a)(p-b)} \quad (z \in \mathbb{U}), \tag{4}$$

then $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$.

Proof. Define a function $w(z)$ by

$$\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} = \frac{p+aw(z)}{p-bw(z)} \quad (z \in \mathbb{U}). \tag{5}$$

Then $w(z)$ is analytic in $\mathbb{U}$ and $w(0) = 0$. By the logarithmic differentiation of (5), we get

$$1+\frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)}-\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} = \frac{p(a+b)zw'(z)}{(p+aw(z))(p-bw(z))}. \tag{6}$$

Now suppose that there exists $z_0 \in \mathbb{U}$ such that

$$\max_{|z|<|z_0|} |w(z)| = |w(z_0)| = 1,$$

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then from Lemma 1, we have (2). Letting $w(z_0) = e^{i\theta}$, from (6), we have

$$\left|1 + \frac{z_0 G''_{\beta,\gamma}(z_0)}{G'_{\beta,\gamma}(z_0)} - \frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}\right| = \left|\frac{p(a+b)ke^{i\theta}}{(p+ae^{i\theta})(p-be^{i\theta})}\right| \geq \frac{p(a+b)}{(p+a)(p-b)}.$$

This contradicts our assumption (4). Therefore $|w(z)| < 1$ holds true for all $z \in \mathbb{U}$. Thus we conclude from (5) that

$$\begin{aligned}\left|\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} - p\right| &= \left|\frac{p+aw(z)}{p-bw(z)} - p\right| \\ &< \frac{p+a-p(p-b)}{p-b} \\ &\leq p-\alpha \quad (z \in \mathbb{U}),\end{aligned}$$

which implies that $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$.

Theorem 12. *Let $f \in \mathcal{A}(p)$. If $G_{\beta,\gamma}(z)$ satisfies anyone of the following conditions:*

$$\left|1 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right| < \frac{p-\alpha}{2p-\alpha}, \tag{7}$$

$$\left|\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\left(1 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right)\right| < p-\alpha, \tag{8}$$

$$\left|\frac{1 + \dfrac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)}}{\dfrac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}} - 1\right| < \frac{p-\alpha}{(2p-\alpha)^2}, \tag{9}$$

$$\left|\frac{1 + \dfrac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - p}{\dfrac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} - p} - 1\right| < \frac{1}{(2p-\alpha)}, \tag{10}$$

$$\mathrm{Re}\left\{\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\left(\frac{1 + \dfrac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - p}{\dfrac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} - p} - 1\right)\right\} < 1, \tag{11}$$

$$(0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}),$$

then $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$.

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Proof. Let $f \in \mathcal{A}(p)$. Define a function $w(z)$ in $\mathbb{U}$ by

$$\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} = p + (p-\alpha)w(z), (0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}), \tag{12}$$

then the function $w(z)$ is analytic in $\mathbb{U}$, and $w(0) = 0$.
It follows from (12) that

$$1 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} = \frac{(p-\alpha)zw'(z)}{p+(p-\alpha)w(z)}. \tag{13}$$

Hence, from (12) and (13), we have

$$\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\left(1 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right) = (p-\alpha)zw'(z), \tag{14}$$

$$\frac{1 + \dfrac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)}}{\dfrac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}} - 1 = \frac{(p-\alpha)zw'(z)}{[p+(p-\alpha)w(z)]^2}, \tag{15}$$

$$\frac{1 + \dfrac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - p}{\dfrac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} - p} - 1 = \frac{zw'(z)}{w(z)}\frac{1}{p+(p-\alpha)w(z)}, \tag{16}$$

and

$$\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\left(\frac{1 + \dfrac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - p}{\dfrac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} - p} - 1\right) = \frac{zw'(z)}{w(z)}. \tag{17}$$

Now, suppose there exists $z_0 \in \mathbb{U}$ such that

$$\max_{|z|<|z_0|} |w(z)| = |w(z_0)| = 1,$$

then from Lemma 1, we have (2). Therefore, letting $w(z_0) = e^{i\theta}$ in each of (13)-(17), we obtain that

$$\left|1 + \frac{z_0G''_{\beta,\gamma}(z_0)}{G'_{\beta,\gamma}(z_0)} - \frac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}\right| = \left|\frac{(p-\alpha)ke^{i\theta}}{p+(p-\alpha)e^{i\theta}}\right| \geq \frac{p-\alpha}{2p-\alpha},$$

$$\left|\frac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}\left(1 + \frac{z_0G''_{\beta,\gamma}(z_0)}{G'_{\beta,\gamma}(z_0)} - \frac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}\right)\right| = \left|(p-\alpha)ke^{i\theta}\right| \geq (p-\alpha),$$

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$$\left|\frac{1+\dfrac{z_0G''_{\beta,\gamma}(z_0)}{G'_{\beta,\gamma}(z_0)}}{\dfrac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}}-1\right| = \left|\frac{(p-\alpha)ke^{i\theta}}{(p+(p-\alpha)e^{i\theta})^2}\right| \geq \frac{p-\alpha}{(2p-\alpha)^2},$$

$$\left|\frac{1+\dfrac{z_0G''_{\beta,\gamma}(z_0)}{G'_{\beta,\gamma}(z_0)}-p}{\dfrac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}-p}-1\right| = \left|\frac{k}{p+(p-\alpha)e^{i\theta}}\right| \geq \frac{1}{(2p-\alpha)},$$

$$\mathrm{Re}\left\{\frac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}\left(\frac{1+\dfrac{z_0G''_{\beta,\gamma}(z_0)}{G'_{\beta,\gamma}(z_0)}-p}{\dfrac{z_0G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}-p}-1\right)\right\} = k \geq 1,$$

$$(0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}),$$

which contradict our assumption (7)-(11), respectively. Therefore $|w(z)| < 1$ holds true for all $z \in \mathbb{U}$. From (12), we have

$$\left|\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}-p\right| = |(p-\alpha)w(z)| < (p-\alpha), (0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}),$$

which implies that

$$\mathrm{Re}\left\{\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right\} = \mathrm{Re}\left\{\frac{\beta\gamma z^3 f'''(z)+(2\beta\gamma+\beta-\gamma)z^2f''(z)+zf'(z)}{\beta\gamma z^2 f''(z)+(\beta-\gamma)zf'(z)+(1-\beta+\gamma)f(z)}\right\} > \alpha,$$

$$(0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < p; p \in \mathbb{N}),$$

and hence $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$.

Remark 1. *By taking* $\gamma = 0$; $\gamma = \beta = 0$; $\gamma = 0$ *and* $\beta = 1$; $\gamma = \beta = \alpha = 0$ *and* $p = 1$; $\gamma = \alpha = 0$ *and* $\beta = p = 1$ *in Theorem 12, we get the results of Irmak and Raina* [3, Theorem 1, Corollary 1-4].

Theorem 13. *Let* $0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < 1, \delta$ *be a complex number,* $\mathrm{Re}\,\delta \geq \dfrac{3-2\alpha}{2-\alpha}$ *and* λ *be a complex number which satisfies the inequality*

$$|\lambda| \leq 1 - \frac{3-2\alpha}{\mathrm{Re}\,\delta(2-\alpha)}. \tag{18}$$

If $F_{\beta,\gamma}(z) := \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}$ *is regular in* $\mathbb{U}$ *and*

Ref.

[3] H. Irmak and R. K. Raina, The starlikeness and convexity of multivalent functions involving certain inequalities, *Rev. Mat. Complut.* (16) (2) (2003), 391–398.

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$$\left|1+\frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)}-\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right|\leq\frac{1-\alpha}{2-\alpha}\quad(z\in\mathbb{U}),\tag{19}$$

then the function

$$F(z)=\left(\delta\int_0^z t^{\delta-1}\frac{G'_{\beta,\gamma}(t)}{G_{\beta,\gamma}(t)}dt\right)^{1/\delta}\tag{20}$$

is univalent in $\mathbb{U}$.

Proof. Define a function

$$h(z)=\int_0^z\frac{F_{\beta,\gamma}(t)}{t}dt,$$

then we have $h(0)=h'(0)-1=0$. Also a simple computation yields $h'(z)=\dfrac{F_{\beta,\gamma}(z)}{z}$ and

$$\frac{zh''(z)}{h'(z)}=\frac{zF'_{\beta,\gamma}(z)}{F_{\beta,\gamma}(z)}-1.\tag{21}$$

From (21), we have

$$\begin{aligned}\left|\frac{zh''(z)}{h'(z)}\right|&\leq\left|\frac{zF'_{\beta,\gamma}(z)}{F_{\beta,\gamma}(z)}\right|+1\\&=\left|1+\frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)}-\frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right|+1.\end{aligned}\tag{22}$$

Hence, from (19) and (22), we have

$$\left|\frac{zh''(z)}{h'(z)}\right|\leq\frac{3-2\alpha}{2-\alpha}.\tag{23}$$

Using (23), we have

$$\begin{aligned}\left|\lambda\left|z\right|^{2\delta}+\left(1-\left|z\right|^{2\delta}\right)\frac{zh''(z)}{\delta h'(z)}\right|&\leq|\lambda|+\left|\frac{zh''(z)}{\delta h'(z)}\right|\\&\leq|\lambda|+\frac{1}{\mathrm{Re}\delta}\frac{3-2\alpha}{2-\alpha}.\end{aligned}$$

Again using (18), we have

$$\left|\lambda\left|z\right|^{2\delta}+\left(1-\left|z\right|^{2\delta}\right)\frac{zh''(z)}{\delta h'(z)}\right|\leq 1.$$

Applying Lemma 3, we obtain that the function $F(z)$ defined by (20) is univalent in $\mathbb{U}$.

We obtain Theorem 14 below, by using Lemma 4 and the same techniques as in the proof of Theorem 13.

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Theorem 14. *Let δ be a complex number,* $\operatorname{Re}\delta > 0$, *$\lambda$ a complex number,* $|\lambda| < 1$, *and $f \in \mathcal{A}$. If*

$$\left|1 + \frac{zG''_{\beta,\gamma}(z)}{G'_{\beta,\gamma}(z)} - \frac{zG'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)}\right| \leq \frac{1-\alpha}{2-\alpha} \quad (z \in \mathbb{U}; 0 \leq \gamma \leq \beta \leq 1; 0 \leq \alpha < 1),$$

then for any complex number η,

$$\operatorname{Re}\eta \geq \operatorname{Re}\delta \geq \frac{3-2\alpha}{(1-|\lambda|)(2-\alpha)},$$

the integral operator

$$F_\eta(z) = \left(\eta \int_0^z t^{\eta-1} \frac{G'_{\beta,\gamma}(t)}{G_{\beta,\gamma}(t)} dt\right)^{1/\eta}$$

is in the class $\mathcal{S}$.

Theorem 15. *Let $p(z)$ be an analytic function in $\mathbb{U}, p(z) \neq 0$ in $\mathbb{U}$ and suppose that*

$$\left|\arg\left(p(z) + \frac{z^2 G'_{\beta,\gamma}(z)}{G_{\beta,\gamma}(z)} p'(z)\right)\right| < \frac{\pi}{2}\alpha \quad (z \in \mathbb{U}), \tag{24}$$

where $0 < \alpha < p, 0 \leq \gamma \leq \beta \leq 1$ and $f(z) \in \mathcal{B}(\gamma, \beta, p, \alpha)$, then we have

$$|\arg(p(z))| < \frac{\pi}{2}\alpha \quad (z \in \mathbb{U}).$$

Proof. Suppose there exists a point $z_0 \in \mathbb{U}$ such that

$$|\arg(p(z))| < \frac{\pi}{2}\alpha, \quad for\ |z| < |z_0|, \quad |\arg(p(z_0))| = \frac{\pi}{2}\alpha.$$

Then, applying Lemma 4, we have

$$\begin{aligned} \arg\left(p(z_0) + \frac{z_0^2 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)} p'(z_0)\right) &= \arg\left(p(z_0)\left(1 + \frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)} \frac{z_0 p'(z_0)}{p(z_0)}\right)\right) \\ &= \arg(p(z_0)) + \arg\left(1 + i\frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)} k\alpha\right). \end{aligned} \tag{25}$$

When $\arg(p(z_0)) = \pi\alpha/2$, since

$$\operatorname{Re}\left(\frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)} k\alpha\right) > 0 \Rightarrow \arg\left(1 + i\frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)} k\alpha\right) > 0,$$

Eq. (25) becomes

$$\arg\left(p(z_0) + \frac{z_0^2 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)} p'(z_0)\right) > \frac{\pi}{2}\alpha. \tag{26}$$

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Similarly, if $\arg(p(z_0)) = -\pi\alpha/2$, since

$$\mathrm{Re}\left(\frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}k\alpha\right) < 0 \Rightarrow \arg\left(1 + i\frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}k\alpha\right) < 0,$$

we obtain that

$$\arg\left(p(z_0) + \frac{z_0^2 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}p'(z_0)\right) = \arg(p(z_0)) + \arg\left(1 + i\frac{z_0 G'_{\beta,\gamma}(z_0)}{G_{\beta,\gamma}(z_0)}k\alpha\right) < -\frac{\pi}{2}\alpha. \tag{27}$$

Thus, we see that (26) and (27) contradict our assumption (24). Consequently, we conclude that

$$|\arg(p(z))| < \frac{\pi}{2}\alpha \quad (z \in \mathbb{U}).$$

References Références Referencias

[1] P. L. Duren, *Univalent Functions*, Grundlehren der Mathematischen Wissenschaften 259, Springer- Verlag, New York Berlin, Heidelberg, and Tokyo, (1983).

[2] A. W. Goodman, *Univalent Functions*, Vols. I and II, Polygonal Publishing House, Washington, New Jersey, (1983).

[3] H. Irmak and R. K. Raina, The starlikeness and convexity of multivalent functions involving certain inequalities, *Rev. Mat. Complut.* (16) (2) (2003), 391–398.

[4] I. S. Jack, Functions starlike and convex of order α, *J. London Math. Soc.* (2) (3) (1971), 469–474.

[5] S. S. Miller and P. T. Mocanu, Differential subordinations and inequalities in the complex plane, *J. Differ. Equations,* (6) (1987), 199–211.

[6] M. Nunokawa, On the order of strongly starlikeness of strongly convex functions, *Proc. Japan Acad.,* (69) (A) (1993), 234–237.

[7] V. Pescar, A new generalization of Ahlfor's and Becker's criterion of univalence, *Bull. Malays. Math. Sci. Soc.,* (2) (19) (1996), 53–54.

[8] V. Pescar, Univalence criteria of certain integral operators, *Acta Cienc. Indica Math.,* (29) (1) (2003), 135–138.

[9] J. K. Prajapat, Some Sufficient Conditions for Certain Class of Analytic and Multivalent Functions, *Southeast Asian Bull. Math.,* (34) (2) (2010), 357–363.

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A Summation Formula Clung to Contiguous Relation

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Abstract - The main aim of the present paper is to evaluate a summation formula in the shadow of contiguous relation and recurrence relation.

Keywords : *gaussian hypergeometric function, contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.*

GJSFR-F Classification : *MSC 2010: 33C60 , 33C70 , 33D15 , 33D50 , 33D60.*

A SUMMATION FORMULA CLUNG TO CONTIGUOUS RELATION

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A Summation Formula Clung to Contiguous Relation

Salahuddin [α] & M. P. Chaudhary [σ]

Abstract - The main aim of the present paper is to evaluate a summation formula in the shadow of contiguous relation and recurrence relation.

Keywords : gaussian hypergeometric function, contiguous function, recurrence relation, bailey summation theorem and legendre duplication formula.

I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ b_1, b_2, \cdots, b_B & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ll} (a_A) & ; \\ (b_B) & ; \end{array} z\right] \equiv {}_AF_B\left[\begin{array}{ll} (a_j)_{j=1}^{A} & ; \\ (b_j)_{j=1}^{B} & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

Contiguous Relation[E. D. p.51(10), Andrews p.363(9.16)] is defined as follows

$$ (a-b)\ {}_2F_1\left[\begin{array}{ll} a,\ b & ; \\ c & ; \end{array} z\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1,\ b & ; \\ c & ; \end{array} z\right] - b\ {}_2F_1\left[\begin{array}{ll} a,\ b+1 & ; \\ c & ; \end{array} z\right] \tag{2} $$

Recurrence relation of gamma function is defined as follows

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{3} $$

Legendre duplication formula[Bells & Wong p.26(2.3.1)] is defined as follows

$$ \sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{4} $$

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$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{5}$$

$$= \frac{2^{(a-1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{6}$$

Bailey summation theorem [Prud, p.491(7.3.7.8)]is defined as follows

$$_2F_1\left[\begin{array}{ll} a, 1-a & ;\ \frac{1}{2} \\ c & ;\ \end{array}\right] = \frac{\Gamma(\frac{c}{2})\ \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-1}\ \Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} \tag{7}$$

II. Main Result of Summation Formula

$$_2F_1\left[\begin{array}{ll} a\ ,\ -a-50 & ;\ \frac{1}{2} \\ c & ;\ \end{array}\right]$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+50}}\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+50}{2})}\Big\{-349281398856053997508091191516200960000a$$

$$+413576826726104639517424992011255808000a^2$$

$$-178735941888370793121003448380707635200a^3$$

$$+356646499044269595131950037578039910 40a^4$$

$$-311554577666453927934685666755362918 4a^5$$

$$+283344646566246923837143793435650 56a^6 + 116490499894914876438539394177521 28a^7$$

$$-338993837247881522237887727181888a^8 - 239678900966770667129844090805 60a^9$$

$$+576081398341880215471369798896a^{10} + 358243559457035648934069037 88a^{11}$$

$$-218753960224314417939862188a^{12} - 308122618562266315643378 65a^{13}$$

$$-294303555355218100065024a^{14}+971497284847430902941 8a^{15}+233658656740810973892a^{16}$$

$$+665467940266912145a^{17} - 33785119600458864a^{18} - 450948140574712a^{19}$$

$$-1349964560868a^{20} + 14581955465a^{21} + 138711936a^{22} + 384578a^{23} + 12a^{24} - a^{25}$$

$$+349281398856054617956492924755640320000c$$

$$-107301280389995420369545074121624780 8000ac$$

$$+826931992648450623755141519452967731200a^2c$$

$$-268178880352284310003820868302667325440a^3c$$

$$+406768324606410709253869873234098647 04a^4c$$

$$-240149011392899319714723147744325785 6a^5c$$

$$-512158336652942331783233013400193 28a^6c+999814862341710703080682425567628 8a^7c$$

$$-244130555659247227353556200283 20a^8c - 191422569930233923761104947152 96a^9c$$

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$$-2322740395571581781405074968 8a^{10}c+2101846567227639169062284848 8a^{11}c$$
$$+2471797320536397310206220 10a^{12}c-1030620721089672491756577 6a^{13}c$$
$$-2543582554481533868164 68a^{14}c+3235065200104977748 08a^{15}c$$
$$+7075513554782201815 0a^{16}c+7728952286853008 64a^{17}c-1182906526632688a^{18}c$$
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$$-1200660410145696123567172867824549888000ac^2$$
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$$+2001624614302528488746646413910031564 8a^4c^2$$
$$-6967514682069854636385203988588687 36a^5c^2$$
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$$-5097772113461259266205 6a^{14}c^2-4648059205944458521 36a^{15}c^2$$
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$$+2770108827393430916552276154324615168 00c^4$$

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$$+5327985666723963443 2a^{13}c^6+638921254888844928a^{14}c^6+463333171357056a^{15}c^6$$

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$$-27119794141440a^{16}c^6 - 134770796160a^{17}c^6 - 98978880a^{18}c^6 + 320320a^{19}c^6$$
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$$-349567381275971095112152252416ac^{13} + 152515214709284023928692408 32a^2c^{13}$$
$$+218825323379159366500614144a^3c^{13} - 171097656613360596857651 20a^4c^{13}$$
$$-122206099379806878105600a^5c^{13} + 623100624833941553152 0a^6c^{13}$$
$$+589589350931243335 68a^7c^{13} - 595218771270402048a^8c^{13} - 7748163223879680a^9c^{13}$$
$$-5155444654080a^{10}c^{13} + 132025614336a^{11}c^{13} + 222265344a^{12}c^{13}$$
$$+856382166548722328430182400 00c^{14} - 125332961811300777583443968 00ac^{14}$$
$$+377096167588144864284377088a^2c^{14} + 999819876703275729446502 4a^3c^{14}$$
$$-340536027759557946900480a^4c^{14} - 492884601422971207680 0a^5c^{14}$$
$$+772227207072792576 00a^6c^{14} + 1166674390020980736a^7c^{14}$$
$$-1915162037452800a^8c^{14} - 63202103132160a^9c^{14} - 134311772160a^{10}c^{14}$$
$$+190513152a^{11}c^{14} + 3176451687815006006961766400c^{15}$$

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$$-36885741202397320035749068 8ac^{15}+692145504817981649688985 6a^2c^{15}$$
$$+2933214703595966221516 80a^3c^{15}-4681522483934003200000a^4c^{15}$$
$$-1074829442705915904 00a^5c^{15}+4952975186240471 04a^6c^{15}$$
$$+13449542683852800a^7c^{15}+22267177205760a^8c^{15}-251477360640a^9c^{15}$$
$$-508035072a^{10}c^{15}+9793532728173182897356 8000c^{16}$$
$$-8859555401078193216552960ac^{16}+8346742267459310557593 6a^2c^{16}$$
$$+6149116495967052627968a^3c^{16}-37154850414966079488a^4c^{16}$$
$$-1515427531741003776a^5c^{16}-821812773519360a^6c^{16}+90493112156160a^7c^{16}$$
$$+255287623680a^8c^{16}-317521920a^9c^{16}+2498697553742732900433920c^{17}$$
$$-1720308251634981257871 36ac^{17}+3203345485930372792 32a^2c^{17}$$
$$+9413702470748130508 8a^3c^{17}-2123582349901824a^4c^{17}-13982628068720640a^5c^{17}$$
$$-42882568028160a^6c^{17}+310648504320a^7c^{17}+784465920a^8c^{17}$$
$$+5236091258727825408 0000c^{18}-2661918961599879577600ac^{18}$$
$$-1083096524325598003 2a^2c^{18}+1040210571393236992a^3c^{18}+3794724189634560a^4c^{18}$$
$$-79649091092480a^5c^{18}-314832322560a^6c^{18}+348651520a^7c^{18}$$
$$+8910092488014823424 00c^{19}-3213018246085921996 8ac^{19}-2727259717679185 92a^2c^{19}$$
$$+7979001383485440a^3c^{19}+44971568660480a^4c^{19}-239798845440a^5c^{19}$$
$$-807403520a^6c^{19}+12108417434910720000c^{20}-2929845467296563 20ac^{20}$$
$$-3358306288533504a^2c^{20}+39263952437248a^3c^{20}+242705498112a^4c^{20}$$
$$-242221056a^5c^{20}+12821040824582144 0c^{21}-1919353142378496ac^{21}$$
$$-24901685608448a^2c^{21}+105054732288a^3c^{21}+530579456a^4c^{21}+1018712555520000c^{22}$$
$$-8291509862400ac^{22}-106212360192a^2c^{22}+96468992a^3c^{22}+5710964326400c^{23}$$
$$-19931332608ac^{23}-201326592a^2c^{23}+20132659200c^{24}$$
$$-16777216ac^{24}+33554432c25\Big\}$$
$$+\frac{1}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+51}{2})}\Big\{+196078146816081941570317208046796800 0000$$
$$-3661026774131950114663981678663434240000a$$
$$+2240090762233257947677314630597746688000a^2$$
$$-6146450144190017312559932398731245568 00a^3$$
$$+7805367577473676912511208521334088704 0a^4$$
$$-3043255274124936913351196421079574016a^5$$
$$-2291793321235228725388631902218397 44a^6$$

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$$+1794658108232009589473761514989267 2a^7+4100175830455883720755561698701 12a^8$$
$$-3878205776943273535070216357144 0a^9-8685521877947829444117895545 04a^{10}$$
$$+3978862691672435241265495841 2a^{11}+1344044442163512163688040062a^{12}$$
$$-9919408027950128138403935a^{13}-9061972227849840295285 24a^{14}$$
$$-9453757494566587215818a^{15}+1515257868272236706 42a^{16}+4294642142413240255a^{17}$$
$$+27694449851967536a^{18}-209906546918288a^{19}-4290754509118a^{20}-23145910865a^{21}$$
$$-7772764a^{22}+379822a^{23}+1262a^{24}+a^{25}+5080828409738663152845985129137438720000c$$
$$-7129727012710533088531986116981686272000ac$$
$$+3464350353104816822235888040121357107200a^2c$$
$$-7574670591177252372322223672702333333760a^3c$$
$$+7233223083743504595418697336473952870 4a^4c$$
$$-1032156143759539534540657430207660544a^5c$$
$$-2659317911026037257375194777011201 28a^6c+9020260880498864853288175289635712a^7c$$
$$+5554065135547915694128485344548 80a^8c-1547736268764409001205727222870 4a^9c$$
$$-8633757239608692744379187680 88a^{10}c+6788342816518488117032677512a^{11}c$$
$$+7713750135336959307369064 10a^{12}c+6748486804230986758938176a^{13}c$$
$$-2531395844938727129224 68a^{14}c-5830435847039153014808a^{15}c$$
$$-1428236382499837625 0a^{16}c+8743657969714599 36a^{17}c+1135391239839771 2a^{18}c$$
$$+32142393315032a^{19}c-382686073770a^{20}c-3536396864a^{21}c-9614228a^{22}c$$
$$+312a^{23}c+26a^{24}c+5383081985040703798697681046680371200000c^2$$
$$-6004634872541150478657109318810730496000ac^2$$
$$+2365807240966142348808314166846518722560a^2c^2$$
$$-4132017256050339287343444801001721364 48a^3c^2$$
$$+2852829640307522592168824098755189964 8a^4c^2$$
$$+3258102646403196560077706203857039 36a^5c^2$$
$$-1148366781035917911559097852269276 16a^6c^2+9164570216941817629181051525509 12a^7c^2$$
$$+2246421734416030825142786225399 04a^8c^2-5794980639997827070372889054 72a^9c^2$$
$$-2590098248842453224531348920 64a^{10}c^2-2411857401944811597347427456a^{11}c^2$$
$$+1365609972997284863138770 48a^{12}c^2+3007808664730215637780216a^{13}c^2$$
$$-8938969098168899560056a^{14}c^2-9099565449526808038 64a^{15}c^2$$
$$-9135116517044266320a^{16}c^2+2375844932490744 0a^{17}c^2+1053172687975408a^{18}c^2$$
$$+6892565247568a^{19}c^2+5758672920a^{20}c^2-108628520a^{21}c^2-393432a^{22}c^2$$
$$-312a^{23}c^2+3264505862992440279812749192190754816000c^3$$

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$$-297536611341771752196395659721057501184 0ac^3$$
$$+960253992988307644756107956860095823872a^2c^3$$
$$-1332551751554746411782964907632521707 52a^3c^3$$
$$+614468222538571557479567032725223833 6a^4c^3$$
$$+269359725495675945790020827523549184a^5c^3$$
$$-2585992610846294503210014552614092 8a^6c^3-3366660623429036388796445716648 96a^7c^3$$
$$+4339332989845055066295008315852 8a^8c^3+6539080230271155980377710831 36a^9c^3$$
$$-3453908461439895041001246873 6a^{10}c^3-8521752368338300382279235 52a^{11}c^3$$
$$+7410746533794561055550816a^{12}c^3+4233015077285572903765 44a^{13}c^3$$
$$+3143627840368997288736a^{14}c^3-4919656540211016032 0a^{15}c^3-96261007825249696 0a^{16}c^3$$
$$-4153459678560192a^{17}c^3+27554114555968a^{18}c^3+330063653920a^{19}c^3+987066080a^{20}c^3$$
$$-32032a^{21}c^3-2912a^{22}c^3+1306433207857823406903619823647850496000c^4$$
$$-9874398547642034648761643843879341916 16ac^4$$
$$+2620269917440798990463566188894529781 76a^2c^4$$
$$-2851624304068926328318220868105825484 8a^3c^4$$
$$+7273438671773418063421281381642485 76a^4c^4$$
$$+7583238508391936869656744448406425 6a^5c^4-3331257014910043344657768801474560a^6c^4$$
$$-1227150768264677670635133411161 60a^7c^4+4411229615802210760721621217792a^8c^4$$
$$+1564802600088025305448394321 28a^9c^4-1847398821875823652282304640a^{10}c^4$$
$$-1095489334008489427993367 20a^{11}c^4-5383050424323759854226 56a^{12}c^4$$
$$+2674570242602048582792 0a^{13}c^4+4166152646305577241 60a^{14}c^4+2514453742838065 60a^{15}c^4$$
$$-40157967472022400a^{16}c^4-326222134221984a^{17}c^4-433735702400a^{18}c^4+5069304240a^{19}c^4$$
$$+20180160a^{20}c^4+16016a^{21}c^4+3732243597969208390973208947100148039 68c^5$$
$$-2358016201059616792305535987368056586 24ac^5$$
$$+5141002856718776898474684949107061555 2a^2c^5$$
$$-4289203021048821568878100681058893824a^3c^5$$
$$+2586707079150425724648488452512153 6a^4c^5$$
$$+1249796877095444270151233917745664 0a^5c^5-2235714042213031871510419533593 60a^6c^5$$
$$-1910577734895766495658122414540 8a^7c^5+1842088450020676184950301422 08a^8c^5$$
$$+1756665023675034681725121696 0a^9c^5+6162777459370454074942608 0a^{10}c^5$$
$$-7253143775870502374120256a^{11}c^5-1014905001040843240328 00a^{12}c^5$$
$$+5815400354118548793 60a^{13}c^5+2216580038827398816 0a^{14}c^5+134566999655088000a^{15}c^5$$

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$$-523553280554304a^{16}c^5 - 8911717214400a^{17}c^5 - 29612142560a^{18}c^5 + 960960a^{19}c^5$$
$$+96096a^{20}c^5 + 799683808620186442108585090901606400 00c^6$$
$$-423882862836991777815637139458621440 00ac^6$$
$$+755887074150096928151349722005818572 8a^2c^6$$
$$-465617071577248976652184917556953088a^3c^6$$
$$-650073841985384293653626432641433 6a^4c^6+136453928532080819872384432896204 8a^5c^6$$
$$-144442729719783641936123729 92a^6c^6 - 178265214489184902245637214822 4a^7c^6$$
$$-952062402336461156634538444 8a^8c^6 + 113927339467013066096272934 4a^9c^6$$
$$+160600774273927778306355 84a^{10}c^6 - 234473419256610302793024a^{11}c^6$$
$$-644741026870258551833 6a^{12}c^6-221415439470753848 32a^{13}c^6+554066927539052928a^{14}c^6$$
$$+5807304449890944a^{15}c^6 + 10777200994560a^{16}c^6 - 91246995840a^{17}c^6 - 403282880a^{18}c^6$$
$$-320320a^{19}c^6 + 1328819200736309379110788409673646080 0c^7$$
$$-591345203938295046050822959804920627 2ac^7$$
$$+855501615332612560783574653706698752a^2c^7$$
$$-364416708076921802557649370071695 36a^3c^7$$
$$-137576889814531746121410515481395 2a^4c^7+102732157511497907394397778116608a^5c^7$$
$$+158001891738888630952127764889 6a^6c^7 - 106543641769594951204734517248a^7c^7$$
$$-189392039348068611303059865 6a^8c^7 + 41752203119379070791542784a^9c^7$$
$$+113922044687324639624601 6a^{10}c^7 - 14849879363590007193 6a^{11}c^7$$
$$-214175604541971218432a^{12}c^7 - 180747698167503667 2a^{13}c^7 + 3695487616364544a^{14}c^7$$
$$+108638058946560a^{15}c^7 + 406111580160a^{16}c^7 - 13178880a^{17}c^7 - 1464320a^{18}c^7$$
$$+175354674035819104102872721758617600 0c^8$$
$$-654066933142146236887760339174686720ac^8$$
$$+758328767934676452355167971518709 76a^2c^8$$
$$-195811648612672646718425794399436 8a^3c^8-147358268557274848028003918954496a^4c^8$$
$$+526873118275418875575353851084 8a^5c^8 + 173665224982627900154389585920a^6c^8$$
$$-389870137662371275061827584 0a^7c^8 - 134002830022645325952697344a^8c^8$$
$$+507697274942649969255936a^9c^8 + 446077745105069579673 60a^{10}c^8$$
$$+297916639594168657920a^{11}c^8 - 353880312229488844 8a^{12}c^8 - 51160438027066368a^{13}c^8$$
$$-125688322805760a^14c^8 + 834253854720a^{15}c^8 + 4144757760a^{16}c^8 + 3294720a^{17}c^8$$
$$+186968064666071811709768865215938560c^9$$
$$-582383037020832891361565390657290 24ac^9$$

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$$+5318058976849284906732137887301632a^2c^9$$
$$-5696699701829759222666116372889 6a^3c^9-10668674846383317356490133553152a^4c^9$$
$$+16385776235769819761092001792 0a^5c^9+10870807761611651383683112960a^6c^9$$
$$-5820396987498723095680614 4a^7c^9-5661742041729373171979264a^8c^9$$
$$-2861466190273045848064 0a^9c^9+1018013160324652011520a^{10}c^9$$
$$+1235091159138431795 2a^{11}c^9-666804227766476 8a^{12}c^9-718218162421760a^{13}c^9$$
$$-3068415201280a^{14}c^9+99573760a^{15}c^9+12446720a^{16}c^9$$
$$+16312215748755220042076200632320000c^{10}$$
$$-4219587600914983878203888841523200ac^{10}$$
$$+2961051703248316680217749859860 48a^2c^{10}+9922142568791259193637538365 44a^3c^{10}$$
$$+4561635143237639032341135 36a^6c^{10}+2147706339848802744467456a^7c^{10}$$
$$-1526457292043825443061 76a^8c^{10}-1746865165348944289792a^9c^{10}$$
$$+1114757067232525516 8a^{10}c^10+253243063437844480a^{11}c^{10}+808365652942848a^{12}c^{10}$$
$$-4412252708864a^{13}c^{10}-25032843264a^{14}c^{10}-19914752a^{15}c^10$$
$$+1175279551008867059362991754444800c^{11}-2505792179952111248229625516523 52ac^{11}$$
$$+1303775337040862916650640762470 4a^2c^{11}+2246228177813201034277504942 08a^3c^{11}$$
$$-2194004013184181580129003110 4a^4c^{11}-1920572676547303205256560 64a^5c^{11}$$
$$+1321155805662826319550873 6a^6c^{11}+1645264364307826882314 24a^7c^{11}$$
$$-2463884816386864545792a^8c^{11}-4732184821216867123 2a^9c^{11}$$
$$-43573855357992960a^{10}c^{11}+2820638145282048a^{11}c^{11}+14058996596736a^{12}c^{11}$$
$$-456228864a^{13}c^{11}-65175552a^{14}c^{11}+7037683640101553676094118297 6000c^{12}$$
$$-1224840849806851834611431899 1360ac^{12}+4470105791783226760521224028 16a^2c^{12}$$
$$+1452503943628013077554869043 2a^3c^{12}-6319681382967326028663357 44a^4c^{12}$$
$$-1168776454040875517673472 0a^5c^{12}+2540845169285487640576 00a^6c^{12}$$
$$+5318789101379491512320a^7c^{12}-1475222761667218636 8a^8c^{12}$$
$$-7487967487131156 48a^9c^{12}-3115570183864320a^{10}c^{12}+14440023736320a^{11}c^{12}$$
$$+95503908864a^{12}c^{12}+76038144a^{13}c^{12}+3516627778658782708865517486080c^{13}$$
$$-4935564280428314788380897443 84ac^{13}+1147439024677907080330031923 2a^2c^{13}$$
$$+6089936934664356697863618 56a^3c^{13}-1282158069381371308736512 0a^4c^{13}$$
$$-3902134669917623107584 00a^5c^{13}+2667015553314395422720a^6c^{13}$$
$$+1056644198086860472 32a^7c^{13}+276072268211453952a^8c^{13}-6870784114360320a^9c^{13}$$
$$-41095750778880a^{10}c^{13}+1333592064a^{11}c^{13}+222265344a^{12}c^{13}$$

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$$+1469074151915386755350528000000c^{14} - 1637576678197774657194557440 0ac^{14}$$
$$-1558007089777484877004 80a^4c^{14} - 8741540168881599283200a^5c^{14}$$
$$-5342112814080000000a^6c^{14} + 1347840118492299264a^7c^{14} + 7486376244019200a^8c^{14}$$
$$-30149341347840a^9c^{14} - 239094005760a^{10}c^{14} - 190513152a^{11}c^{14}$$
$$+5129614710826717498100940800c^{15} - 4455309208563458960911237 12ac^{15}$$
$$+1147256477210179174137856a^2c^{15} + 4257153230265102578483 20a^3c^{15}$$
$$-1020576870203392000 00a^4c^{15} - 136127891020146278400a^5c^{15}$$
$$-679623761601232896a^6c^{15} + 10469764576051200a^7c^{15} + 78278043893760a^8c^{15}$$
$$-2540175360a^9c^{15} - 508035072a^{10}c^{15} + 1493491449435283584450560 00c^{16}$$
$$-986379753622086267961 3440ac^{16} - 5222574995094126093926 4a^2c^{16}$$
$$+7339982431006425874432a^3c^{16} + 3916744879010886451 2a^4c^{16}$$
$$-1444942810753204224a^5c^{16} - 11290288210575360a^6c^{16} + 40198910115840a^7c^{16}$$
$$+398172487680a^8c^{16} + 317521920a^9c^{16} + 3608154907313382851870720c^{17}$$
$$-1755471525235388464496 64ac^{17} - 2069455529116885843968a^2c^{17}$$
$$+9357133375015433011 2a^3c^{17} + 869553767374258176a^4c^{17} - 9699927376527360a^5c^{17}$$
$$-96696930140160a^6c^{17} + 3137863680a^7c^{17} + 784465920a^8c^{17}$$
$$+7176885424659365888000 00c^{18} - 2464805021517715865600ac^{18}$$
$$-4218388008212312883 2a^2c^{18} + 846613217445675008a^3c^{18} + 10375434466754560a^4c^{18}$$
$$-33104810475520a^5c^{18} - 436860354560a^6c^{18} - 348651520a^7c^{18}$$
$$+1161722032816966860800c^{19} - 2651930844183343923 2ac^{19} - 5709481725835345 92a^2c^{19}$$
$$+4991774684610560a^3c^{19} + 74643648020480a^4c^{19} - 2422210560a^5c^{19} - 807403520a^6c^{19}$$
$$+1504648843860377600 0c^{20} - 20840356834705408 0ac^{20} - 5304540362440704a^2c^{20}$$
$$+15332673585152a^3c^{20} + 303260762112a^4c^{20} + 242221056a^5c^{20} + 1521081314076262 40c^{21}$$
$$-1093436182626304ac^{21} - 32701203611648a^2c^{21} + 1061158912a^3c^{21} + 530579456a^4c^{21}$$
$$+1155698524160000c^{22} - 3053243596800ac^{22} - 120682708992a^2c^{22} - 96468992a^3c^{22}$$
$$+6204214476800c^{23} - 201326592ac^{23} - 201326592a^2c^{23} + 20971520000c^{24}$$
$$+16777216ac^{24} + 33554432c^{25}\Big\}\Big] \tag{8}$$

Derivation of main result (8):

Substituting $b = -a - 50, z = \frac{1}{2}$ in given result (2),we get

$$(2a+50)\ {}_2F_1\left[\begin{matrix} a\ , & -a-50 \\ c & \end{matrix}\ ;\ \frac{1}{2}\right]$$
$$= a\ {}_2F_1\left[\begin{matrix} a+1\ , & -a-50 \\ c & \end{matrix}\ ;\ \frac{1}{2}\right] + (a+50)\ {}_2F_1\left[\begin{matrix} a\ , & -a-49 \\ c & \end{matrix}\ ;\ \frac{1}{2}\right]$$

Notes

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Now using same parallel method which is used in Ref[6], we can prove the main result.

References Références Referencias

1. Andrews, L.C.(1992) ; *Special Function of mathematics for Engineers,second Edition*, McGraw-Hill Co Inc., New York.

2. Arora, Asish, Singh, Rahul , Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems *Journal of Rajasthan Academy of Physical Sciences.*, 7(2008), 335-342.

3. Bells, Richard, Wong, Roderick ; *Special Functions , A Graduate Text.* Cambridge Studies in Advanced Mathematics, 2010.

4. Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; *Integrals and Series Vol. 3: More Special Functions.* Nauka, Moscow, 1986. Translated from the Russian by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.

5. Rainville, E. D.; The contiguous function relations for ${}_pF_q$ with applications to Bateman's $J_n^{u,v}$ and Rice's $H_n\ (\zeta, p, \nu)$, *Bull. Amer. Math. Soc.*, 51(1945), 714-723.

6. Salahuddin, Chaudhary, M.P ; A New Summation Formula Allied With Hypergeometric Function, *Global Journal of Science Frontier Research*, 11(2010),21- 37.

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A Logit Regression Analysis of Homeowners in Nigeria

By O.Y. Halid & F.I. Akinnitire
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Abstract - This paper studied the application of logit regression analysis to homeownership in Ado-Ekiti area of Ekiti State (Nigeria). The performance of the logit model in terms of classification of homeowners with respect to the average monthly income of some individuals was examined. The data of homeownership and income was fitted to the model by the WLS techniques. Result showed that the odds ratio in favour of owning a house by an individual whose average monthly income is 0.158 (N Million) was 1.0387. Also the probability of owning a house by the individual was 0.51.

Keywords : logit regression, home ownership, average monthly income, odds ratio, WLS-weighted least squares, OLS-ordinary least squares.

GJSFR-F Classification : MSC 2010: 62G08

A LOGIT REGRESSION ANALYSIS OF HOMEOWNERS IN NIGERIA

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A Logit Regression Analysis of Homeowners in Nigeria

O.Y. Halid [α] & F.I. Akinnitire [σ]

Abstract - This paper studied the application of logit regression analysis to homeownership in Ado-Ekiti area of Ekiti State (Nigeria). The performance of the logit model in terms of classification of homeowners with respect to the average monthly income of some individuals was examined. The data of homeownership and income was fitted to the model by the WLS techniques. Result showed that the odds ratio in favour of owning a house by an individual whose average monthly income is 0.158 (N Million) was 1.0387. Also the probability of owning a house by the individual was 0.51.

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I. Introduction

Feeding, clothing and shelter constitute the fundamental human needs. Among the three, shelter is the most complex, challenging and expensive. Apart from providing shelter, housing is a sure way of creating jobs, eradicating poverty, ensuring social security and propagating sustainable economic development.

In Nigeria, the housing sector is the second highest employer of labour next only to agriculture. Research has consistently shown the importance of the housing sector on the economy and the long social and financial benefits to individual homeowners.

Homeownership brings substantial social benefits for families, communities and the country as a whole. Because of these benefits, policy makers have promoted homeownership through a number of channels. Homeownership has been an essential element of the Nigerian dream for decades and continues to be even today.

Homeownership has significant impact on social outcomes, specifically educational achievement, civil participation, health benefits, public assistance, property maintenance and improvement.

In general, research supports the view that homeownership brings substantial social benefits. Because of these extensive social benefits, policies that support homeownership are well justified.

Apart from all these advantages, homeownership also provides one with pride of ownership, freedom of control, privacy, strong credit base, financial stability, appreciating asset to mention a few.

The immense economic benefits of homeownership are also well documented. For instance, United Nations Centre for Human Settlement estimate Nigeria's current housing deficit at 16 million units. Considering that an average household is between five and six persons, it is inferred by experts that 80 - 96 million housing units will be achieved within

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a period 30 - 50 years provided construction of 200,000 - 250,000 new housing units is carried out yearly. This is disheartening in the view of the fact that current national housing production figures stand at less than 10,000 per year, not to mention the estimated cost which runs into tens of trillions of Naira.

Inspite of the growing trend of homeownership in Nigeria, certain impediments such as high unemployment rate and moderate GDP growth of between 7% and 9%, inflation and high interest rate which all degenerate from unwholesome macro-economic environment and lack of financing systems are still lingering.

Land use acts, high cost of building materials, high cost of land in urban areas, poor quality of construction are also impediments to homeownership.

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II. The Logit Regression Model

The logit regression analysis is a technique which allows for estimating the probability that an event occur or not by predicting a binary dependent outcome from a set of independent variable(s). The applications of the logit model to different areas had been previously seen in sources such as Ojo (1989), Gujarati (2003), Ogunleye and Fagbohun (2009) and others too many to mention.

Let x_i be a random variable (say income) and $y=1$ indicating an individual owns a house $y=0$ if otherwise, then

$$P_i = E(Y=1 \mid X_i) = \frac{1}{1+e^{-(\beta_1+\beta_2 X_i)}} \tag{1}$$

for simplicity, we set $Z_i = \beta_1 + \beta_2 X_i$,

$$P_i = \frac{1}{1+e^{-Z_i}} = \frac{e^{Z_i}}{1+e^{Z_i}} \tag{2}$$

(2) is called the cumulative logistic distribution function,
where Z_i ranges from $-\infty$ to $+\infty$, P_i ranges from 0 to 1, called the probability of owning a house.
P_i is non linearity related to Z_i and β_i's are the model coefficients.
Since P_i is the probability of owning a house, then

$$1-P_i = \frac{1}{1+e^{Z_i}} \tag{3}$$

called the probability of not owning a house.
Then, we write

$$\frac{P_i}{1-P_i} = \frac{1+e^{Zi}}{1+e^{-Zi}} = e^{Z_i} \tag{4}$$

called the odds ratio in favour of owning a house (the ratio of the probability that an individual will own a house to the probability of not owning a house).
Taking the natural logarithm of (4), we get

$$L_i = ln\left(\frac{P_i}{1-P_i}\right) = Z_i = \beta_1 + \beta_2 X_i + \mu_i \tag{5}$$

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called logit model. L_i is called the logit and μ_i is the stochastic error term.

III. Estimation

The estimate of model coefficient of logit regression models depends on the data at hand. This is categorized as follows.

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Case I

Suppose the data at hand is on individuals, the OLS technique becomes infeasible since if we have $P_i=1$, if an individual owns a house, and $P_i=0$, if he does not, then the logit

$$L_i = \begin{cases} ln\left(\frac{1}{0}\right), \text{ if an individual owns a house} \\ ln\left(\frac{0}{1}\right), \text{ if an individual does not own a house} \end{cases} \quad (6)$$

Clearly, these expressions are meaningless and hence such data cannot be used in estimation of (5) by the OLS method.

As a result of this, the maximum likelihood method may be used.

Case II

For a grouped data on several individuals grouped according to income level and number of individuals owning a house at each income level X_i, there are N_i individuals, n_i among whom are homeowner $(n_i \le N_i)$ so that

$$\hat{P}_i = \frac{n_i}{N_i} \quad (7)$$

This is the relative frequency which can be used as the true P_i corresponding to each X_i.

If N_i is fairly large, $\hat{P}_i$ will be a reasonably good estimate of P_i using the estimated $\hat{P}_i$, the logit estimate in (5) can be obtained by

$$\hat{L}_i = ln\left(\frac{\hat{P}_i}{1-\hat{P}_i}\right) = Z_i = \hat{\beta}_1 + \hat{\beta}_2 X_i \quad (8)$$

which is a fairly good estimate of the true logit L_i assuming the N_i at each X_i is reasonably large.

Since the properties of the stochastic error term μ_i is unknown, and N_i is fairly large, X_i is independently distributed binomial variable so that

$$\mu_i \square N\left[0, \frac{1}{N_i P_i (1-P_i}\right] \quad (9)$$

which implies that μ_i follows the normal distribution with mean zero and variance $\frac{1}{N_i P_i (1-P_i)}$.

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Consequently, the logit is estimated using the weighted least square (WLS) procedure to resolve the problem of heteroscedasticity.

This gives rise to

$$\sqrt{w_i}\, L_i = \beta_1 \sqrt{w_i} + \beta_2 \sqrt{w_i}\, X_i + \sqrt{w_i}\mu_i \tag{10}$$

which can be written as

$$L_i^* = \beta_1 \sqrt{w_i} + \beta_2 X_i^* + \upsilon_i \tag{11}$$

where weights $w_i = N_i P_i (1 - P_i)$, L_i^* is the weighted L_i, X_i^* weighted X_i and υ_i weighted μ_i.

The odds ratio in favour of owning a house by an individual with average income X_i is given by

$$\frac{\hat{P}_i}{1 - \hat{P}_i} \tag{12}$$

where $\hat{P}_i$ is the estimated probability of owning a house, while the estimated logit is given by

$$ln\left(\frac{\hat{P}_i}{1 - \hat{P}_i}\right) \tag{13}$$

The probability of an individual with average monthly income X_i owning a house is

$$\hat{P}_i = \frac{e^{-L_i^*}}{1 + e^{-L_i^*}} \tag{14}$$

where L_i^* is as defined in (11).

IV. Analysis

A questionnaire was administered to 100 inhabitants of different areas of Ado-Ekiti, Ekiti State.

These 100 respondents were classified into 5 groups of 20 individuals, each based on their average monthly income.

Out of each (N_i) 20 individual, 14, 7, 5, 6 and 9 were homeowners (n_i) giving rise to respective relative frequencies 0.70, 0.35, 0.25, 0.30 and 0.45.

The respective weights (w_i) are 4.20, 4.55, 3.75, 4.20 and 4.95 with corresponding average monthly income 0.19, 0.15, 0.15, 0.18 and 0.12 (N million).

These data was fitted to the logit model in (11) by the WLS technique using SAS 9.3 so that

$$L_i^* = -2.4979\sqrt{w_i} + 4.6432\, X_i^* \tag{15}$$

is the estimated regression curve with coefficient of determination (R^2) value 0.9228.

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The odds ratio in favour of owning a house by an individual whose average income is 0.158(N million) is 1.0387 while the probability of owning a house by such individual is 0.51. Also, an estimated logit of 0.038 was also obtained.

V. Conclusion

Notes

The coefficient of determination value 0.9228 was an indication of a goodness of fit. This also indicates a strong relationship between the income and the probability of owning a house.

The calculated t-value of 32.44 is hugely in excess of the tabulated value of 5.01 and hence this leads to the conclusion that income of individuals will influence the probability of being homeowners.

The probability 0.51 of owning a house by an individual whose average monthly income is 0.158 (N million) was obtained. This shows that an increase in monthly income may increase the probability of owning a house.

The odds ratio of 1.0387 in favour of owning a house gives a slight advantage over the chance of not owning a house in the state.

VI. Recommendation

Government should make some amendments on the land use act to make more land available for residential purposes in certain 'newly created' states such as Ekiti. This will enhance rapid development in the state.

Government through its agencies should control the activities of land owners, middlemen, estate valuers and other play makers involved in land issues.

This will control the land prices and make more land available for residential purposes. This will also reduce cases of land disputes.

Non-governmental organisations, mortgage banks and other private investors should invest more into housing schemes in the state and the country at large.

This will make more people to become homeowners and make them enjoy the full benefits of home ownership.

References Références Referencias

1. Boehm T.P. and Schlottman A.M. (1999). "Does homeownership by parents have an economic impact on their children". Journal of housing Economics. Vol 8, pp 217-232.
2. Damodar N. Gujarati (2003). "Basic Econometrics" Mc Graw Hill Pub. Co Inc New York U.S. (4th Edition) pp. 595 – 607.
3. Gyourko, J. and P. Linneman (1997), "The changing influences of education income family structure and race on homeownership by age over time", Journal of Housing Research. Vol 8, pp 1-25.
4. Wojtek J. Krzanowski (1998), "An Introduction to Statistical Modelling" Oxford University Press Inc. N. Y
5. Ojo M.O. (1989), "Analysis of Some Prison Data", Journal of Applied Statistics Vol 16, (6) pp 377 – 383.
6. Ogunleye S.O. and Fagbohun A.B. (2009), "A Logit Regression Analysis of some Tuberculosis Data", International Journal of Numerical Mathematics Vol 4(1) pp 1- 14.

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The Development of Discrete Version of Laplace Transformation (Sigma (σ) Transformation) Obtained from the Relationship between Laplace and Fourier Transformations

By Dr. Umana Thompson Itaketo
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Abstract - The relationships between Laplace and Fourier transformations are studied very closely. It is observed that Fourier transformation can be obtained from Laplace transformation but the reverse is not true. Based on this, a generic mathematical analysis leads to an expression relating Laplace transformation to Fourier transformation. Further mathematical analysis from that expression leads to something quite new: The Discrete Version of Laplace transformation, which the author calls sigma (σ) transformation.

Keywords : laplace transformation, fourier transformation, relationships, analysis, discrete, sigma (σ) transformation.

GJSFR-F Classification : MSC 2010: 44A10

THE DEVELOPMENT OF DISCRETE VERSION OF LAPLACE TRANSFORMATION SIGMA TRANSFORMATION OBTAINED FROM THE RELATIONSHIP BETWEEN LAPLACE AND FOURIER TRANSFORMATIONS

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The Development of Discrete Version of Laplace Transformation (Sigma (σ) Transformation) Obtained from the Relationship between Laplace and Fourier Transformations

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Abstract - The relationships between Laplace and Fourier transformations are studied very closely. It is observed that Fourier transformation can be obtained from Laplace transformation but the reverse is not true. Based on this, a generic mathematical analysis leads to an expression relating Laplace transformation to Fourier transformation. Further mathematical analysis from that expression leads to something quite new: The Discrete Version of Laplace transformation, which the author calls sigma (σ) transformation.

Keywords : laplace transformation, fourier transformation, relationships, analysis, discrete, sigma (σ) transformation.

I. Introduction

It is a common knowledge in Mathematics that Laplace transformation is about converting any time-dependent function, f(t), to a complex domain function, F(s), (where s = σ + jw) (Ogata, 2010) and vice-versa. Also, it is equally known in Mathematics that Fourier transformation is about converting a time-dependent function, f(t), to a frequency-dependent function, F(w), and vice-versa. Mathematical expressions representing these two statements are given by:

$$F(s) = \int_{0}^{\infty} e^{-st} f(t)\, dt \qquad (1)$$

(Stephenson, 2011, 2nd ed.)

(Equation for Laplace transformation).

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$$F(t) = \mathcal{L}^{-1} F(s) = \frac{1}{2\pi e^{st}} \int_{\sigma jw}^{\sigma + jwt} F(s)\, ds \qquad (2)$$

(Stephenson, 2011, 2nd ed.)
(Equation for inverse Laplace transformation).

$$F(jw) = \int_{-\infty}^{\infty} e^{-jwt} f(t)\, dt \qquad (3)$$

(Gabel, 2010)
(Equation for Fourier transformation).

$$F(t) = \mathcal{L}^{-1} F(jw) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(jw) e^{jwt}\, dw \qquad (4)$$

(Gabel, 2010)
(Equation for inverse Fourier transformation).

Taking a hard look at those four (4) equations, one would have some impressions that there are some relationships between Laplace and Fourier transformations which have not yet been explored. This paper attempts to uncover, link and exploit such relationships.

II. Establishing a Relationship Between Laplace and Fourier Transformation

Looking at equations (1) and (3), it will immediately be noticed that Fourier transformation, F(jw), is a special case of Laplace transformation F(s), when S = jw, covering the entire time spectrum -∞ to +∞. (Recall that in the complex plane, S = σ + jw. Hence when the real component σ = 0, S = 0 + jw, and F(s) = F(jw)). This is demonstrated below:

Refer to equation (1),

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\, dt \qquad \text{(Eqn. (1) recalled).}$$

But S = σ + jw

$$\therefore F(\sigma + jw) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt \qquad (5)$$

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If $\sigma = 0$, equation (5) becomes:

$$F(jw) = \int_0^{\infty} e^{-jwt} f(t)\, dt \qquad (6)(a)$$

Equation (6)(a) is a one-sided Fourier transformation in the right half of the time axis. Equation (3) equally expresses Fourier transformation but across the entire time axis ($-\infty$ to $+\infty$). It can hence be confirmed that Fourier transformation is a special case of Laplace transformation when $\sigma = 0$. Now, if in equation (5),

$$jw = 0,\ F(\sigma) = \int_0^{\infty} e^{-(\sigma)t} f(t)\, dt \qquad (6)(b).$$

It should be noted that equation (6)(b) does not represent Laplace transformation. Hence, the reverse of how equation (6)(a) was obtained is not true. This will further be explained later.

III. The Law of Linearity Applied to Laplace and Fourier Transformations

If σ and jw are linearly-related, and the law of linearity applied to equation (5), $F(\sigma + jw)$ in equation (5) becomes:

$$F(s) = F(\sigma + jw) = F(\sigma) + F(jw) \text{ (Kreyszig, 2005)} \qquad (7)$$

But from (5),

$$F(\sigma + jw) = \int_0^{\infty} e^{-(\sigma + jw)t} f(t)\, dt \qquad \text{(eqn. (5) recalled).}$$

$$\therefore F(\sigma) + F(jw) = \int_0^{\infty} e^{-(\sigma + jw)} f(t)\, dt \qquad (8)$$

Also, from equation (3),

$$F(jw) = \int_{-\infty}^{\infty} e^{-jwt} f(t)\, dt \qquad \text{(eqn. (3) recalled)}$$

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Substituting equation (3) in (8), we have:

$$F(\sigma) + \int_{-\infty}^{\infty} e^{-jwt} f(t)\, dt = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt \qquad (9)$$

Notes

Equation (9) then becomes:

$$F(\sigma) + \left(\int_{0}^{\infty} e^{-jwt} f(t)\, dt + \int_{\infty}^{0} e^{-jwt} f(t) dt \right) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt$$

$$\therefore F(\sigma) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt - \int_{0}^{\infty} e^{-jwt} f(t)\, dt - \int_{-\infty}^{0} e^{-jw} f(t)\, dt$$

$$F(\sigma) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt - \int_{0}^{\infty} e^{-jwt} f(t)\, dt - \int_{-\infty}^{0} e^{-jwt} f(t)\, dt$$

$$F(\sigma) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt - \left(\int_{0}^{\infty} e^{-jwt} f(t)\, dt + \int_{-\infty}^{0} e^{-jwt} f(t)\, dt \right)$$

$$F(\sigma) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt - \left[|\tfrac{1}{2}F(jw)| + |\tfrac{1}{2}F(jw)| \right] \qquad \text{(Ejimanya, 2005).}$$

$$F(\sigma) = \int_{0}^{\infty} e^{-(\sigma + jw)t} f(t)\, dt - F(jw)$$

$$F(\sigma) = F(S) - F(jw) \qquad (10)$$

Equation (10) goes to prove the linearity property of the complex spectrum, s(=σ+ jw) earlier postulated in equation (7), on which Laplace transformation is based.

However, it is pertinent to mention here that Laplace transformation is expressed by F(s), not F(σ). What then does F(σ) signify?

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IV. The Significance of F(σ)

The expression F(σ) can be viewed as the real axis component of Laplace transformation while F(jw) (Fourier transformation), can be viewed as the imaginary component of Laplace transformation. This implies that F(σ) and F(jw) are complementary to each other to produce F(σ). Hence making use of equation (10), a table could be developed for F(σ) from the knowledge of F(s) and F(jw).

Now, given a function F(t), its F(s) and F(jw) can be determined or obtained from tables. A difference of that, that is F(s) – F(jw) will give F(σ) which is the real axis component of Laplace transformation of the given expression. It should be noted that as at today, a table of F(σ) does not exist.

As earlier mentioned above, it would strike readers compulsively that Fourier transformation, F(jw) of any function, is the imaginary component of Laplace transformation because it can be obtained by setting σ = 0 in s = σ + jw as earlier discussed. Fortunately, this has been developed and presented in tables as Fourier transformation. However, by setting jw = 0 in s = σ + jw, the tables for the resulting expression, F(s) = F(σ + 0) = F(σ), has never been developed nor interpretation given to its meaning.

A word of caution in Laplace transformation: One may argue that the claim that F(σ) has never been developed is not true. That is, the individual could express F(σ) as:

$$F(\sigma) = \int_0^{\infty} e^{-\sigma t} f(t)\, dt \qquad (11)$$ (Nagrath *et al*., 2009 2nd ed.)

It should be noted that equation (11)does not express Laplace transformation. The right hand side of equation (11) only resembles Laplace transformation. Reason: The right hand side of any Laplace transformation <u>MUST</u> contain e^{-st}. It should be noted that $e^{-\sigma t}$ in equation (11) is not the same as e^{-st}. In every Laplace expression, the "s" component (signifying a complex spectrum: s = σ + jw), must be there, otherwise it is not Laplace transformation.

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V. Applications of F(σ)

As already mentioned above, F(σ) signifies the real axis component of Laplace expression. It has been shown in equation (10) that this component can be obtained from the expression F(σ) = F(s) – F(jw). It will not be mathematically true that F(σ) for any function can be obtained from a Laplace transformation of that function simply by setting jw = 0. The reason for this claim is that Laplace transformation of a function has an "s" (a complex function given by s = σ + jw), embedded in it. The "s" is usually applied in all the processes of obtaining the final result of F(s). It will therefore be mathematically untrue to go to the final expression of any Laplace transformation expression and simply put jw = 0, obtain an expression, and claim that expression to represent F(σ).

The following example will justify this argument:

Let's take a function $f(t) = te^{-at}$.

From Laplace tables, the Laplace transform, F(s) of that function is

$$F(s) = \left(\frac{1}{s+a}\right)^2 = \frac{1}{(s+a)^2} \qquad \text{(Distefanno III, } \textit{et al.}\text{, 2009)} \qquad (12)$$

Also, from Fourier tables, the Fourier transform F(jw) of that function is

$$F(jw) = \left(\frac{1}{jw+a}\right)^2 = \frac{1}{(a+jw)^2} \qquad \text{(Distefanno III } \textit{et al.}\text{, 2009)} \qquad (13)$$

Bye-the-way, equation (13) goes to confirm the earlier postulation made in equation (6) that F(jw) is a special case of Laplace transformation when σ is set to zero, that is when σ = 0.

Now, back to the argument. In equation (12), it is known that s = σ + jw. Hence

$$F(s) = F(\sigma + jw) = \frac{1}{(s+a)^2} = \frac{1}{[(\sigma + jw) + a]^2}$$

$$= \frac{1}{[(\sigma + a) + jw]^2} \qquad (14)$$

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If jw is set to zero in both sides of equation (14) in order to determine F(σ), we will obtain:

$$F(\sigma) = \frac{1}{(\sigma + a)^2} \qquad (15)$$

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Now, let's carry out the operation in equation (10), that is F (σ) = F(s) – F(jw) on the same function, $f(t) = te^{-at}$.

From Laplace transform tables, F(s) for that expression $= \frac{1}{(s + a)^2}$,

(already stated in equation (12)). Also, from Fourier transform tables,

$F(jw) = \frac{1}{(a + jw)}$, (equally stated in equation (13).

From equation (10),

$$F(\sigma) = F(s)$$

$$= \frac{1}{(s + a)^2} - \frac{1}{(a + jw)} \qquad (16)$$

Substituting for s = σ

$$F(s) = \frac{1}{(\sigma + jw + a)^2} - \frac{1}{(a + jw)^2}$$

$$F(s) = \frac{(a + jw)^2 - (\sigma + jw + a)^2}{(\sigma + jw + a)^2 (a + jw)^2}$$

$$F(s) = \frac{(a^2 + 2jwa - w^2) - [(\sigma + a) + jw]^2}{[(\sigma + a) + jw]^2 [a + jw]}$$

$$F(s) = \frac{[(a^2 - w^2) + j (2wa)] - [(\sigma + a)^2 + 2jw(\sigma + a) + w^2]}{[(\sigma + a) + jw]^2 [a + jw]}$$

$$F(s) = \frac{(a^2 - w^2 + j2wa) - [(\sigma^2 + 2\sigma a + a^2) + 2jw\sigma + 2jwa + w^2]}{[(\sigma + a) + jw]^2 [a + jw]^2}$$

$$F(s) = \frac{\cancel{a^2} - w^2 + j\,\cancel{2wa} - \sigma^2 - 2\sigma a - \cancel{a^2} - 2jw\sigma - \cancel{2jwa} - w^2}{[(\sigma + a) + jw]^2 (a + jw)}$$

$$F(s) = \frac{-w^2 - \sigma^2 - 2\sigma a - 2jw\sigma - w^2}{[(\sigma + a) + jw]^2 [a + jw]}$$

$$F(s) = \frac{-[2w^2 + \sigma (\sigma + 2a)] - j2\sigma w}{[(\sigma + a) + jw]^2 [a + jw]^2}$$

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But along the real axis, w = 0.

$$\therefore F(s) = F(\sigma + jw) = F(\sigma) = \frac{-\sigma(\sigma + 2a)}{(\sigma + a)^2 (a)^2} \qquad (17)$$

It can be seen that the RHS of equation (17) and that of (15) are not the same. Equation (17) was obtained from first principles whereas (15) was not.

Similar procedure can be applied to develop what the author chooses to call "sigma (σ) transformation", which could otherwise be called "Real axis translation of Laplace transformation, F(s)." From such approach, a comprehensive table, like Laplace and Fourier tables, could be developed for sigma (σ) transformation, F(σ), by considering various f(t)'s and their respective F(s) and F(jw).

Sigma (σ) has values through the entire spectrum of real numbers, that is from $-\infty$ to $+\infty$. The value for "a" can be obtained from a given expression, such as $f(t) = te^{-at}$. In the expression $f(t) = te^{-at}$, if a = 2, for instance, $f(t) = te^{-2t}$. At a point along the real axis, say σ= 3, using equation (17), we have:

$$F(\sigma) = F(3) = \frac{-3(3 + 2(2)}{(3 + 2)^2 (2)^2} = \frac{-21}{100}$$

$$F(3) = \frac{-21}{100},$$

can be interpreted as the "Discrete Laplace transformation" of the expression, $f(t) = te^{-2t}$ at a point σ = 3 along the real axis.

The development of Discrete Laplace transformation, as presented, may not have applications now but since Science and Technology are continuously developing, it is likely to have in the near future.

VI. Discussion

Fourier transformation, F(jw), is frequency transformation of a continuous – time signal, f(t). Also, it will be recalled that z-transformation, by formulation and definition, is a discrete transformation process applied to discrete events. The z-transformation expression is given by:

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$$F(z) = \sum_{k=0}^{\infty} f(k)\, z^{-k} \text{ (Ogata, 2009)} \qquad (18)$$

where z is defined by $z = e^{-ST}$.

Notes

A very similar analogy here is that just as the Laplace transformation is the transformation of a time-based signal to the complex (s) plane, the sigma (σ) transformation shall one day provide information about the discrete components of Laplace transformation along the real axis.

VII. Conclusion

Knowledge in all spheres of human endeavour, including Science and Technology, is evolving day by day. In this particular adventure into the intricacies and properties of Laplace and Fourier transformations, a new expression has been established through the relationship between the two; the sigma (σ) transformation. It is hoped that sooner or later, this new expression shall be put to use in Science, Engineering and Technology for the benefit of mankind.

References Références Referencias

1. Disteffano III, J. J. *et al*. (2009), "Feedback and Control Systems," Schaum's Outline Series. McGraw Hill Book Company, New York, USA (pp. 361 – 362).
2. Ejimanya, J. I. (2005), "Communication Electronics," Prints Konsult, Lagos, Nigeria, (pp. 126 – 132).
3. Gabel, R. A. *et al*. (2010), "Signals & Linear Systems," New York, USA (pp. 296 – 326).
4. Kreyszig, E. (2005), "Advanced Engineering Mathematics," John Wiley & Sons, Toronto, Canada, (pp. 65 – 66).
5. Nagrath, I. J. *et al*. (2009, 2nd ed.), "Control Systems Engineering," Wiley Eastern Ltd., New Delhi, India, (pp. 677 – 678).
6. Ogata, K. (2009), "Discrete-Time Control systems," Prentice Hall, Englewoods Cliffs, New Jersey, 07632, USA, (pp. 24 – 39).
7. Ogata, K. (2010), "Modern Control Engineering, 2nd ed.", Prentice Hall, Simon & Schuster Ltd., Alexandra, Block A, PasirPangjang Road, Singapore, (p. 13).
8. Stephen, G. (2011, 2nd ed.), "Mathematical Methods for Science Students," English Language Book Society, Longman, England, (pp. 449 – 450).

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Analytic and Numeric Solution of Linear Partial Differential Equation of Fractional Order

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Abstract - The existence and uniqueness solution of the Cauchy problem are discussed and proved in a Banach space E due to Bielecki method and Picard method depending on the properties we expect a solution to possess. Moreover, some properties concerning the stability of solution are obtained. The product Nyström method is used as a numerical method to obtain a linear system of algebraic equations. Also, many important theorems related to the existence and uniqueness solution of the algebraic system are derived. Finally, an application is given and numerical results are obtained.

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ANALYTIC AND NUMERIC SOLUTION OF LINEAR PARTIAL DIFFERENTIAL EQUATION OF FRACTIONAL ORDER

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Analytic and Numeric Solution of Linear Partial Differential Equation of Fractional Order

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Abstract - The existence and uniqueness solution of the Cauchy problem are discussed and proved in a Banach space *E* due to Bielecki method and Picard method depending on the properties we expect a solution to possess. Moreover, some properties concerning the stability of solution are obtained. The product Nyström method is used as a numerical method to obtain a linear system of algebraic equations. Also, many important theorems related to the existence and uniqueness solution of the algebraic system are derived. Finally, an application is given and numerical results are obtained.

Keywords : linear partial differential equation of fractional order, semigroup, linear algebraic system, product nyström method.

I. Introduction

The use of semigroups methods for partial differential equations has had a long history starting with the works of Feller [1], Hille [2], and Yosida [3]. The basic results of the semigroup theory may be considered as a natural generalization of theorem of M. Stone on one-parameter group of unitary operators in a Hilbert space (see Yosida [4]). Also, the semigroups play a special role in applications, for example they describe how densities of initial states evolve in time. Moreover, there are equations which generate semigroups. These equations appear in such diverse areas as astrophysics-fluctuations in the brightness of the Milky Way [5], population dynamics [6,7], and in the theory of jump processes.

In [8], Mijatovie and Pilipovie introduced and analyzed α-times integrated semigroups for $\alpha \in (\frac{1}{2},1)$. In [9], El-Borai studied the Cauchy problem in a Banach space E for a linear fractional evolution equation. In his paper, the existence and uniqueness of the solution of the Cauchy problem were discussed and proved. Also, the solution was obtained in terms of some probability densities. In [10], El-Borai discussed the existence and uniqueness solution of the nonlinear Cauchy problem.

In this work, we treat the following Cauchy problem of the fractional evolution equation

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Ref.

1. W. Feller, *The parabolic differential equations and the associated semi-groups of transformations*, Ann. of Math.55 (1952), 468-519.

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$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = Au(x,t) + B(t)u(x,t) \quad ,$$

with the initial condition: $u(x,0) = u_0(x)$,

In a Banach space *E*. Here $u(x,t)$ is an *E*-valued function on $E \times [0,T]$, $T < \infty$, A is a linear closed operator defined on a dense set S_1 in *E* into *E*, $\{B(t), t \in [0,T]\}$ is a family of linear closed operators defined on a dense set $S_2 \supset S_1$ in E into *E*, $u_0(x) \in E$ and $0 < \alpha \le 1$.

II. Linear Fractional Evolution Equation

Consider the Cauchy problem of the fractional evolution equation

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = Au(x,t) + B(t)u(x,t) \;\; ; \; 0 < \alpha \le 1 \; , \tag{2.1}$$

with the initial condition : $$u(x,0) = u_0(x) \; , \tag{2.2}$$

in a Banach space *E*, where $u(x,t)$ is an *E*-valued function on $E \times [0,T]$, $T < \infty$, A is a linear closed operator defined on a dense set S_1 in E into E, $\{B(t), 0 \le t \le T\}$ is a family of linear closed operators defined on a dense set $S_2 \supset S_1$ in E into *E*, and $u_0(x) \in E$.

It is assumed that A generates an analytic semigroup $Q(t)$. This condition implies:

$$\|Q(t)\| \le k \quad for \;\; t \ge 0 \;, and \; \|AQ(t)\| \le \frac{k}{t} \;\; for \;\; t > 0 \; , \tag{2.3}$$

where $\|\cdot\|$ is the norm in *E* and k is a positive constant (Zaidman [11]).

Let us suppose that $B(t)g$ is uniformly Hölder continuous in $t \in [0,T]$, for every $g \in S_1$; that is

$$\| B(t_2)g - B(t_1)g \| \le k_1 (t_2 - t_1)^{\beta} \; , \tag{2.4}$$

for all $t_2 > t_1$, $t_1, t_2 \in [0,T]$, where k_1 and β are positive constants, $\beta \le 1$.
We suppose also that there exists a number $\gamma \in (0,1)$, such that

$$\| B(t_2) Q(t_1) h \| \le \frac{k_2}{t_1^{\gamma}} \| h \| \; , \tag{2.5}$$

where $t_1 > 0$, $t_2 \in [0,T]$, $h \in E$ and k_2 is a positive constant (El-Borai [9,12,13]).
(Notice that $Q(t)h \in S_1$ for each $h \in E$ and each $t > 0$) .

Following Gelfand and Shilov ([14],[15]) , we can define the integral of order $\alpha > 0$ by

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\theta)^{\alpha - 1} f(\theta) d\theta \, .$$

If $0 < \alpha < 1$, we can define the derivative of order α by

$$\frac{d^{\alpha} f(t)}{d t^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(\theta)}{(t-\theta)^{\alpha}} d\theta \quad , \quad \left(f'(\theta) = \frac{d f(\theta)}{d\theta} \right)$$

where f is an abstract function with values in E.

Ref.

11. S.D. Zaidman, *Abstract Differential Equations, Research Notes in Mathematics*, Pitman Advanced Publishing Program, San Francisco, London Melbourne, 1979.

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Let $C_E(E\times[0,T])$ be the set of all continuous functions $u(x,t)\in E$. We define on $C_E(E\times[0,T])$ a norm by $\|u(x,t)\|_{C_E(E\times[0,T])}=\max_{x,t}\|u(x,t)\|_E$, $\forall\, t\in[0,T]$, $x\in E$. By a solution of the Cauchy problem (2.1), (2.2), we mean an abstract function $u(x,t)$ such that the following conditions are satisfied:

(a) $u(x,t)\in C_E(E\times[0,T])$ and $u(x,t)\in S_1$ for all $t\in[0,T], x\in E$.

(b) $\dfrac{\partial^\alpha u(x,t)}{\partial t^\alpha}$ exists and is continuous on $E\times[0,T]$, where $0<\alpha<1$.

(c) $u(x,t)$ satisfies (2.1) with the initial condition (2.2) on $E\times[0,T]$.

Now, it is suitable to rewrite the Cauchy problem (2.1), (2.2), in the form

$$u(x,t)=u_0(x)+\frac{1}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}Au(x,\theta)\,d\theta + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}B(\theta)u(x,\theta)\,d\theta\,, \tag{2.6}$$

where the solution of (2.6) is equivalent to the solution of Cauchy problem (2.1), (2.2)

III. The Existence and Uniqueness Solution of Linear Fractional Evolution Equation

In this section , the existence and uniqueness solution of (2.6) and consequently its equivalent Cauchy problem (2.1), (2.2), will be discussed and proved in a Banach E by two different ways. The first way is due to "Bielecki method", and the second is "Picard method" depending on the properties we expect a solution to possess.

a) Modified Bielecki Method

Here, we will generalized the technique of Bielecki method to obtain the existence and uniqueness solution of (2.6) in *E*, for $0<\alpha\le 1$.
It's suitable to consider the following lemma.

Lemma 1:

If $\lambda>1$ and $0<\delta<1$, then

$$\int_0^t (t-\eta)^{\delta-1}d\eta \le \left(\frac{1}{\lambda}\right)^{\delta-1}t\ , \tag{3.1}$$

and

$$\int_0^t e^{\lambda\eta}(t-\eta)^{\delta-1}d\eta \le \left(\frac{1}{\lambda}\right)^{\delta}\left[1+\frac{1}{\delta}\right]e^{\lambda t}. \tag{3.2}$$

Theorem 1 :

If *A* and *B* are linear bounded operators in a Banach space *E*, and $0<\alpha\le 1$, then (2.6) has a unique solution in *E*.

Proof:

Let K be an operator defined by

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$$Ku(x,t)=u_0(x)+\frac{1}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}Au(x,\theta)\,d\theta+ \frac{1}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}B(\theta)\,u(x,\theta)\,d\theta\ . \tag{3.3}$$

Hence, we have

$$\|Ku(x,t)\|\le\|u_0(x)\|+\frac{1}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}\|Au(x,\theta)\|d\theta + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}\|B(\theta)\,u(x,\theta)\|d\theta\ . \tag{3.4}$$

Since A and B are bounded operators, there exists positive constants L and M such that

$$\|Au(x,t)\|\le L\,\|u(x,t)\|\ ,and\ \|B(t)u(x,t)\|\le M\,\|u(x,t)\|. \tag{3.5}$$

In the light of (3.5), (3.4) takes the form

$$\|Ku(x,t)\|\le\|u_0(x)\|+\frac{(L+M)}{\Gamma(\alpha)}\int_0^t (t-\theta)^{\alpha-1}\|u(x,\theta)\|d\theta\ . \tag{3.6}$$

Using (3.1) in (3.6), we get

$$\|Ku(x,t)\|\le\|u_0(x)\|+\frac{(L+M)}{\Gamma(\alpha)}(\frac{1}{\lambda})^{\alpha-1}T\|u(x,t)\|\ ,\ (T=\max_{0\le t\le T} t)\ . \tag{3.7}$$

Inequality (3.7) shows that, the operator K maps the ball $B_r\subset E$ into itself, where

$$r=\frac{\sigma}{1-\delta},\quad (\sigma=\|u_0(x)\|,\ \delta=\frac{(L+M)}{\Gamma(\alpha)}(\frac{1}{\lambda})^{\alpha-1}T)$$

Since $r>0$, $\sigma>0$, therefore $\delta<1$. Also, the inequality (3.7) involves the boundedness of the operator K.

For the two functions $u(x,t)$ and $v(x,t)$ in E, the formula (3.3) leads to

$$\|Ku(x,t)-Kv(x,t)\|\le\frac{1}{\Gamma(\alpha)} \left\{\int_0^t (t-\theta)^{\alpha-1}\|A(u(x,\theta)-v(x,\theta))\|\,d\theta+ \int_0^t (t-\theta)^{\alpha-1}\|B(\theta)\,(u(x,\theta)-v(x,\theta))\|\,d\theta\right\}. \tag{3.8}$$

Using (3.5) in (3.8), we have for $\lambda>1$

$$\|Ku(x,t)-Kv(x,t)\|\le\frac{(L+M)}{\Gamma(\alpha)} \max_{x,t}\left\{e^{-\lambda(t+x)}\|u(x,t)-v(x,t)\|\right\}\int_0^t (t-\theta)^{\alpha-1}e^{\lambda(\theta+x)}\,d\theta.$$

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Using (3.2), the above inequality becomes

$$\max_{x,t}\left\{e^{-\lambda(t+x)}\left\|Ku(x,t)-Kv(x,t)\right\|\right\}\leq \sigma\max_{x,t}\left\{e^{-\lambda(t+x)}\left\|u(x,t)-v(x,t)\right\|\right\}, \tag{3.9}$$

where

$$\sigma=\frac{(L+M)}{\Gamma(\alpha)}\left(\frac{1}{\lambda}\right)^{\alpha}\left[1+\frac{1}{\alpha}\right].$$

Inequality (3.9) can be adapted in the form

$$d(Ku(x,t),Kv(x,t))\leq\sigma\, d(u(x,t),v(x,t)).$$

If we choose λ sufficiently large, then $\sigma<1$, and d is a contraction mapping. By Banach fixed point theorem, K has a unique fixed point which is the unique solution of (2.6).

b) Semigroup Method

To obtain the solution of the Cauchy problem (2.1), (2.2) in the dense set S_1 in E, we can follow the work of El-Borai [9]. Hence, the Cauchy problem (2.1), (2.2), and (2.6) are equivalent to the following integral equation

$$u(x,t)=\alpha\int_0^t\int_0^\infty \theta(t-\eta)^{\alpha-1}\zeta_\alpha(\theta)Q((t-\eta)^\alpha\theta)w(x,\eta)\,d\theta\,d\eta+ \int_0^\infty \zeta_\alpha(\theta)Q(t^\alpha\theta)u_0(x)\,d\theta, \tag{3.10}$$

where $\zeta_\alpha(\theta)$ is a probability density function defined on $(0,\infty)$, and $w(x,t)=B(t)u(x,t),\ \ (x,t)\in(E\times[0,T])$.

The integral equation (3.10) represents a Volterra equation of the second kind with Abel kernel, where the first term of the R.H.S is known and continuous. The integral equation will be solved numerically in the next section.

Now, we will prove that (3.10) has a unique solution which represents the required solution of the Cauchy problem (2.1), (2.2).

Theorem 2:
The Cauchy problem (2.1), (2.2) has a unique solution in $C_E(E\times[0,T])$.
The proof of this theorem depends on the following lemmas.

Lemma 2:
Under the condition (2.5), (3.10) has a solution in the space $C_E(E\times[0,T])$.

Proof:
Consider the following integral equation

$$w(x,t)=\alpha\int_0^t\int_0^\infty \theta(t-\eta)^{\alpha-1}\zeta_\alpha(\theta)B(t)Q((t-\eta)^\alpha\theta)w(x,\eta)\,d\theta\,d\eta + \int_0^\infty \zeta_\alpha(\theta)\,B(t)\,Q(t^\alpha\theta)u_0(x)\,d\theta. \tag{3.11}$$

Ref.

9. M.M.EL-Borai, *Some probability densities and fundamental solutions of fractional evolution equation*, Chaos, Solitons & Fractals 14(2002), 433-440.

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Using the method of successive approximations, we set

$$w_{n+1}(x,t)=\int_0^\infty \zeta_\alpha(\theta)\ B(t)Q(t^\alpha\theta)u_0(x)\,d\theta+$$
$$\alpha\int_0^t\int_0^\infty \theta(t-\eta)^{\alpha-1}\zeta_\alpha(\theta)B(t)Q((t-\eta)^\alpha\theta)w_n(x,\eta)\,d\theta\,d\eta\,.$$

Thus, we have

$$\|w_2(x,t)-w_1(x,t)\|\le$$
$$\alpha\int_0^t\int_0^\infty \theta(t-\eta)^{\alpha-1}\zeta_\alpha(\theta)\left\|B(t)Q((t-\eta)^\alpha\theta)[w_1(x,\eta)-w_0(x,\eta)]\right\|d\theta\,d\eta\,,$$

where $w_0(x,t)$ is the zero element in E.
In view of the condition (2.5), we get

$$\|w_2(x,t)-w_1(x,t)\|\le$$
$$\alpha\int_0^t\int_0^\infty \theta(t-\eta)^{\alpha-1}\zeta_\alpha(\theta)\frac{k_2}{((t-\eta)^\alpha\theta)^\gamma}\|w_1(x,\eta)-w_0(x,\eta)\|\,d\theta\,d\eta.$$

The above inequality for $u_0(x)\in S_1$ can be adapted in the form

$$\|w_2(x,t)-w_1(x,t)\|\le\frac{\mu t^\nu}{\nu}, \tag{3.12}$$

where,

$$\mu=\alpha k_2\int_0^\infty \theta^{1-\gamma}\zeta_\alpha(\theta)\sup_{t,\theta}\left\|\int_0^\infty \zeta_\alpha(\theta)B(\eta)Q(\eta^\alpha\theta)u_0(x)\,d\theta\right\|d\theta\,, \tag{3.13}$$
$$\nu=\alpha(1-\gamma).$$

By induction, we obtain

$$\|w_{n+1}(x,t)-w_n(x,t)\|\le\frac{\mu^n t^{n\nu}(\Gamma(\nu))^n}{\Gamma(n\nu+1)}\,.$$

Thus, the series $\sum_{i=0}^\infty\|w_{i+1}(x,t)-w_i(x,t)\|$ converges uniformly on $E\times[0,T]$, under the condition, $\mu t^\nu\Gamma(\nu)<1$.

Since $w_{n+1}(x,t)=\sum_{i=0}^n[w_{i+1}(x,t)-w_i(x,t)]$, it follows that the sequence $\{w_n(x,t)\}$ converges uniformly in the space $C_E(E\times[0,T])$ to a continuous function $w(x,t)$ which satisfies (3.11), consequently $u(x,t)\in C_E(E\times[0,T])$.

Lemma 3:
Under the condition (2.5), (3.10) has a unique solution in the space $C_E(E\times[0,T])$.

Proof:

For the two functions $w_1(x,t)$ and $w_2(x,t)$ in the space $C_E(E\times[0,T])$, the formula (3.11) with the aid of condition (2.5), leads to

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$$\| w_2(x,t) - w_1(x,t) \| \le \mu \int_0^t (t-\eta)^{\nu-1} \| w_2(x,t) - w_1(x,t) \| d\eta .$$

Consequently,

$$\| w_2(x,t) - w_1(x,t) \| \le \mu \, \rho \int_0^t e^{\lambda(\eta+x)} (t-\eta)^{\nu-1} \, d\eta \ , \tag{3.14}$$

where, $\rho = \max_{x,t} [e^{-\lambda(t+x)} \| w_2(x,t) - w_1(x,t) \|]$, and $\lambda > 1$.

Using (3.2) in (3.14), we get

$$\| w_2(x,t) - w_1(x,t) \| \le \mu \rho (\frac{1}{\lambda})^{\nu} [1 + \frac{1}{\nu}] e^{\lambda(t+x)} .$$

Thus, we have

$$\max_{x,t} [e^{-\lambda(t+x)} \| w_2(x,t) - w_1(x,t) \|] \le \mu \, \rho \, (\frac{1}{\lambda})^{\nu} \, [1 + \frac{1}{\nu}] .$$

We can choose λ sufficiently large such that

$$\mu \, (\frac{1}{\lambda})^{\nu} \, [1 + \frac{1}{\nu}] = \mu_1 < 1 .$$

Thus,

$$\rho = \max_{x,t} [e^{-\lambda(t+x)} \| w_2(x,t) - w_1(x,t) \|] = 0 \ .$$

This completes the proof of the lemma.

Lemma 4:

Under the conditions (2.4) and (2.5), the solution $w(x,t)$ of (3.11) satisfies a uniform Hölder condition. (El-Borai [9])

Proof of Theorem 2:

By virtue of lemmas (3) and (4), we deduce that, the function $u(x,t) \in S_1$ and represents the unique solution of Cauchy problem (2.1), (2.2) in the space $C_E(E \times [0,T])$.

Corollary 1:

The integral equation (3.10) has a unique solution in the Banach space $C_{\Re}(\Re \times [0,T])$. Now, we will prove the stability of the solutions of the Cauchy problem (2.1), (2.2). In other words, we will show that the Cauchy problem (2.1), (2.2) is correctly formulated.

Theorem 3:

Let $\{u_n(x,t)\}$ be a sequence of functions, each of which is a solution of (2.1) with the initial condition $u_n(x,0) = g_n(x)$, where $g_n(x) \in S_1 \ (n = 1,2,...)$. If the sequence $\{g_n(x)\}$ converges to an element $u_0(x) \in S_1$, the sequence $\{A g_n(x)\}$ converges and the sequence $\{B(t) g_n(x)\}$ converges uniformly on $E \times [0,T]$. Then, the sequence of solutions $\{u_n(x,t)\}$ converges uniformly on $E \times [0,T]$ to a limit function $u(x,t)$, which is the solution of the Cauchy problem (2.1), (2.2).

Proof:

Consider the sequences $\{z_n(x,t)\}$ and $\{u_n^*(x,t)\}$, where

$$\frac{\partial^{\alpha} u_n^*(x,t)}{\partial t^{\alpha}} - A u_n^*(x,t) = z_n(x,t) \ ,$$

Ref.

9. M.M.EL-Borai, *Some probability densities and fundamental solutions of fractional evolution equation*, Chaos, Solitons & Fractals 14(2002), 433-440.

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$$u_n^*(x,t) = u_n(x,t) - g_n(x) \ , \quad u_n(x,0) = g_n(x) \ ,$$

$$u_n^*(x,t) = \alpha \int_0^t \int_0^\infty \theta (t-\eta)^{\alpha-1} \zeta_\alpha(\theta) Q((t-\eta)^\alpha \theta) z_n(x,\eta) d\theta d\eta \ ,$$

and

$$z_n(x,t) = B(t) u_n^*(x,t) + B(t) g_n(x) + A g_n(x).$$

In view of the conditions (2.5) and (3.13), we get

$$\| z_n(x,t) - z_m(x,t) \| \le \mu \int_0^t (t-\eta)^{\nu-1} \| z_n(x,\eta) - z_m(x,\eta) \| d\eta$$
$$+ \| B(t) g_n(x) - B(t) g_m(x) \| + \| A g_n(x) - A g_m(x) \|.$$

Given $\varepsilon > 0$, we can find a positive integer $N = N(\varepsilon)$, such that

$$\| z_n(x,t) - z_m(x,t) \| \le$$
$$\mu \int_0^t (t-\eta)^{\nu-1} \| z_n(x,\eta) - z_m(x,\eta) \| d\eta + (1-\mu_1)\varepsilon \ ,$$

for all $n \ge N, m \ge N$ and $(x,t) \in E \times [0,T]$.
Using (3.2), the above inequality takes the form

$$(1-\mu_1) e^{-\lambda(t+x)} \| z_n(x,t) - z_m(x,t) \| \le (1-\mu_1) e^{-\lambda(t+x)} \varepsilon \ .$$

Thus, for sufficiently large λ, we get

$$\max_{x,t} [e^{-\lambda(t+x)} \| z_n(x,t) - z_m(x,t) \|] \le \varepsilon \ .$$

Since E is a complete space, it follows that the sequence $\{z_n(x,t)\}$ converges uniformly on $E \times [0,T]$ to a continuous function $z(x,t)$, so the sequence $\{u_n^*(x,t)\}$ converges uniformly on $E \times [0,T]$ to a continuous function $u^*(x,t)$. It can be proved that $z(x,t)$ satisfies a uniform Hölder condition on $[0,T]$, thus $u^*(x,t) \in S_1$.

IV. The Numerical Solution of Linear Fractional Evolution Equation

In this section, we will use the product Nyström method (Linz [17], and Dzhuraev [18]), to obtain numerically, the solution of the Cauchy problem (2.1), (2.2), in the Banach space $C_{\Re}(\Re \times [0,T])$, where $\| u(x,t) \|_{C_{\Re}(\Re \times [0,T])} = \max_{x,t} | u(x,t) |$, $\forall t \in [0,T]$, $-\infty < x < \infty$. For this, the integral equation (3.10) can be written in the form

$$u(x,t) = f^*(x,t) + \alpha \int_0^t p(t,\eta) \, Q^*(t,\eta) \, B(\eta) u(x,\eta) \, d\eta, \tag{4.1}$$

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where,

$$f^*(x,t) = \int_0^\infty \zeta_\alpha(\theta)\, Q(t^\alpha \theta) u_0(x)\, d\theta \ , \tag{4.2}$$

$$Q^*(t,\eta) = \int_0^\infty \theta\, \zeta_\alpha(\theta)\, Q((t-\eta)^\alpha \theta)\, d\theta \ , \tag{4.3}$$

and the bad kernel

$$p(t,\eta) = (t-\eta)^{\alpha-1} \quad , \ (0<\alpha<1,\ 0 \le \eta \le t \le T \ ; \ T<\infty). \tag{4.4}$$

Here, the unknown function $u(x,t) \in C_{\Re}(\Re \times [0,T])$, while $f^*(x,t)$, $Q^*(t,\eta)$ and $p(t,\eta)$ are known functions and satisfy the following conditions:

(1) $f^*(x,t)$ is a continuous function in $(\Re \times [0,T])$.

(2) $Q^*(t,\eta)$ with its partial derivatives are continuous functions in $[0,T]$.

(3) $p(t,\eta)$ is a badly behaved function of its arguments such that:

(a) for each continuous function $u(x,t)$ and $0 \le t_1 \le t_2 \le t$, the integrals

$$\int_{t_1}^{t_2} p(t,\eta) Q^*(t,\eta)\, B(\eta) u(x,\eta)\, d\eta \ ,$$

and

$$\int_0^t p(t,\eta) Q^*(t,\eta) B(\eta) u(x,\eta)\, d\eta \ ,$$

are continuous functions in $(\Re \times [0,T])$.

(b) $p(t,\eta)$ is absolutely integrable with respect to η for all $0 \le t \le T$.

Remark 1:

By virtue of corollary (1), the integral equation (4.1) has a unique solution in the Banach space $C_{\Re}(\Re \times [0,T])$.

Now, we will apply the product Nyström method, to obtain numerically, the solution of (4.1). Therefore, putting $t = t_i = \eta_i = x_i = x$, $t_i = ih$, $h = t_{i+1} - t_i$ $(i = 0,1,\ldots,N \text{ and } N \text{ is even})$, and using the following notations

$$u_{i,i} = u(t_i, x_i)\ , \quad Q^*_{i,j} = Q^*(t_i, \eta_j)\ , \quad f^*_{i,i} = f^*(t_i, x_i),$$
$$B_i = B(\eta_i),$$

we get the following linear algebraic system

$$u_{i,i} = f^*_{i,i} + \alpha \sum_{j=0}^{N} w_{i,j}\, Q^*_{i,j}\, B_j\, u_{j,j}\ , \quad (i = 0,1,2,\ldots,N) \tag{4.6}$$

where,

$$\begin{aligned} &w_{i,0} = \beta_1(t_i)\ , \quad w_{i,2j+1} = 2\gamma_{j+1}(t_i) \\ &w_{i,2j} = \alpha_j^*(t_i) + \beta_{j+1}(t_i)\ , \quad w_{i,N} = \alpha^*_{\frac{N}{2}}(t_i). \end{aligned} \tag{4.7}$$

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And,

$$\alpha_j^*(t_i)=\frac{1}{2h^2}\int_{t_{2j-2}}^{t_{2j}} p(t_i,\eta)(\eta-\eta_{2j-2})(\eta-\eta_{2j-1})\,d\eta\ ,$$

$$\beta_j(t_i)=\frac{1}{2h^2}\int_{t_{2j-2}}^{t_{2j}} p(t_i,\eta)(\eta_{2j-1}-\eta)(\eta_{2j}-\eta)\,d\eta\ ,$$

$$\gamma_j(t_i)=\frac{1}{2h^2}\int_{t_{2j-2}}^{t_{2j}} p(t_i,\eta)(\eta-\eta_{2j-2})(\eta_{2j}-\eta)\,d\eta\ . \tag{4.8}$$

Evaluating the integrals of (4.8), where $p(t,\eta)=(t-\eta)^{\alpha-1}$, and introducing the results in the values of $w's$, we get

$$w_{i,0}=\frac{-h^\alpha}{2\alpha(\alpha+1)(\alpha+2)}\Big\{[2|i-2|+\alpha+2]|i-2|^{\alpha+1}-$$
$$-[2|i|^2-3(2+\alpha)|i|+2(\alpha+1)(\alpha+2)]|i|^\alpha\Big\},$$

$$w_{i,2j+1}=\frac{2h^\alpha}{\alpha(\alpha+1)(\alpha+2)}\Big\{(\alpha+2)[|i-2j-2|^{\alpha+1}$$
$$+|i-2j|^{\alpha+1}]+|i-2j-2|^{\alpha+2}-|i-2j|^{\alpha+2}\Big\},$$

$$w_{i,2j}=\frac{-h^\alpha}{2\alpha(\alpha+1)(\alpha+2)}\Big\{(\alpha+2)|i-2j+2|^{\alpha+1}$$
$$+(\alpha+2)|i-2j-2|^{\alpha+1}+6(\alpha+2)|i-2j|^{\alpha+1}$$
$$+2|i-2j-2|^{\alpha+2}-2|i-2j+2|^{\alpha+2}\Big\},$$

and

$$w_{i,N}=\frac{-h^\alpha}{2\alpha(\alpha+1)(\alpha+2)}$$
$$\Big\{2(\alpha+1)(\alpha+2)|i-N|^\alpha+3(\alpha+2)|i-N|^{\alpha+1}+$$
$$(\alpha+2)|i-N+2|^{\alpha+1}+2|i-N|^{\alpha+2}-2|i-n+2|^{\alpha+2}\Big\}\ . \tag{4.9}$$

The linear algebraic system (4.6) represents $(N+1)$ equations in $u_{i,i}$. Therefore, the approximate solution of $u(x,t)$ can be written in the vector form

$$(I-\alpha W)U=F^*\ , \tag{4.10}$$

where,

$$W=\begin{bmatrix} 1-\alpha\, w_{0,0}Q_{0,0}^*\,B_0 & -\alpha w_{0,1}Q_{0,1}^*\,B_1 & \dots & -\alpha w_{0,N}Q_{0,N}^*\,B_N \\ -\alpha\, w_{1,0}Q_{1,0}^*\,B_0 & 1-\alpha w_{1,1}Q_{1,1}^*\,B_1 & \dots & -\alpha w_{1,N}Q_{1,N}^*\,B_N \\ \vdots & \vdots & \vdots & \vdots \\ -\alpha\, w_{N,0}Q_{N,0}^*\,B_0 & -\alpha w_{N,1}Q_{N,1}^*\,B_1 & \dots & 1-\alpha w_{N,N}Q_{N,N}^*\,B_N \end{bmatrix},$$

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$$U = \begin{bmatrix} u_{0,0} \\ u_{1,1} \\ \cdot \\ \cdot \\ \cdot \\ u_{N,N} \end{bmatrix}, \text{ and } F^* = \begin{bmatrix} f^*_{0,0} \\ f^*_{1,1} \\ \cdot \\ \cdot \\ \cdot \\ f^*_{N,N} \end{bmatrix}$$

Notes

When $det(W) \neq 0$, the algebraic system (4.6) has a unique solution in the form

$$U = [I - \alpha W]^{-1} \; F^*, \tag{4.11}$$

where I is the identity matrix.

Theorem 4:

The algebraic system (4.6) has a unique solution in the Banach space ℓ^∞, under the following conditions

$$\sup_i \left| f^*_{i,i} \right| \leq q \,, \quad (q \text{ is a constant}) \;. \tag{4.12}$$

$$\sup_i \sum_{j=0}^{N} \left| w_{i,j} \, Q^*_{i,j} \right| \leq q^* \,, \quad (q^* \text{ is a constant}) \;. \tag{4.13}$$

$$\sup_i \left| B_i \, u_{i,i} \right| \leq M \sup_i \left| u_{i,i} \right| \,, \quad (M \text{ is a constant}) \;. \tag{4.14}$$

Proof :

Let Y be the set of all functions $U = \{u_{i,i}\}$ in ℓ^∞ such that $\|U\|_{\ell^\infty} \leq \rho^*$, ρ^* is a constant. Define the operator $\tilde{T}$ by

$$\tilde{T} U = F^* + \alpha W U \,, \tag{4.15}$$

where , $\left\| \tilde{T} U \right\|_{\ell^\infty} = \sup_i \left| \tilde{T} \; u_{i,i} \right|$, $\forall \; i = 0,1,2,\ldots$.

The formulas (4.6) and (4.15) lead to

$$\left| \tilde{T} u_{i,i} \right| \leq \sup_i \left| f^*_{i,i} \right| + \alpha \sup_i \sum_{j=0}^{N} \left| w_{i,j} \; Q^*_{i,j} \right| \sup_j \left| B_j u_{j,j} \right| \;, \forall i = 0,1,2,\ldots \;.$$

In view of the conditions (4.12) and (4.14), the above inequality takes the form

$$\left\| \tilde{T} U \right\|_{\ell^\infty} \leq q + \lambda^* \|U\|_{\ell^\infty} \,, \; (\lambda^* = \alpha q^* M) \;. \tag{4.16}$$

Inequality (4.16) shows that, the operator $\tilde{T}$ maps the set Y into itself, where

$$\rho^* = \frac{q}{1 - \lambda^*} \;.$$

Since $\rho^* > 0$, $q > 0$, therefore $\lambda^* < 1$. Also, the inequality (4.16) involves the boundedness of operator $\tilde{T}$.

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For the two functions U and V in ℓ^{∞}, the formulas (4.6) and (4.15) lead to

$$\left|\tilde{T}u_{i,i}-\tilde{T}v_{i,i}\right|\le\alpha\sup_{i}\sum_{j=0}^{N}\left|w_{i,j}Q_{i,j}^{*}\right|.\sup_{j}\left|B_{j}(u_{j,j}-v_{j,j})\right|.$$

The above inequality, with the aid of conditions (4.13) and (4.14), can be adapted in the form

$$\left\|\tilde{T}U-\tilde{T}V\right\|_{\ell^{\infty}}\le\lambda^{*}\left\|U-V\right\|_{\ell^{\infty}}.$$

Therefore, $\tilde{T}$ is a continuous operator in ℓ^{∞}, then under the condition $\lambda^{*}<1$, $\tilde{T}$ is contractive. Hence, by Banach fixed point theorem, $\tilde{T}$ has a unique fixed point which is the unique solution of the linear algebraic system in the Banach space ℓ^{∞}.

Theorem 5:

If the conditions (4.13) and (4.14) of Theorem (4) are verified, and the sequence of functions $\left\{F_{m}^{*}\right\}=\left\{(f_{i,i}^{*})_{m}\right\}$ converges uniformly to the function $F^{*}=\left\{f_{i,i}^{*}\right\}$ in the Banach space ℓ^{∞}. Then, the sequence of approximate solutions $\{U_{m}\}=\left\{(u_{i,i})_{m}\right\}$ converges uniformly to the exact solution $U=\left\{u_{i,i}\right\}$ of the linear algebraic system (4.6) in ℓ^{∞}.

Proof:

In the light of (4.6), we get

$$\left|u_{i,i}-(u_{i,i})_{m}\right|\le\alpha\sup_{i}\sum_{j=0}^{N}\left|w_{i,j}Q_{i,j}^{*}\right|\sup_{j}\left|B_{j}(u_{j,j}-(u_{j,j})_{m})\right|$$
$$+\sup_{i}\left|f_{i,i}-(f_{i,i})_{m}\right|\quad,\ \forall\, i=0,1,2,\ldots\ .$$

Using the conditions (4.13) and (4.14), we have

$$\left\|U-U_{m}\right\|_{\ell^{\infty}}\le\frac{1}{1-\lambda^{*}}\left\|F^{*}-F_{m}^{*}\right\|_{\ell^{\infty}}\ ;\qquad(\lambda^{*}<1).$$

Since $\left\|F^{*}-F_{m}^{*}\right\|_{\ell^{\infty}}\to 0$ as $m\to\infty$, so that $\left\|U-U_{m}\right\|_{\ell^{\infty}}\to 0$.

This complete the prove of the theorem.

When $N\to\infty$, it is natural to expect that the sum $\sum_{j=0}^{N}w_{i,j}\,Q_{i,j}^{*}\,B_{j}\,u_{j,j}$; $0\le i,j\le N$, becomes $\int_{0}^{t}p(t,\eta)\,Q^{*}(t,\eta)\ B(\eta)u(x,\eta)\,d\eta$. Consequently, the solution of the algebraic system (4.6) is the same solution of the integral equation (4.1).

Theorem 6:

If the sequence of continuous functions $\left\{f_{n}^{*}(x,t)\right\}$ converges uniformly to the function $f^{*}(x,t)$, and the functions $Q^{*}(t,\eta)$, $p(t,\eta)$ satisfy, respectively, the conditions (2) and (3-b). Then, the sequence of approximate solutions $\{u_{n}(x,t)\}$ converges uniformly to the exact solution of (4.1) in the Banach space $C_{\Re}(\Re\times[0,T])$.

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Proof:
The formula (4.1) with its approximate solution give

$$\max_{x,t}\left|u(x,t)-u_n(x,t)\right|\le \max_{x,t}\left| f^*(x,t)-f_n^*(x,t)\right|$$

$$+\alpha\int_0^t \left|p(t,\eta)\right|\left|Q^*(t,\eta)\right|.\max_{x,\eta}\left|B(\eta)\,(u(x,\eta)-u_n(x,\eta))\right| d\eta\,, \tag{4.17}$$

$$\forall\, 0\le\eta\le t\le T\,,\quad -\infty<x<\infty\,.$$

In view of the conditions (2) and (3-b), there exist two constants c_1 and c_2, such that

$$\left|Q^*(t,\eta)\right|\le c_1 \quad ,and \int_0^t \left|p(t,\eta)\right| d\eta \le c_2\,. \tag{4.18}$$

Hence, the inequality (4.17) with the aid of (4.18) and (3.5), takes the form

$$\left\|u(x,t)-u_n(x,t)\right\|_{C_{\Re}(\Re\times[0,T])} \le \frac{1}{(1-c^*)}\left\| f^*(x,t)-f_n^*(x,t)\right\|_{C_{\Re}(\Re\times[0,T])},$$

$$\left(c^*=\alpha\, c_1 c_2\, M\right).$$

Since

$$\left\| f^*(x,t)-f_n^*(x,t)\right\|_{C_{\Re}(\Re\times[0,T])}\to 0 \quad as \quad n\to\infty,$$

$$\text{hence}\quad \left\| u(x,t)-u_n(x,t)\right\|_{C_{\Re}(\Re\times[0,T])}\to 0.$$

Definition 1:

The product Nyström method is said to convergent of order r in [a,b] if and only if for sufficiently large N, there exists a constant $c>0$ independent of N, such that

$$\left\|\phi(x)-\phi_N(x)\right\|_\infty \le c\; N^{-r}\,.$$

Definition 2:

The consistency error R_N of the product Nyström method is determined by the following equation

$$R_N=\left|\int_0^{t_N} p(t,\eta)Q^*(t,\eta)B(\eta)u(x,\eta)\,d\eta-\sum_{j=0}^{N} w_{i,j}\,Q_{i,j}^*\,B_j u_{j,j}\right|. \tag{4.19}$$

Also , (4.19) gives

$$u(x,t)-u_N(x,t)=\sum_{j=0}^{N} w_{i,j}\,Q^*(t_i,\eta_j)[B_j(u(x_j,\eta_j)-u_N(x_j,\eta_j))]+R_N\,, \tag{4.20}$$

where $u_N(x,t)$ is the approximate solution of (4.1).

Theorem 7:
Assume that, the hypothesis of Theorem (5) are verified, then

$$\lim_{N\to\infty} R_N=0\,. \tag{4.21}$$

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Proof:
The formula (4.20) leads to

$$|R_N| \le \sup_i |u_{i,i} - (u_{i,i})_N|$$
$$+ \sup_i \sum_{j=0}^{N} \left| w_{i,j} Q^*_{i,j} \right| \sup_j \left| B_j (u_{j,j} - (u_{j,j})_N) \right|.$$

In view of the conditions (4.13) and (4.14), the above inequality takes the form

$$\| R_N \|_{\ell^\infty} \le \| U - U_N \|_{\ell^\infty} + q^* M \| U - U_N \|_{\ell^\infty} \quad , \ \forall \ N = 1,2,\ldots \ .$$

Since $\| U - U_N \|_{\ell^\infty} \to 0 \ \ as \ \ N \to \infty$ (see Theorem (5), it follows that $\| R_N \|_{\ell^\infty} \to 0$.

Application I:

In (4.1), let $0 < \alpha < 1$, $Q^*(t,\eta) = 1$, $B(\eta) = I$, where I is the identity operator. Hence, we get a linear Volterra integral equation of the second kind with Abel kernel

$$u(x,t) = x\,(1 - t^\alpha) + t\,(1 - \frac{t^\alpha}{\alpha + 1}) + \alpha \int_0^t (t - \eta)^{\alpha - 1} u(x,\eta)\, d\eta, \tag{4.22}$$

where the exact solution $u(x,t) = x + t$.

The results are obtained numerically in the flowing Table which lists various values of $x, t \in [0,0.8]$ together with the values of the exact and approximate solutions and the error of (4.22). Also, we can see from this table that:

1. The exact and approximate solutions are coincident for $x = t = 0$.
2. As x and t are increasing through [0,0.8], the error is also increasing for $\alpha = 0.98$, $\alpha = 0.8$ and $\alpha = 0.4$.
3. The maximum value of the error is 0.421056 which occurs at $x = t = 0.8$ for $\alpha = 0.8$.

		$\alpha = 0.4$		$\alpha = 0.8$				$\alpha = 0.98$	
x=t	*Exact*	Appr. Sol.	Error	Appr. Sol.	Error	*x=t*	*Exact*	Appr. Sol.	Error
0	0	0	0	0	0	0	0	0	0
0.08	0.16	0.154992	0.005008	0.155284	0.004716	0.08	0.16	0.156646	0.003354
0.16	0.32	0.306929	0.013071	0.302817	0.017183	0.16	0.32	0.306367	0.013633
0.24	0.48	0.456241	0.023759	0.442848	0.037152	0.24	0.48	0.448667	0.031333
0.32	0.64	0.602977	0.037023	0.575147	0.064853	0.32	0.64	0.582967	0.057033
0.4	0.8	0.747068	0.052932	0.699325	0.100675	0.4	0.8	0.708623	0.091377
0.48	0.96	0.88839	0.07161	0.814898	0.145102	0.48	0.96	0.824933	0.135067
0.56	1.12	1.02678	0.09322	0.921307	0.198693	0.56	1.12	0.93113	0.18887
0.64	1.28	1.162041	0.117959	1.017924	0.262076	0.64	1.28	1.02638	0.25362
0.72	1.44	1.293951	0.146049	1.104056	0.335944	0.72	1.44	1.10978	0.33022
0.8	1.6	1.422257	0.177743	1.178944	0.421056	0.8	1.6	1.180347	0.419653

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Notes

References Références Referencias

1. W. Feller, *The parabolic differential equations and the associated semi-groups of transformations,* Ann. of Math.55 (1952), 468-519.
2. E. Hille, *On the Integration Problem for Fokker-Planck's equation,* in "10th Congress of Scandinavian Mathematics," Trondheim, 1950.
3. K. Yosida, *An operator - theoretical treatment of temporally,* homogeneous Markoff process, J. Math. Soc. Japan 1 (1949), 244 – 253.
4. K. Yosida, "*Functional Analysis*" Springer-Verlag Berlin Heidelberg, New York, (1974).
5. S. Chandrasekhar and G.Münch, *The theory of fluctuations in brightness of the Milky – Way,* Astrophys. J. 125, 94-123.
6. M.C.Mackey and Rudnicki, *Global stability in a delayed partial differential equation describing cellular replication,* J.Math. Biol. 33 (1994), 89-109.
7. J.A.J.Metz and O.Diekmann, "*The Dynamics of Physiologically Structured Populations*" Springer lecture Notes in Biomathematics, Vol.68, Springer-Verlag, New York, 1986.
8. M.Mijatovie, S.Pilipovie and F.Vujzovie, *α -Times Integrand semi-groups ($\alpha \in \Re^+$),* J.Math. Anal. Appl.210 (1997), 790-803.
9. M.M.EL-Borai, *Some probability densities and fundamental solutions of fractional evolution equation,* Chaos, Solitons & Fractals 14(2002), 433-440.
10. M.M.El-Borai, *Semigroups and some nonlinear fractional differential equations,* J.Appl.Math. Comput. 149 (2004), 823-831.
11. S.D. Zaidman, *Abstract Differential Equations, Research Notes in Mathematics,* Pitman Advanced Publishing Program, San Francisco, London Melbourne, 1979.
12. M.M.El-Borai, *Semigroups and incorrect problems for evolution equations,* 16th IMACS WORLD CONGRESS LAUSANNE, August 2000.
13. M.M.El-Borai, O.L. Moustafa and F.H.Micheal, *On the correct formulation of a nonlinear differential equations in Banach space,* Int.J.Math. 22 (1) (1999).
14. I.M. Gelfand and G.E.Shilov, *Generalized Functions,* Vol.1, Nauka, Moscow, 1959.
15. W.R.Schneider and W.Wayes, *Fractional diffusion and wave equation,* J.Math. Phys. 30 (1989).
16. Peter Linz, *Analytic and Numerical Methods for Volterra Equations,* SIAM, Philadelphia, 1985.
17. A. Dzhuraev, *Methods of Singular Integral Equations,* London, New York, 1992.

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Multipliers of Distributions Spaces

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Abstract - In this paper we consider multipliers of distributions spaces. We obtain some characterizations of various multipliers spaces similar to those of multipliers of Banach algebras.

Keywords : *distribution, multiplier, convolution.*

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Multipliers of Distributions Spaces

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Abstract - In this paper we consider multipliers of distributions spaces. We obtain some characterizations of various multipliers spaces similar to those of multipliers of Banach algebras.

Keywords : distribution, multiplier, convolution.

I. Preliminaries

Let G be a locally compact group and let $L_1(G)$ denote the Lebesgue space with respect to the Haar measure on G. An $L_1(G)-$multiplier is any continuous linear transformation from $L_1(G)$ into itself which commutes with translations. Many investigations have been carried out to characterize the $L_1(G)-$multipliers. The theory has been extended to other group algebras, general Banach algebras, some locally convex linear spaces and $\mathcal{C}^*-$algebras. See [8], [1] and references therein. For further informations on the theory of multipliers we refer to [8]. In this paper we study multipliers of distributions. Some results in this area involving temperate distributions can be found in [5] and [7]

a) *Definition and notations of distributions spaces*

In this section we recall some facts about distributions. For more details we refer to [2], [3] and [4]. As usual $\mathcal{D}$ denotes the set of infinitely differentiable complex valued functions on $\mathbb{R}^n$ having compact support. The set $\mathcal{D}$ is topologized in the following way. The subset $\mathcal{D}_K$ of $\mathcal{D}$ whose elements have their supports in the compact subset K of $\mathbb{R}^n$ is a locally convex vector space with the semi-norms $P_{m,K}$ defined by

$$P_{m,K}(\phi) = \sup_{x\in K, |q|\leq m} |D^q(\phi)| \tag{1}$$

where $m \in \mathbb{N}$, $q = (q_1, \ldots, q_n) \in \mathbb{N}^n$, $|q| = \sum_{i=1}^{n} q_i$, $x = (x_1, \ldots, x_n)$ and $D^q = \dfrac{\partial^{|q|}}{\partial x_1^{q_1} \ldots \partial x_n^{q_n}}$. Then $\mathcal{D}$ is the strict inductive limit of the $\mathcal{D}_K$'s, when K runs over an increasing sequence of compact sets whose union is $\mathbb{R}^n$. Thus a net

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Ref.

[1] Akemann, C. A., Pedersen, G. K. and Tomiyama, J., *Multipliers of $\mathcal{C}^*-$algebras.* J. Funct. Anal. **13** (1973), 277-301.

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$\{\phi_\alpha\}$ converges to 0 in $\mathcal{D}$ means that all the ϕ_α's have their supports in a fixed K and for any $q \in \mathbb{N}^n$ the net $\{D^q\phi_\alpha\}$ converges to 0 uniformly in K.
A *distribution* on $\mathbb{R}^n$ is a continuous linear functional on $\mathcal{D}$. The vector space of distributions is naturally denoted by $\mathcal{D}'$. Let U be a distribution. We say that U vanishes on a subset Ω of $\mathbb{R}^n$ if $U(\phi) = 0$ whenever $supp\phi \subset \Omega$. The support of U is by definition the smallest closed subset of Ω of $\mathbb{R}^n$ such that U vanishes off Ω.
Let $\mathcal{E}$ denotes the set of infinitely differentiable complex functions on $\mathbb{R}^n$. It is endowed with the locally convex topology with semi-norms $P_{K,m}$ defined as in (1) where K now is no longer fixed. It was shown that the topological dual $\mathcal{E}'$ is identical with the space of distributions with compact support [2, page 38].
Let $\mathcal{S}$ be the subset of $\mathcal{E}$ which contains the functions ϕ such that

$$\lim_{\|x\|\to\infty} |x^k D^q\phi(x)| = 0, \forall k \in \mathbb{N} \tag{2}$$

with $x^k = x_1^k x_2^k \dots x_n^k$. $\mathcal{S}$ is called the space of rapidly decreasing infinitely differentiable functions whereas its topological dual $\mathcal{S}'$ is called the space of slowly increasing distributions or temperate distributions.
Finally denote by $\mathcal{O}_M$ the subset of $\mathcal{E}$ such that

$$\phi \in \mathcal{O}_M \text{ if and only if } \lim_{\|x\|\to\infty} |\varphi(x)D^q\phi(x)| = 0,\ \forall\varphi \in \mathcal{S} \tag{3}$$

and by $\mathcal{O}'_C$ the subset of $\mathcal{D}'$ such that

$$U \in \mathcal{O}'_C \text{ if and only if } (1+\|x\|^2)^{\frac{k}{2}}U \text{ is bounded in } \mathbb{R}^n, k \in \mathbb{N}, \tag{4}$$

which means $(1+\|x\|^2)U^{\frac{k}{2}}(\phi_\alpha)$ converges to 0 whenever ϕ_α and all its derivatives belong to $L_\infty(\mathbb{R}^n)$ and (ϕ_α) such as its derivatives converges to 0 in $L_\infty(\mathbb{R}^n)$. For the various topologies see [4].

In the sequel $\mathcal{D}'$ and $\mathcal{S}'$ will be endowed with their strong dual topologies and $\mathcal{O}'_C$ (which is not the dual of $\mathcal{O}_M$) will carry its usual topology except otherwise stated.

b) Operations on Distributions

The translation $\tau_h U$ is defined by

$$\tau_h U(\phi) = U(\tau_{-h}\phi),\ U \in \mathcal{D}', \phi \in \mathcal{D}. \tag{5}$$

The partial derivative $\dfrac{\partial U}{\partial x_k}$ is defined by

$$\frac{\partial U}{\partial x_k}(\phi) = -U(\frac{\partial \phi}{\partial x_k}),\ U \in \mathcal{D}', \phi \in \mathcal{D}. \tag{6}$$

It is shown in [4, pages 77 and 88] that

$$\frac{\partial U}{\partial x_k} = \lim_{h_k\to 0} \frac{\tau_{-h}U - U}{h_k} \tag{7}$$

Ref.

[2] Grubb, G. *Distributions and operators.* Springer, New York, (2009).

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where $h = (0, \ldots, 0, h_k, 0, \ldots, 0) \in \mathbb{R}^n$ and also that

$$\frac{\partial \tau_h U}{\partial h_k} = -\frac{\partial \tau_h U}{\partial x_k}. \tag{8}$$

The multiplication αU is defined by

$$\alpha U(\phi) = U(\alpha\phi),\ U \in \mathcal{D}', \alpha \in \mathcal{E}, \phi \in \mathcal{D}. \tag{9}$$

The Fourier transform $\mathcal{F}U$ is defined by

$$\mathcal{F}U(\phi) = U(\mathcal{F}\phi),\ U \in \mathcal{S}', \phi \in \mathcal{S}. \tag{10}$$

The convolution $U * V$ is defined by

$$U * V(\phi) = U(V(\tau_{-y}\phi)) \tag{11}$$

where V acts on ϕ as a function of x and U on the result as a function of y.

Convolution is not always meaningful, but it makes sense for instance if one the distributions at least has compact support. When it is defined, convolution is bilinear, commutative and the mapping $(U, V) \mapsto U * V$ is hypocontinuous in the sense that if one of U or V varies in a bounded set of $\mathcal{D}'$ or $\mathcal{E}'$ and the other converges to 0, then $U * V$ converges to 0. Moreover

$$\tau_h(U * V) = \tau_h U * V = U * \tau_h V \tag{12}$$

$$\text{and} \quad \frac{\partial(U * V)}{\partial x_k} = \frac{\partial U}{\partial x_k} * V = U * \frac{\partial V}{\partial x_k}. \tag{13}$$

We recall the following well-known lemma from [4, page 268].

Lemma 1.1 *The Fourier transform is an isomorphism between $\mathcal{O}'_C$ and $\mathcal{O}_M$, and changes convolution into multiplication in $\mathcal{S}'$ i.e. if $U \in \mathcal{S}'$, $W \in \mathcal{O}'_C$ then $\mathcal{F}U \in \mathcal{S}'$, $\mathcal{F}W \in \mathcal{O}_M$ and*

$$\mathcal{F}(W * U) = \mathcal{F}W\mathcal{F}U. \tag{14}$$

II. Main Results

Definition 2.1 *Let $\mathcal{V}$ or $\mathcal{W}$ be any one of $\mathcal{D}'$, $\mathcal{S}'$, $\mathcal{O}'_C$ or $\mathcal{E}'$. We call $(\mathcal{V}, \mathcal{W})-$multiplier any continuous linear transformation $T : \mathcal{V} \to \mathcal{W}$ that commutes with translations.*

We denote by $\mathcal{L}(\mathcal{V}, \mathcal{W})$ the space of continuous linear transformation from $\mathcal{V}$ to $\mathcal{W}$ and by $\mathcal{M}(\mathcal{V}, \mathcal{W})$ the subset of $\mathcal{L}(\mathcal{V}, \mathcal{W})$ consisting of the $(\mathcal{V}, \mathcal{W})-$multipliers. We write $\mathcal{L}(\mathcal{V})$ and $\mathcal{M}(\mathcal{V})$ for $\mathcal{L}(\mathcal{V}, \mathcal{W})$ and $\mathcal{M}(\mathcal{V}, \mathcal{W})$ when $\mathcal{V} = \mathcal{W}$ respectively.

Ref.

[4] Schwartz, L. , *Théorie des distributions*. Hermann, Paris, (1966).

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Theorem 2.2 *Let T be a linear continuous operator in $\mathcal{D}'$. Then the following statements are equivalent:*

i) $T \in \mathcal{M}(\mathcal{D}')$.

ii) T *commutes with partial derivatives.*

iii) There exists a unique W *in* $\mathcal{E}'$ *such that*

$$TU = W * U \text{ for all } U \text{ in } \mathcal{D}'. \tag{15}$$

Proof.

1. i)$\Rightarrow$ ii)
   Suppose $T \in \mathcal{M}(\mathcal{D}')$. Then $T(\tau_h U) = \tau_h(TU)$, $\forall\, U \in \mathcal{D}'$, $\forall\, h \in \mathbb{R}^n$.
   We have

$$\begin{aligned} T(\frac{\partial U}{\partial x_k}) &= T(\lim_{h_k \to 0} \frac{\tau_{-h}U - U}{h_k}) \\ &= \lim_{h_k \to 0} T(\frac{\tau_{-h}U - U}{h_k}) \\ &= \lim_{h_k \to 0} \frac{\tau_{-h}TU - TU}{h_k} = \frac{\partial TU}{\partial x_k}. \end{aligned}$$

   Hence T commutes with partial derivatives.

2. ii) $\Rightarrow$ i). This implication can be found in [4, page 163]. For the convenience of the reader, we reproduce it here.
   Assume that ii) holds and consider the function ψ define by

$$\psi(h) = [T(\tau_h U)](\tau_h \phi). \tag{16}$$

   We are going to prove that for $U \in \mathcal{D}'$ and $\phi \in \mathcal{D}$ fixed, the function ψ is independent of $h = (h_1, h_2, \ldots, h_n)$ in $\mathbb{R}^n$. For $k = 1, 2, \ldots, n$, we have

$$\begin{aligned} \frac{\partial \psi}{\partial h_k}(h) &= \frac{\partial}{\partial h_k}\{[T(\tau_h U)](\tau_h \phi)\} \\ &= [\frac{\partial}{\partial h_k} T(\tau_h U)](\tau_h \phi) + [T(\tau_h U)](\frac{\partial}{\partial h_k}(\tau_h \phi)) \\ &= T[\frac{\partial}{\partial h_k}(\tau_h U)](\tau_h \phi) + [T(\tau_h U)](\frac{\partial}{\partial h_k}(\tau_h \phi)) \\ &= T[-\frac{\partial}{\partial x_k}(\tau_h U)](\tau_h \phi) - [T(\tau_h U)](\frac{\partial}{\partial x_k}(\tau_h \phi)) \end{aligned}$$

Ref.

[4] Schwartz, L. , *Théorie des distributions.* Hermann, Paris, (1966).

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$$= -\frac{\partial}{\partial x_k}[T(\tau_h U)](\tau_h\phi) - [T(\tau_h U)](\frac{\partial}{\partial x_k}(\tau_h\phi))$$

$$= [T(\tau_h U)](\frac{\partial}{\partial x_k}(\tau_h\phi)) - [T(\tau_h U)](\frac{\partial}{\partial x_k}(\tau_h\phi)) = 0,$$

using successively the fact that T commutes with partial derivatives and is linear, and formulas (8) and (6). We conclude from $\frac{\partial\psi}{\partial h_k}(h) = 0$ for $k = 1, 2, \ldots, n$ that ψ is a constant equal to $\psi(0)$. That means that $[T(\tau_h U)](\tau_h\phi) = TU(\phi)$. Now, $T(\tau_h U)(\tau_h\phi) = \tau_{-h}T(\tau_h U)(\phi)$ by definition. Thus $\tau_{-h}T(\tau_h U) = TU$ and $T(\tau_h U) = \tau_h(TU)$. Hence i) holds.

3. iii) $\Rightarrow$ i).
For any $W \in \mathcal{E}'$, the mapping $U \mapsto T_W(U) = W * U$ defines a continuous operator in $\mathcal{D}'$ which commutes with translations according to (12) and because $(U, W) \mapsto W * U$ is hypocontinuous.

4. i) $\Rightarrow$ iii).
Let T be an element of $\mathcal{M}(\mathcal{D}')$. We shall show first that if $T(U * V)$ and $TU * V$ are defined then they are equal. In fact, for $\phi \in \mathcal{D}$, we have $(TU * V)(\phi) = (V * TU)(\phi) = V(TU(\tau_{-y}\phi)) = V(\tau_y TU(\phi)) = V(T\tau_y U(\phi)) = V * U(T^*\phi) = T(U * V(\phi))$ where T^* is the adjoint of T. Consequently $TU * V = T(U * V)$. Applying this equality to $U = \delta * U$ where δ is the Dirac distribution at the origin, we obtain $TU = T\delta * U$. Put $T\delta = W$ to have $TU = W * U$. Indeed W belongs to $\mathcal{E}'$. (See the remark in [4, page 163]).

Examples.

1. The identity operator T is a multiplier; the element W of $\mathcal{E}'$ associated with it is δ since $U = \delta * U$.

2. Translations τ_h are multipliers; the W of $\mathcal{E}'$ corresponding is δ_h, the point mass at $h \in \mathbb{R}^n$, since $\tau_h U = \delta_h * U$.

3. Distributional derivatives are continuous. They are multipliers because they commute with one another. Then we have $\frac{\partial U}{\partial x_k} = \frac{\partial \delta}{\partial x_k} * U$.

Theorem 2.3 *Let the set $\mathcal{L}(\mathcal{D}')$ be endowed with its strong topology i.e. the topology of convergence on bounded sets of $\mathcal{D}$. Then*

1. *The mapping $T \mapsto W = T\delta$ from $\mathcal{M}(\mathcal{D}')$ into $\mathcal{E}'$ equipped with its strong topology is a bicontinuous isomorphism in the topology induced by $\mathcal{L}(\mathcal{D}')$ on $\mathcal{M}(\mathcal{D}')$ i.e. $\mathcal{M}(\mathcal{D}')$ is isomorphic to $\mathcal{E}'$.*

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2. *$\mathcal{M}(\mathcal{E}', \mathcal{D}')$ is isomorphic to $\mathcal{D}'$.*

3. *$\mathcal{M}(\mathcal{O}'_C, \mathcal{S}')$ is isomorphic to $\mathcal{S}'$.*

4. *$\mathcal{M}(\mathcal{S}')$ is isomorphic to $\mathcal{O}'_C$.*

Proof.

Let us prove the assertion 1.

The mapping $\mathcal{M}(\mathcal{D}') \to \mathcal{E}'$, $T \mapsto T\delta$ is obviously linear. It is surjective because each $W \in \mathcal{E}'$ defines a multiplier T_W such that $T_W\delta = W * \delta = W$. It is injective, for if $T\delta = S\delta$, $T, S \in \mathcal{M}(\mathcal{D}')$, then for any U in $\mathcal{D}'$, we have $TU = T\delta * U = S\delta * U = SU$, so $T = U$.

Let us show now that it is continuous. Suppose (T_α) tends to 0 in $\mathcal{M}(\mathcal{D}')$. Then $(T_\alpha\delta * U)$ tends to 0 in $\mathcal{D}'$ uniformly on $U \in B'$ where B' is a bounded set in $\mathcal{D}'$. Put $T_\alpha\delta = W_\alpha$. Then for every bounded set B in $\mathcal{D}$, $\sup\limits_{\phi\in B, U\in B'} |W_\alpha * U(\phi)|$ tends to 0 in $\mathbb{R}$. Now $W_\alpha * U(\phi) = W_\alpha(U(\tau_{-y}\phi))$ and $U(\tau_{-y}\phi)$ belongs to $\mathcal{E}$. Moreover $(U(\tau_{-y}\phi))_{\phi\in B, U\in B'}$ is bounded in $\mathcal{E}$ i.e. is bounded on every compact subset K of $\mathbb{R}^n$. Hence for every bounded set B_0 in $\mathcal{E}$ of the form $B_0 = (U(\tau_{-y}\phi))$, $\sup\limits_{B_0\ni\psi} |W_\alpha(\psi)|$ tends to 0, with respect to α. Now let B_1 be an arbitrary bounded set in $\mathcal{E}$. We are going to prove that $\sup\limits_{\psi\in B_1} |W_\alpha(\psi)|$ tends to 0. For any compact $K \subset \mathbb{R}^n$, put $B_K = \phi_K B_1 = \{\phi_K\, \psi : \psi \in B_1\}$ where $\phi_K \in \mathcal{D}$ with $supp\phi_K \supset K$ and $\phi_K \equiv 1$ on K. The set B_K is bounded in $\mathcal{D}$. Then $\sup\limits_{\psi_K\in B_K} |W_\alpha(\psi_K)| = \sup\limits_{\psi_K\in B_K} |W_\alpha * \delta(\psi_K)| = \sup\limits_{\psi_K\in B_K} |W_\alpha(\delta(\tau_{-y}\psi_K))|$ which tends to 0 according to what is pointed out above.

If $\sup\limits_{\psi\in B_1} |W_\alpha(\psi)|$ does not converge to 0, then there would exist $\varepsilon > 0$ such that for every α there would exist $\beta > \alpha$ such that $|W_\beta(\psi)| > \varepsilon$ for every $\psi \in B_1$. But for every $\varepsilon > 0$ there exists α_0 such that $\alpha > \alpha_0$ implies $|W_\alpha(\psi_K)| < \varepsilon$ for every compact $K \subset \mathbb{R}^n$ and every $\psi_K \in B_K$. Now $W_\beta(\psi) = W_\beta(\psi_{K_\beta})$ where $K_\beta = supp W_\beta$ is compact. Let β be such that $\beta > \alpha > \alpha_0$, so that $|W_\beta(\psi_{K_\beta})| > \varepsilon$. This is a contradictory. We conclude that (W_α) tends to 0 in $\mathcal{E}'$. Then $T \mapsto T\delta$ is continuous.

The inverse mapping is also continuous. In fact, let (W_α) be a net in $\mathcal{E}'$ which converges to 0. Then for U in a bounded set of $\mathcal{D}'$, $((W_\alpha * U)$ converges to 0 in $\mathcal{D}'$ (hypocontinuity of the convolution). That is the mapping $W \mapsto T_W$ from $\mathcal{E}'$ into $\mathcal{M}(\mathcal{D}')$ is continuous.

The assertion 1 is completely proved.

The assertions 2., 3. and 4. are proved similarly.

Corollary 2.4 *$T \in \mathcal{M}(\mathcal{S}')$ if and only if there exists a unique $\phi \in \mathcal{O}_M$ such that $\mathcal{F}(TU) = \phi\mathcal{F}(U)$. Moreover the mapping $T \mapsto \phi$ is a topological isomorphism between $\mathcal{M}(\mathcal{S}')$ and $\mathcal{O}_M$.*

Proof. The corollary follows from Theorem 2.3, assertion 4. and Lemma 1.1 with $\mathcal{F}W = \phi$.

Notes

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Notes

Remark.

Some writers define a $(\mathcal{V},\mathcal{W})-$multiplier as a function ϕ such that $\phi V \in \mathcal{W}$ for every $V \in \mathcal{V}$. Let us denote in this case the multipliers spaces by $M(\mathcal{V},\mathcal{W})$. It was pointed out in [4, page 246] that $M(\mathcal{S}') = M(\mathcal{S}',\mathcal{S}')$ is precisely $\mathcal{O}_M$. Then according to Corollary 2.4, we can say that $M(\mathcal{S}')$ is topologically isomorphic to $\mathcal{M}(\mathcal{S}')$. See [6] for some extension results.

Theorem 2.5 *1. $T \in \mathcal{M}(\mathcal{D}')$ and $U \in \mathcal{E}'$ imply $TU \in \mathcal{E}'$. Moreover, every $T \in \mathcal{M}(\mathcal{E}')$ has a unique extension to an element of $\mathcal{M}(\mathcal{D}')$.*

2. $T \in \mathcal{L}(\mathcal{E}')$ belongs to $\mathcal{M}(\mathcal{E}')$ if and only if $TU*V = T(U*V)$, $U,V \in \mathcal{E}'$.

Proof.

1. If $T \in \mathcal{M}(\mathcal{D}')$ then by Theorem 2.2, $TU = W*U$ for some $W \in \mathcal{E}'$. Hence $TU \in \mathcal{E}'$ whenever $U \in \mathcal{E}'$ because $supp(W*U)$ is closed and included in $suppW + suppU$ which is compact.
   Since $\mathcal{E}'$ is a dense subspace of $\mathcal{D}'$ then $T \in \mathcal{M}(\mathcal{E}')$ has a unique continuous extension $\hat{T}$ to $\mathcal{D}'$ and $\hat{T} \in \mathcal{M}(\mathcal{D}')$.

2. $T\tau_h = \tau_h T$ implies $T(U*V) = TU*V,\ U,V \in \mathcal{E}'$; this is contained in the proof of Theorem 2.2 above.
   Conversely, suppose $T(U*V) = TU*V,\ U,V \in \mathcal{E}'$. Then $T(\tau_h U) = T(\delta*\tau_h U) = T\delta*\tau_h U = \tau_h(T\delta*U) = \tau_h(TU)$ i.e. $T \in \mathcal{M}(\mathcal{E}')$.

One has the following immediate consequence.

Corollary 2.6 *A continuous linear operator belongs to $\mathcal{M}(\mathcal{D}')$ if and only if its restriction to $\mathcal{E}'$ commutes with convolution, i.e.*

$$T \in \mathcal{M}(\mathcal{D}') \Leftrightarrow T_{|\mathcal{E}'}(U*V) = T_{|\mathcal{E}'}U*V.$$

References Références Referencias

[1] Akemann, C. A., Pedersen, G. K. and Tomiyama, J., *Multipliers of $\mathcal{C}^*-$algebras*. J. Funct. Anal. **13** (1973), 277-301.

[2] Grubb, G. *Distributions and operators*. Springer, New York, (2009).

[3] Khoan, V.-K. *Distributions, Analyse de Fourier et Opérateurs aux dérivées partielles*. Tome 1 and Tome 2, Vuibert, Paris, (1972).

[4] Schwartz, L. , *Théorie des distributions*. Hermann, Paris, (1966).

[5] Kučera, J., *On multipliers of temperate distributions*. Czechoslovak Math. J. **21** (1971), 610-618.

[6] Kučera, J., *Extension of the L. Schwartz space O_M of multipliers of temperate distributions*. J. Math. Anal. Appl. **56** (1976), 368-372.

[7] Kučera, J., Bosch, C., *Multipliers of temperate distributions*. Mathematica Bohemica **130**, No 3, (2005), 225-229.

[8] Larsen, R. , *An Introduction to the Theory of Multipliers*. Springer-Verlag,Berlin, (1971).

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Content

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- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
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- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

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- In spite of position, each table must be titled, numbered one after the other and complete with heading
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- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
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- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

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- Submit to work done by specific persons (including you) in past tense.
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References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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